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Philos
Logic

THE

PRINCIPLES OF LOGIC

BY

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PREFACE.

THE deductive part of this book may require a few words of explanation.

In most text-books on logic deduction and induction are treated from two very different standpoints. Induction is regarded as a means for the investigation of nature, and its canons are frankly objective or 'material'. They tell us how things must be related to each other in one respect if they are also related in some other respect (*e.g.*, how two events must be related in time if one is the cause of the other); and they do not say anything about the relation of our thoughts to each other as mere thoughts, or give any rules for the arrangement and manipulation of the words in which these thoughts are expressed.

Deduction, on the other hand, is usually defined as the "science of the laws of thought"—as one writer says, its "object-matter is thought",—and it is treated altogether from the subjective or 'formal' standpoint, as though mere thoughts as such could be consistent or inconsistent, wholly regardless of the nature of the object to which they refer; and then, when it comes to working out the details of the subject from this standpoint, the 'laws of thought' are treated practically as though they were laws for the right arrangement and manipulation of words. Hence we have rules of the syllogism and allied rules for conversion and obversion which say nothing whatever about the things under discussion and their relations to each other, but tell us only how we must or must not arrange our words in discussing them.

In this book I have tried to treat deduction from the objective standpoint that everybody assumes in the treatment of induction. Consequently I have omitted the traditional rules of the syllogism and put in their place a direct statement of the principles on which we reason in the different figures, adding certain 'cautions' that must be observed if the principles are not to be misapplied; and I have treated conversion and obversion in much the same way.

The fourth figure of the syllogism seemed to me not to represent any distinct principle of reasoning, and therefore to have no proper place in the objective treatment of logic; but I have explained the traditional way of dealing with it. The 'algebra' of logic I have omitted altogether. Readers interested in it are referred to the "Johns Hopkins Studies in Logic" (Little, Brown & Co., Boston).

However imperfect my own treatment of deduction from the objective standpoint may be, I believe that the standpoint itself is not only more correct philosophically than the subjective, but also better pedagogically; for we do far more to make a student clear-headed by teaching him to look a situation in the face and analyze it than by giving him any amount of dexterity in the reduction of arguments to a given verbal form.

My indebtedness to other authors is apparent. I am no less indebted to individuals—particularly to colleagues in other departments who have given me valuable suggestions on matters related to their special subjects; but most of all to Dr. W. T. Marvin, to whom I read the whole book while it was still in manuscript.

H. A. A.

March 21, 1902.

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LOGIC.

CHAPTER I.

INTRODUCTION.

THE business of Logic is to help us to think clearly and objectively, express ourselves plainly and accurately, reason correctly, and estimate aright the statements and arguments of others.

We cannot fully understand what is meant by the statement that logic helps us to think clearly unless we distinguish carefully between two kinds of thinking.

Two kinds of thinking.

The first kind makes some statement or asks some question; the second consists in a mere play of mental images, such as takes place when we follow quite passively the successive notes of a piece of music, hearing each as it passes but doing nothing more, or when the music runs through our head afterwards in the same way, or when half-dozing we watch the pictures that float before us, or when in the same passive way we feel yet do not note the sensations that come from our limbs as we walk or row or take some other such mechanical exercise. But just as soon as we recognize that the music is or is not beautiful, that one note is following another, that some one is playing, or that we are hearing, feeling, or imagining something, then we get back for the moment at least to the first kind of thinking; for to the mere passive images

we have added something else and there is now an active affirmation or denial.

The second kind of thinking—that which consists in the mere play of sensations or images—serves no purpose beyond the pleasure or recreation of the thinker; and logic has nothing to do with it. Consequently, when we speak of clear thinking we do not refer to the vividness or continuity of such passive images, but rather to the definiteness and consistency of active affirmations, denials, or questions.

When we affirm or deny anything we are said to form a belief, to come to a conclusion, or to pass a *judgment;* and the sentence in which the judgment is expressed is called a proposition. *Propositions* can thus be defined as sentences expressing judgments. According to this definition interrogative sentences are not propositions, for a question implies the absence of a judgment. On the other hand negative sentences are propositions; for it is quite as much an act of judging to deny that something is the case, *e.g.*, to say or think 'The day is not hot', as to affirm it—to say or think 'The day is hot'. There is thus a vast difference between saying that a thing is not so and not saying that it is so, in spite of the fact that we often say 'I do not think so' when we mean 'I think not'.

Judgments and propositions.

Judgments always have reference to something other than themselves. Like the eye which always looks outward and never sees itself, they are always concerned with some object or other which lies beyond them, and never with themselves. This is most important. Perhaps it can be made clear by an illustration from grammar. When I say "The word 'good' is an adjective", I am speaking of that word as it occurred somewhere else; for as I use it here it is a noun. Words never refer to themselves, but to something else which they name or 'mean'. The same is true of judgments. When I say "The house is on fire", I do not mean to state anything about myself or about my judgment with

reference to the house. I do not even mean to say that I judge that the house is on fire. I do judge it; and the statement expresses my judgment. But to *express* a judgment is to tell something about the object thought about or judged of—in this case the house—not about the thought or judgment itself, or the person who passes it, or the words in which it is expressed. When we wish to see our own eyes we do not look at them directly, but at their image in a mirror. So when we wish to know our own thought we must get a new thought with the thought in question as its object, *e.g.*, '*I thought* that the house was on fire', or '*I recognized* that the house was on fire'. In these propositions I am talking about my own thought or judgment; in the former I was not. But even here the thought of which I am speaking is not the thought that I am expressing, for I am speaking of the past thought and expressing the present thought about it.

That about which a judgment is passed, *i.e.*, that about which something is asserted (affirmed or denied), is called the Subject or Object of our thought. If it is a real thing or person, it is also called the subject of the relations that are affirmed of it, and the word used to point it out is usually the subject of the sentence or proposition in which the judgment is expressed. Thus, in the example just given, the real house is the subject of the state we call being on fire, for it is the house—not something else—that is blazing; the real house is the subject of the judgment, for it is the real house that I am thinking and speaking about; but it is the word 'house' that is the subject of the sentence. It is not the word 'house' that is on fire or of which I am thinking or speaking, and it is not the real house that is part of a sentence. Thus the subject of a sentence is not that about which something is stated in the sentence, but it is *the name of* that about which something is stated.

We should never confuse these three different meanings of the word Subject—what we are thinking about, or the

subject of thought; the thing that is in a certain state, or the subject of that state; and the name of what we are thinking and speaking about, or the subject of a sentence.

There are plenty of logicians and other writers who have not made or recognized this distinction. Locke, for example, who wrote more than two hundred years ago, tells us in his celebrated "Essay Concerning Human Understanding" that knowledge is "the perception of the agreement or disagreement of our ideas". This definition takes no account whatever of the things beyond our ideas to which they are supposed to refer. So Jevons tells us in his "Lessons in Logic", a text-book that is very widely used, that an act of judgment "consists in comparing together two notions or ideas of objects derived from simple apprehension, so as to ascertain whether they agree or differ". But when we judge that the house is on fire we do not compare our idea of the house with our idea of fire, find that in some way or other the two ideas 'agree', and then use the copula 'is' to indicate the agreement and to fasten the two agreeing ideas together. On the contrary we are usually wholly absorbed in the house and its fate and do not think about our own ideas at all. When we say that a person can tell 'only about his own ideas of things' it would be more accurate to say that he can express only his own opinions. But to *express* an opinion about a thing is to tell not about the opinion itself—about one's own thoughts and their relations to each other—but about the thing as one believes it to be, its states and its relations to other things. If we really agreed with Locke when he says or implies that everything we know is our own idea instead of something beyond it, we should have to agree also with Hume when he says that we "can form ideas which shall be no greater [*i.e.*, larger] than the smallest atom of the animal spirits of an insect or a thousand times less than a mite", merely because we can think of things so small; when he speaks of perceptions or sensations as "composed of parts"

A different view of judgments.

because the things that we perceive or feel are so composed; and when he seriously discusses "the infinite divisibility of our *ideas* of space and time." *

If there is no distinction between thought and the object thought of, Hume is right enough in talking of large and small ideas; but if there is a distinction, we must not assume that a thought of a large or complex object is any larger than the thought of one that is small or simple. A good photograph of a single brick involves just as complex a chemical process on the sensitive plate as that of a whole house; to pick a single grape is just as complex an act as to pick a whole bunch; and in the same way to think of a mob is not to have a mob of thoughts. Some acts of thinking are undoubtedly more complex than others; but the complexity of the thought in no way corresponds with that of the object thought about. †

* See his "Treatise of Human Nature", Book I, Part II, Sec. I. David Hume lived from 1711 to 1776. He was probably the greatest of all British philosophers.

† As for the 'simple apprehension' spoken of by Jevons and so many other writers, it has little or nothing to do with an act of judgment. We can divide a sentence into subject and predicate, or, if we like, into subject, predicate, and copula; but we cannot make any such division in the judgment which the sentence expresses. "Simple apprehension", says Jevons, "is the act of mind by which we merely become aware of something, or have a notion, idea, or impression of it brought into the mind. The adjective *simple* means apart from other things, and *apprehension* the taking hold by the mind. Thus the name or term *Iron* instantaneously makes the mind think of a strong and very useful metal, but does not tell us anything about it, or compare it with anything else." (*Elementary Lessons in Logic*, p. 11.) But let the reader spend five minutes trying to think of such a metal without making some statement about it. He can say 'Iron is strong,' or 'Iron is useful,' or 'Iron is a metal,' or 'This is iron,' and during the five minutes he will doubtless make an immense number of other statements; but when he tries to think of it without discovering some relation or passing some judgment he will probably find himself mechanically repeating some word applicable to iron and really thinking about something else; or it may be that with the word upon his lips or some visual image in his phantasy

Since all thinking has reference to some reality beyond itself, we think clearly when we discern the object that we are thinking about without confusion, and we reason correctly when we see how one relation of a thing involves another. In order to think clearly and reason correctly it is therefore necessary to look outwards continually beyond ourselves and beyond the words used by others towards the things that we or they are thinking about, in order to see these things and all their essential relations as they are. Unless we do this we cannot succeed either in expressing ourselves plainly and accurately or in forming a right estimate of the reasoning of others. Thus the habit of closely examining the reality beyond us and of testing all our thoughts and words with reference to that reality is necessary for all the aims of logic.

Logical thinking is objective.

Logic is often defined as the science of the laws of thought; but if what we have been saying is correct, it would be far more appropriate to say that it points out the laws of things which all thought should respect, or that it deals with the mutual implications of the relations of things. The special sciences and metaphysics also study relations of things and the way in which one involves another, but with a somewhat different purpose. Each of the special sciences is concerned with some one group of things and relations, and when it inquires how one relation involves another it is for the sake of gaining more knowledge about the particular things and relations in question. Its aim is thus the attainment of wider or more exact knowledge in some one special field. Metaphysics, on the other hand, inquires into the most fundamental and general relations of all things, and tries to find out what the inmost nature of any thing must be in order that all of these relations should belong to it together. Logic, like metaphysics,

Scope of logic.

he will find consciousness itself flickering and disappearing.—If simple apprehensions exist, they form no part of our knowledge, of our coherent thought, or of our reasoning. All these involve judgments.

has a very general aim; it too inquires into the most fundamental relations of things and the way in which one involves another. But its inquiry is not so profound as that of metaphysics; it does not ask what the inmost nature of things must be in order that these relations should exist together in them; and the knowledge that it does try to gain about relations and their mutual implications it regards as a means, not as an end. We cannot reason at all in science or anything else unless we have some idea of them, and we cannot reason correctly unless our idea of them is essentially correct; but it may be correct enough to enable us to reason well about most subjects without being nearly so profound as advanced metaphysical inquiries try to make it. Thus in so far as logic tries to make us reason correctly by giving us correct conceptions of things and the way in which their relations involve each other, it is a kind of simple metaphysics studied for a practical end.

There is a sense, however, in which it is perfectly true that logic deals with 'laws of thought'. A law of thought tells how people actually do think, just as a law of astronomy tells how heavenly bodies actually move, and the real science of the laws of thought is therefore psychology; but inasmuch as there are certain natural ways of thinking that lead to various kinds of logical blunders, it is necessary to understand them in order to understand why we make the blunders. Thus in so far as logic deals with various kinds of fallacies which we naturally commit and tries to explain their origin, it is touching on the field of psychology and dealing with 'laws of thought'.

Truth impersonal.

Every judgment, true or false, asserts something about some supposed reality beyond itself, and the difference between the true and the false is that the state of affairs asserted by the former really exists and that asserted by the latter does not. Whether it does or does not depends altogether upon the nature of things and the presence or absence of the conditions that might naturally

produce it. It does not depend at all upon the judgment about it. Whether the sun is shining or not at a certain place depends altogether upon the time of day and the presence or absence of clouds, fog, and an eclipse. If such conditions as these are all favorable, the sun is certainly shining whether I happen to think so or not; and if I say it is not shining when it really is, my statement is false no matter who I am or how sincere I may be in making it; so likewise if I have nervous prostration or a broken leg, I have it no matter who says that I have or that I have not. Hence it is arrant nonsense to say that something may be 'true for one person and false for another'. One person may believe that a statement is true and another may believe that it is not, but the facts are what they are wholly regardless of these conflicting beliefs, and one of the persons must be wrong. If every possible statement could really be true for one person and false for another, then every one would always be right in what he thought no matter what it was, and there would be no such thing as an error or mistake, and therefore no distinction between correct thinking and incorrect. All logic is based on the assumption that there is such a distinction, and therefore from the standpoint of logic no blunder could be more fundamental or destructive than that which is involved in the serious belief that something may be 'true for one person and false for another'.

The notion that some fact might exist 'for' one person and not 'for' another doubtless arises from the existence of individual differences in matters of taste and a certain confusion about their meaning. If a picture pleases me, I say it is beautiful; and if it displeases you, you say it is ugly; and all that either of us has any right to mean by the statement is that the picture does please him or displease him. Each statement thus tells about the relation between two things, the picture and the beholder; and because of the difference between the two beholders both statements understood in this way may be perfectly true; the picture really is beautiful

for me and really is ugly for you, and there is nothing more to be said—*de gustibus nil disputandum.* But these words 'beautiful' and 'ugly' and others like them have the same grammatical form as words like 'square' and 'round' which really tell about the thing itself, quite regardless of its relations to the beholder; and this helps to make us ignore the difference between them and assume that somehow or other we can describe the thing itself (as we do with such words as 'round' and 'square'), while at the same time the truth of the description depends (as it does with such words as 'beautiful' and 'ugly') upon who it is that gives it.

Somewhat like the statement that something may be true for one person and false for another is the statement that it may be true in one science and false in another. Towards the end of the middle ages the monkish philosophers found themselves reaching conclusions that were quite contrary to the doctrine of the Church which they were bound to accept. So they said that there was a difference between theology and philosophy, that a doctrine might be true in one though false in the other, and that they accepted all the teachings of the Church as true in theology, though they might reject a part of them as false in philosophy. By this subterfuge they tried to give an excuse for continuing their thinking as freely as possible and yet save their heads by remaining in nominal subservience to the Church. Of course this doctrine of a 'double truth' was nothing but a subterfuge, and it disappeared when men gained the right to exercise their own individual judgment in matters of belief.

Corollary.

Since the reality with which all thought is concerned is something different from the thought itself, we have no right to assume without evidence that there is any relation between them beyond the bare relation of knower and object known. We have no right to assume that our thoughts are like things—*e.g.*, that our thought of the moon is round like the moon itself—or that they have the same history or are subject to the same laws. We have

no right to assume, for example, that distant events are any more vague than those of the present simply because our ideas of them are more vague, or that things were vague and chaotic before they were definite because our ideas of them were.

Facts are as independent of our feelings as they are of our ideas. Hence when we are trying to find out what the facts really are we must not ask instead what we should like them to be and assume that we have answered the first question when we have only answered the second. Yet obvious as this is, the tendency to confuse the facts as they are with what we should like them to be is exceedingly strong. Indeed it is so strong that hardly any one can overcome it altogether. To do so—to look facts squarely in the face and accept them as they are, no matter how pleasant or unpleasant they may be—is one of the very first conditions of greatness, and it is always a mighty aid to success in any career. Moreover it is something which does not require any unusual mental ability. But it does require intellectual honesty; and because very few of us are willing to be absolutely honest in our thought those who are so often seem heroic. The '*Appeal to Consequences*', on the other hand—the argument which really invites one to accept a certain view merely because the view itself or something else that it involves is more pleasant to believe in than the contrary—is thoroughly contemptible, and yet it is something to which we are so accustomed that it takes a strong man with a great love for truth to show us how contemptible it really is. I quote the following from the account of the discussion of evolution at the Oxford meeting of the British Association in 1860 in the Life of Professor Huxley (vol. i. pp. 197–8): "The Bishop spoke thus 'for full half an hour with inimitable spirit, emptiness, and unfairness.' 'In a light, scoffing tone, florid and fluent, he assured us there was nothing in the idea of evolution; rock-pigeons were what rock-pigeons had always been.'" Then "he rhetori-

Fact and feeling.

cally invoked the aid of *feeling*, and said, 'If any one were willing to trace his descent through an ape as his *grandfather*, would he be willing to trace his descent similarly on the side of his *grandmother*?'" "On this Mr. Huxley slowly and deliberately arose. A slight tall figure, stern and pale, very quiet and very grave, he stood before us and spoke those tremendous words—words which no one seems sure of now, nor, I think, could remember just after they were spoken, for their meaning took away our breath, though it left us in no doubt as to what it was. He was not ashamed to have a monkey for his ancestor; but he would be ashamed to be connected with a man who used great gifts to obscure the truth. No one doubted his meaning, and the effect was tremendous. One lady fainted and had to be carried out; I, for one, jumped out of my seat."

No one can think clearly and reason correctly or be relied upon by others as fair-minded and impartial who believes that any view of things is right if it is not true, or who does not strive with all his might to see things as they really are in spite of all his wishes.

Our feelings tend to influence our judgment not only by making us believe what it is pleasant to believe, but also by making us believe whatever happens to fit in with the emotion of the moment. Leslie Stephen says: 'We are not unhappy because we believe in hell; but we believe in hell because we are unhappy.' When we are despondent the world seems dark and sad, when we are happy it seems bright and glad, when we are in love it is easy to find "Helen's beauty in a brow of Egypt", and when we are angry or irritated it is hard to believe that anger or irritation is out of place.

> "Speak roughly to your little boy,
> And beat him when he sneezes.
> He only does it to annoy,
> Becauses he knows it teases."

This certainly is the logic of the emotions, and it is hard

enough to overcome it—until the mood is over; and if we must wait until then to see things as they are, we should also make it a rule to wait before we express or act upon our judgments. Hence the wisdom of counting one hundred before displaying anger, and of the regulation which obtains, I believe, in the British navy requiring that no officer shall punish a man until twenty-four hours after the supposed offence.

In our effort to see things as they are in spite of our wishes and emotions we often have to resist an appeal to them made wittingly or unwittingly by some one else. When any one discusses the question at issue on its own intrinsic merits he is said to reason *to the point*, or, as the old logicians would say, his is an *Argumentum ad Rem;* but when one party to a discussion takes advantage of the weakness of another and tries to persuade him that something is true by appealing to his wishes or his emotions he uses one form of the *Argumentum ad Hominem.* Since the essential purpose of this so-called argument is to leave a person in a certain mood which will affect his judgment, it makes very little difference how it is done. It may be by gentle or inflammatory speeches or it may be without speech at all—by feeding him or embarrassing him or getting him out in the moonlight.

The *Argumentum ad Populum* is essentially the same as this form of the Argumentum ad Hominem except that it is addressed to a crowd. The real arguments of successful political speakers are generally very weak. They carry their point and get the votes merely by gaining the sympathy of the audience: by getting it to feel in harmony with the speaker and out of harmony with his opponents; and it does not make much difference whether this is done by solid arguments, impassioned appeals, ridicule and abuse of the other side, or funny stories.

CHAPTER II.

THE MEANINGS OF WORDS.

In almost all our thought and our communication with others we use words. A word is "a mark which may raise in our mind a thought like to some thought which we had before, and which, being pronounced to others, may be to them a sign of what thought the speaker had before in his mind". Unfortunately, however, the thing or the relation which a given word is used to mark is not always the same. and if we assume that it is in any particular case when in fact it is not, we are bound to misunderstand each other, to make some egregious blunder in our own reasoning, or not to notice such blunders in the reasoning of others.

To speak first of the blunders of interpretation. These often arise when a student is beginning the study of any science and takes it for granted that words which are used in a purely technical sense are used in the popular sense to which he happens to be accustomed. The word 'phenomenon' as used in science merely means something that we perceive or appear to perceive, but the student assumes that it means something strange or miraculous. When the psychologist speaks of 'imagining' something he merely means forming a mental picture of it; but the student may assume that he means believing something that is not so. 'Immediate' in science means direct or without the assistance of anything else; but the student will probably assume that it means without any

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delay. A 'particular' proposition in logic is one that tells about some undesignated part (cf. 'particle') of a class; but the student who reads his book in a hurry assumes that it is one that tells about some individual in particular. When a student misinterprets statements in this way he is almost certain to misconceive the meaning of the whole paragraph or chapter in which they occur, or to gain no definite idea from it whatever. This may be partly the fault of the author, for if a book is intended as an elementary text-book, it is his business not to use words in these new senses without saying something about it. But it is also largely the fault of the student himself. He knows that the author is trying to convey some definite meaning; and if he took the trouble to inquire what that meaning really is, instead of being satisfied with his work when he has read the words or learned to jumble some of them together, he would see very easily that some of these words must be used in a strange sense.

The same trouble occurs also very frequently in history and literature. If a book was written more than a century ago, many of its words will have been used in a sense with which we are no longer familiar; and here again unless we are very careful we are likely to misunderstand the author's meaning completely. What, for example, is the meaning of the italicised words in the following passages from the Bible? "I may *tell* all my bones: they look and stare upon me" (Ps. 22:17). "I *prevented* the dawning of the morning, and cried: I hoped in thy word" (Ps. 119:147). "But unto thee have I cried, O Lord; and in the morning shall my prayer *prevent* thee" (Ps. 88:13). "That I may show all thy praises within the *ports* of the daughter of Sion" (Ps. 9:14, Prayer-book version). "My daughter is grievously *vexed* with a devil" (Matt. 15:22). What also is meant by the word 'let', by the word 'meat' in the phrase 'meat and drink', by 'rod' and 'staff' in Ps. 23, and by the phrase 'What have I to do with thee?' What did the

Biblical writers mean by a 'prophet', by 'cherubim', and by a 'penny' as the word is used in the story of the Prodigal Son? What is meant in Magna Charta when it says, "No free man shall be taken or imprisoned . . . but by lawful judgment of his *peers*"?*

If words do not mean anything when they are taken in a sense with which we are familiar, we can be sure that the author was either writing nonsense or using them in some sense with which we are not familiar. But even when they do mean something when taken in our ordinary sense, that may not be what the author meant them to mean. Hence students of historical methods say that we must not read some old writings for the purpose "of extracting information from it without any thought of first ascertaining exactly what was in the author's mind". If we do, we are sure to give the author's words our meaning instead of his. Therefore we must make it a rule to understand the exact meaning of what is said "before asking what can be extracted from it for the purpose of history",† or for any other purpose. The Bible, for example, is full of the deepest truths; but most of us read it without finding them simply because the rhythm is pleasant and the words are familiar and it never occurs to us to inquire whether or not the men who wrote them meant to say anything that we have not thought already, and if they did, what it is.

This finding of the meaning, even where it seems plain enough already, is no mere perfunctory matter. To be sure

* A student of philosophy should pay particular attention to the meaning of such words as 'Idea', 'Perception', 'Impression', 'Reflection', as used by Locke, by Berkeley, and by Hume; the phrase 'Moral Philosophy' as used by Hume and his contemporaries; 'Conceive' as used in different contexts by Herbert Spencer; 'Substanz', 'Wirklichkeit', 'Realität', 'Noumenon', and 'Ding an sich' as used by Kant, and the like.

† Langlois-Seignobos, "Introduction to the Study of History", pp. 143–146 (Henry Holt & Co., 1898).

that we have done it aright we should have studied the language of the time and country as well as of the author himself. But the essence of the method is always the same —to find a meaning or a set of meanings for a word which will enable us to give a reasonable interpretation to every passage in which it occurs.*

But we need not go to science and literature to find words misunderstood. Such misunderstandings occur continually in every-day life, and often do great mischief.

How words become ambiguous.

There are various ways in which words may become *ambiguous*, or get several meanings that are liable to be confused. Jevons gives three of them as follows:

1. From the accidental confusion of different words; *e.g.*, the adjective *mean* may signify *medium* or *average* (from the French *moyen*), or *despicable* (from the Anglo-Saxon *gemœne*); *light* may signify the opposite of *heavy* (from the same root as *levis*) or the opposite of *dark* (from the same root as *lux*).

2. From the transfer of meaning from the original objects to others associated with them; *e.g.*, the words *house*, *court*, *church*, all mean either a place or those that meet there.

3. From the transfer of meaning to analogous objects. The word *sweet* is applied to sounds and innumerable other things that give pleasant feelings, though none of these feelings is similar in any other respect to sweet tastes. Similarly the *foot* of a mountain, the *hand* of a clock and the *leg* of a table do not bear any close resemblance to human limbs, but in certain respects they answer the same purpose. As Whately puts it, "leg : animal :: supporting stick : table". It is by the same kind of analogy that recent writers speak of society as an *organism*.

* "'These studies of words', said Fustel de Coulanges, 'have a great importance in historical science. A badly interpreted term may be a source of serious error.' And, in fact, simply by a methodical application of interpretative criticism to a hundred words or so, he succeeded in revolutionizing the study of the Merovingian epoch." (Op. cit., p. 150.)

The more difference there is between a word's different meanings the more likely we are to discover the ambiguity before it has done much harm. When any one speaks of 'the church' we are not likely to confuse a building with the group of people that worships in it. The context soon shows which he means. But we very well might confuse different larger and smaller groups of worshippers. When we are told that such and such is the custom or practice of 'the Church' it might be hard to tell whether the speaker was referring to the communicants or voting members of a certain particular congregation, to the congregation as a whole, to the denomination, to a particular group of denominations, excluding Roman Catholics, Unitarians, or others that the speaker regarded as heretical, to the Western Church in all its branches, or to the whole body of those who call themselves Christians. Even when we are sure which of these bodies he means we may still be unable to say what proportion of the individuals in the body he intends to include when he says that such is the practice of 'the Church'. One person might say that something was the practice of 'the Church' if it were done habitually by a third of the individual members; another might not say so unless it were done by two-thirds or three-quarters; and still a third might deny that anything was the practice of 'the Church' even if it were done habitually by all the members so long as it was not done officially by the body as an organized whole under the direction of the proper officers.

What ambiguities are most dangerous.

Words which are ambiguous because we cannot tell how much or how little they are intended to include are called Vague. The word Church in the last example was vague because it did not show precisely what individuals the speaker intended to include in the group that he used it to indicate. In other cases the ambiguity is about relations: a word is used to indicate a group of them, but we cannot tell precisely what they are. This vagueness is characteristic of many of

our commonest words, and is explained by the way in which we begin to use them.

"In Geometry . . . we learn the definitions of the words used, *point*, *line*, *parallel*, etc., before we proceed to use them. But in common speech, we learn words first in their application to individual cases. Nobody ever defined *good* to us, or *fair*, or *kind*, or *highly educated*. We hear the words applied to individual objects; we utter them in the same connection; we extend them to other objects that strike us as like without knowing the precise points of likeness that the convention of common speech includes. The more exact meaning we learn by induction from individual cases. *Ugly*, *beautiful*, *good*, *bad*—we learn the words first as applicable to things and persons: gradually there arises a more or less definite sense of what the objects so designated have in common. The individual's extension of the name proceeds upon what in the object has most impressed him when he caught the word; this may differ in different individuals; the usage of neighbors corrects individual eccentricities."

The more complex and intangible the object or relation which a word is used to indicate the greater is the danger of misunderstanding from its ambiguities. "Take such words as *monarchy*, *tyranny*, *civil freedom*, *freedom of contract*, *landlord*, *gentleman*, *prig*, *culture*, *education*, *temperance*, *generosity*. . . . Let two men begin to discuss any proposition in which any such word is involved, and it will often be found that they take the word in different senses. If the relation expressed is complex, they have different sides or lines of it in their minds; if the meaning is an obscure quality, they are guided in their application of it by different outward signs.

"Monarchy, in its original meaning, is applied to a form of government in which the will of one man is supreme, to make laws or break them, to appoint or dismiss officers of state and justice, to determine peace or war, without control

of statute or custom. But supreme power is never thus uncontrolled in reality; and the word has been extended to cover governments in which the power of the titular head is controlled in many different modes and degrees. The existence of a head, with the title of King or Emperor, is the simplest and most salient fact; and wherever this exists the popular concept of a monarchy is realized. The President of the United States has more real power than the Sovereign of Great Britain; but the one government is called a Republic and the other is called a Monarchy. People discuss the advantages and disadvantages of monarchy without first deciding whether they take the word in its etymological sense of unlimited power, or its popular sense of titular kingship, or its logical sense of power definitely limited in certain ways. And often, in debate, monarchy is really a singular term for the government of Great Britain.

"*Culture*, *religious*, *generous*, are names for inward states or qualities: with most individuals some simple outward sign directs the application of the word—it may be manner, or bearing, or routine observances, or even nothing more significant than the cut of the clothes or of the hair. Small things undoubtedly are significant, and we must judge by small things when we have nothing else to go by; but instead of trying to get definite conceptions for our moral epithets, and suspending judgment till we know that the use of the epithet is justified, the trifling superficial sign becomes for us practically the whole meaning of the word, We feel that we must have a judgment of some sort at once: only simple signs are suited to our impatience.

"It was with reference to this state of things that Hegel formulated his paradox that the true abstract thinker is the plain man who laughs at philosophy as what he calls abstract and unpractical. He holds decided opinions for or against this or the other abstraction, *freedom*, *tyranny*, *revolution*, *reform*, *socialism*, but what these words mean and within

what limits the things signified are desirable or undesirable he is in too great a hurry to pause and consider." *

Here is an example of some of the mischief done by ambiguous words in economics: "The discussion of 'the relations of labor and capital' has not hitherto been very fruitful. It has been confused by ambiguous definitions, and it has been based upon assumptions. . . .

"Let us first examine the terms.

"(1) Labor means properly *toil*, irksome exertion, expenditure of productive energy.

"(2) The term is used, secondly, by a figure of speech, and in a collective sense, to designate the body of *persons* who, having neither capital nor land, come into the productive organization offering productive services in exchange for means of subsistence. These persons are united by community of interest into a group, or class, or interest, and when interests come to be adjusted, the interests of this group will undoubtedly be limited by those of other groups.

"(3) The term labor is used, thirdly, in a more restricted, very popular and current, but very ill-defined way, to designate a limited sub-group among those who live by contributing productive efforts to the work of society. Every one is a laborer who is not a person of leisure. Public men, or other workers, if any, who labor but receive no pay, might be excluded from the category, and we should immediately pass, by such a restriction, from a broad and philosophical to a technical definition of the labor class. But merchants, bankers, professional men, and all whose labor is, to an important degree, mental as well as manual, are excluded from this third use of the term labor. The result is, that the word is used, in a sense at once loosely popular and strictly technical, to designate a group of laborers who separate their interests from those of other laborers. Whether farmers are included under 'labor' in this sense or not I

* Minto's Logic, pp. 83–87 (Scribners, 1893).

have not been able to determine. It seems that they are or are not, as the interests of the disputants may require. . . .

"(1) Capital is any product of labor which is used to assist production.

"(2) This term also is used, by a figure of speech, and in a collective sense, for the *persons* who possess capital, and who come into the industrial organization to get their living by using capital for profit. To do this they need to exchange capital for productive services. These persons constitute an interest, group, or class, although they are not united by any such community of interest as laborers, and, in the adjustment of interests, the interest of the owners of capital must be limited by the interests of other groups.

"(3) Capital, however, is also used in a vague and popular sense which it is hard to define. In general it is used, in this sense, to mean employers of laborers, but it seems to be restricted to those who are employers on a large scale. It does not seem to include those who employ only domestic servants. Those also are excluded who own capital and lend it, but do not directly employ people to use it.

"It is evident that if we take for discussion 'capital and labor', if each of the terms has three definitions, and if one definition of each is loose and doubtful, we have everything prepared for a discussion which shall be interminable and fruitless, which shall offer every attraction to undisciplined thinkers, and repel everybody else." *

Here is another example from the same source: "There is no possible definition of 'a poor man'. A pauper is a person who cannot earn his living; whose productive powers have fallen positively below his consumption; who cannot, therefore, pay his way. . . . But he is not the 'poor man'. The 'poor man' is an elastic term, under which any number of social fallacies may be hidden. Neither is there any possible definition of 'the weak'. Some are weak in one

* Sumner, "What Social Classes Owe to Each Other," pp. 81–84. (New York: Harper & Bros.)

way, and some in another; and those who are weak in one sense are strong in another. . . .

"Under the names of the poor and the weak, the negligent, shiftless, inefficient, silly, and imprudent are fastened upon the industrious and prudent as a responsibility and a duty. On the one side, the terms are extended to cover the idle, intemperate, and vicious, who, by the combination, gain credit which they do not deserve, and which they could not get if they stood alone. On the other hand, the terms are extended to include wage-receivers of the humblest rank, who are degraded by the combination. The reader who desires to guard himself against fallacies should always scrutinize the terms 'poor' and 'weak' as used, so as to see which or how many of these classes they are made to cover." *

When two persons use words in different senses it is not surprising that one should reject a statement which the other accepts. What is surprising is that we fail so often to discover that our difference is not about the external facts at all, but only about the meaning of the words. Often a discussion does end in our looking up words in a dictionary. But it ought not to end there; for if we fully realized the difference between the words that we happened to use and the things and relations that we used them to indicate, we should come back from the dictionary and ask what was the real point of difference between us with reference to these things and relations. For surely it was these and not the meaning of words that we meant to discuss.

Blunders of inference.

Vague and ambiguous words not only make us misunderstand others. They do worse, and make us misunderstand ourselves and commit bad blunders in consequence. Since a word is a mark intended "to raise in our mind a thought like to some thought which we had before", we naturally take it for granted that any thought which the word does raise in our mind is the one

* Sumner, l. c., pp. 20, 21.

which we had before when we used it; and if we have concluded that a statement is true we suppose it to be true absolutely, without its ever occurring to us that it might be perfectly true when the words are taken in one sense and utterly false when they are taken in another. But if words are ambiguous and we take them in one sense when we are inquiring into their truth and then take them in a different sense when we are asking what can be inferred from their truth, it is evident that we may easily reach some conclusion which we never would have reached if the two different thoughts had not happened to be expressed or suggested by the same word. Sometimes these plays on words are so obvious that they are only intended to amuse a hearer. Sometimes they are intended to puzzle him; it is perfectly evident that the conclusion does not follow; but the words make it seem as though it should; and the puzzle is to explain why. Often, however, the words really do deceive both speaker and hearer. As Bacon says: "Men imagine that their minds have the command of language; but it often happens that language bears rule over their minds." Here are some examples of various kinds.

We know that we ought to control our tempers, and we happen to have expressed this by the statement that it 'is wrong to be irritated', and to remember the words. A physician finds that a patient who needs absolute quiet is being constantly disturbed by various noises, and in stating the fact he happens to say in his technical language that the patient is constantly 'irritated'. This is the word used in the formula that we happen to remember: it is wrong to be irritated; and so, without stopping to ask what the physician means by it, we jump to the conclusion that the patient is constantly doing something wrong.

'Lincoln says you cannot fool all the people all the time; but Mr. G. at his bargain-sales fools all the people and he fools them every time; therefore Lincoln was wrong.' Here the trouble lies in the ambiguity of the phrase 'all the

people' and in the assumption that 'all the time' means the same as 'every time'. 'All the people' in the second sentence means all the people at the bargain-sale; but Lincoln meant by it all the people in the country. 'Every time' as it is used here means every time Mr. G. has a bargain-sale; but 'all the time' as Lincoln used the phrase means for a long continuous period in the life of a nation.

'A saving man is a blessing to the community; a miser is a saving man; therefore a miser is a blessing to the community.' Here the word 'saving' may be ambiguous or it may not. If we understand by 'a saving man' in the first sentence any one whatever who saves his money instead of spending it, then the conclusion follows from the facts assumed. But it is more likely that any one who grants the truth of the first statement is thinking of 'a saving man' as one who exercises reasonable economy in contrast to a spendthrift. If this is so, we make him say more than he intended if we extend the meaning of the word 'saving' so as to include those who exercise unreasonable economy; and thus the argument is unfair.

'A teacher must know how to teach; Mr. R., who has just taken up the work, does not know how to teach; therefore Mr. R. cannot be a teacher.' Here there are several ambiguities. 'A teacher' may mean any one who earns his living in a schoolroom, or it may mean a 'true' or ideal teacher. When we say he *must* know how to teach we may mean that if he does not he is not a teacher (as when we say a square must have four sides); we may mean that he ought to know; or we may mean that if any one does not know he will not be allowed to attempt the work. 'Know how to teach' is ambiguous in two respects. In the first place, to 'know how' may mean to understand the principles involved, it may mean to have a certain amount of practical skill whether one understands the principles or not, and it may mean to have both principles and skill. In the second place the phrase 'know how' is vague because it does not tell us

how much knowledge the speaker has in mind. When he says that a teacher must 'know how' to teach, does he mean that he must be able to do it according to the latest approved principle, or does he merely mean that he must be able to do it as well as any intelligent person could? Even 'teach' is ambiguous. Does it mean merely to give instruction in certain subjects, or does it mean also to keep order in the schoolroom and instil good morals and manners? The phrase 'cannot be' in the conclusion of the argument has much the same ambiguities as 'must' in the first part of it. Does it mean 'certainly is not', 'will not be permitted to be', or 'cannot become'? If the premises mean that an ideal teacher has a certain amount of knowledge or skill and that Mr. R. has not, the conclusion follows that Mr. R. is not an ideal teacher; but it does not follow that Mr. R. is not earning his living in a schoolroom, that he is excluded from the teaching profession, or that he cannot become an ideal teacher; and so with the other meanings of the various sentences.

The following curious arguments are taken from a book written to prove that a lie is never justifiable, and have doubtless imposed upon a large number of readers as well as the author himself.*

A) "Truth is, so to speak, the very substratum of Deity. . . . As there is no God but the true God, so without truth there is and can be no God."

B) "As Christ is Truth, those who are in Christ must never violate the truth. . . . This would seem to be explicit enough to shut out the possibility of a justifiable lie."

C) "We cannot conceive of God as God, unless we conceive of him as the true God, and the God of truth. If there is any falsity in him, he is not the true God. Truth is of God's very nature. To admit in our thought that a lie is of God, is

* Trumbull, "A Lie Never Justifiable", pp. 135, 145-6, 223, 224. The first extract is given as a quotation from Hodge's "Systematic Theology", and the second as a summary of Martensen.

to admit that falsity is in him, or, in other words, that he is a false God."

D) "A lie is the opposite of truth, and a being who will lie stands opposed to God, who by his very nature cannot lie. Hence he who lies takes a stand, by that very act, in opposition to God. Therefore if it be necessary at any time to lie, it is necessary to desert God and be in hostility to him so long as the necessity for lying continues."

Let us examine these arguments. In A the author supposes his reader to take for granted that 'There is no God but the true God'. From this it is supposed to follow that 'Without truth there is and can be no God'; from this that 'Truth is, so to speak, the very substratum of Deity'; and from this that a lie is never justifiable under any circumstances. But what do these words all mean? When he expects every one to admit that 'there is no God but the true God', he must mean by it that there is no God but the god that *really exists*, whatever we may call him, and whatever his nature may be. If the next step follows from this, 'Without truth there is and can be no God', these words must be taken to mean that if there is no *reality*—if there is nothing real—there is and can be no real God. This same thought is expressed in the next set of words: 'Truth'—that is to say, reality—'is the very substratum of Deity'. But from this fact that God would not exist if nothing real existed it surely does not follow that a lie is never justifiable. The only reason that it seems to follow is that the word 'truth' is ambiguous, and when the author says that 'truth' is the very substratum of Deity he forgets that when this was proved it meant that *reality* was the substratum of Deity, and now he takes it to mean that always making true statements and disapproving of every false statement is 'the very substratum of Deity'; and from this it follows that a lie is never justifiable. Hence we have to take the words in one sense in order that they should follow from what every one is supposed to admit, and take them in quite another sense in order

that the conclusion should follow from them. So long as we think only of the words it is possible to be deceived by the argument. When we ask their precise meaning and look carefully at the things about which they are supposed to tell it is not.

In B what is meant by the statement that Christ is 'Truth'? It certainly does not mean that he is an accurate statement. Hence this must be taken more or less metaphorically to mean that in his life there is to be found a revelation of the deepest spiritual truths, that he is a true or genuine manifestation of God, or something of that sort. Christ then revealing the true spiritual life, it follows that 'those who are in Christ must never violate the truth', that is, it follows that those who are in communion with him should not do anything inconsistent with that highest spiritual life. But why does this 'shut out the possibility of a justifiable lie' unless we assume what was to be proved, that a lie never is justifiable, or in accordance with the highest spiritual life? Here again it is only the ambiguity of the word 'truth' that makes the conclusion seem to follow. Unless the statement that those who are in Christ must never violate the truth means what I have said, it does not follow from what was said about Christ; and unless the sense is then twisted and the statement is taken to mean that they must never tell a lie under any circumstances whatever, the conclusion does not follow from it.

In C there is the same play on the words 'true' and 'truth' as in A, and a similar play on the words 'false' and 'falsity'. This becomes very apparent if we omit these ambiguous words altogether and substitute for them the synonyms that seem to express the meaning most accurately in each individual case. The passage will then read something like this: 'We cannot conceive of God as God unless we conceive of him as the *real* God, and the God *of reality* (or *who is trustworthy*, or *who never approves of a lie*). If there is any *unreality* (or *untrustworthiness*) in him, he is not

the *real* (or *trustworthy*) God. *Reality* (or *trustworthiness*) is of God's very nature. To admit in our thought that a lie is of (*i.e.*, approved under any circumstances by) God, is to admit that *unreality* is in him, or, in other words, that he is a fictitious God.' So long as we do not make any attempt to interpret the words the conclusion may seem to be proved; but when we drop the words and consider only the relations that they ought to be used to point out it is perfectly evident that it is not.

In D the ambiguity lies in the word 'opposed' and its derivatives. When we say that 'a lie is the opposite of truth' we mean that a false statement is unlike a true one and can be contrasted with it just as large can be contrasted with small; and if God 'by his very nature cannot lie', then a being who can and will lie 'stands opposed to God' in the sense that the two can be contrasted, just as a large man can be contrasted with a small one. It does not follow in the slightest that there is any 'hostility' between them. The only reason that the conclusion appears to follow is that the words 'opposite', 'opposed' and 'opposition' are all derived from the same root, and we therefore assume when they are being used that they all have essentially the same meaning without stopping to ask what that meaning is until the verbal manipulation is over. Then we are told that 'in opposition' means 'in hostility', and we cannot deny it. As a piece of reasoning this sort of verbal jugglery is almost on a plane with that which proves something, I have forgotten what, by saying that a beehive is a bee-holder, a beholder a spectator, and a specked 'tater a bad vegetable.

This last example shows the danger of so-called Paronymous terms. When a word is not ambiguous in itself we may introduce an ambiguity by substituting for it some other word derived from the same root and so constructed that it seems to point to exactly the same relations, when it does not. We have a perfect right to say, 'Romeo is in love; lovers are impetuous; therefore Romeo is im-

Paronymous terms.

petuous'; because the words 'in love' and 'lover' point to precisely the same state of mind. But we have no right to say, 'Schemers are not to be trusted; this man has a scheme; therefore he is not to be trusted'; for one is not a 'schemer' unless his 'schemes' are rather dishonest, and are so habitually. So likewise a man who is once *drunken* or who once *lies* is not necessarily a *drunkard* or a *liar*. Words like these, which appear to correspond in meaning but really do not, are quite common, *e.g.: art, artful; design, designing, faith, faithful; presume, presumption; king, kingly; probable, probability; child, childish.*

How to deal with ambiguities.

When we are confronted with arguments which turn upon some ambiguous word we do not usually see what the trouble is all at once; but we often do have a vague feeling that there is something wrong; and this ought to be sufficient to make us go back and examine all the words on which the argument seems to turn. The surest way to test the argument is to put it into new words altogether, taking care that each one of them shall indicate something perfectly definite. Since all correct inference is based upon the nature and relations of the things in question, and not upon the relations of the words in which we happen to speak of them, this method of getting rid of the questionable words is perfectly legitimate and reasonable. When this has been done and we realize exactly what the relations in question are, we may then go back to the old words and show how they seemed to indicate first one and then another. If we wish to avoid making such blunders ourselves, we should pursue much the same method, and make sure that our argument depends upon the relations of things themselves and not upon the accidents of language by seeing whether it will seem to hold no matter what language we use to denote these relations.

When words are vague or otherwise ambiguous, it is usually wise for the person who uses them in a given connection to announce the exact meaning which he intends them to

express. When he does this he is said to define them. A Definition, therefore, is a statement of the meaning of a word as used in a given connection, or a statement that tells what qualities or other relations an object must have in order that the word defined should be properly applied to it. By the aid of definitions the same word can be used in different senses in different connections without confusion.

Definition.

When a word of doubtful meaning has not been defined by the user the only course for the hearer or reader is to define it for himself by comparing the various passages in which it occurs. He must try to find some definite meaning which will fit them all. This is what the makers of every dictionary have to do. By a definite meaning I mean of course a definite relation that the word is used to denote, not merely a definite set of phrases to put after it in a definition whether they mean anything or not. We have not found the meaning for the word 'toves' merely because we have learned to say, "They are something like badgers—they are something like lizards—and they are something like corkscrews". If a word has several distinct meanings, or if the meaning is vague, the effort to find one definite meaning which will fit the context in every case will soon make this plain.

To find a definite meaning for every word and every statement in the books that one studies is no easy task, and when any one first undertakes to do it he will probably be disappointed to find how little ground he can cover. But unless a person seeks one he cannot understand the book thoroughly, and unless he does it often enough to feel confidence in his own work he cannot tell the difference between a book that means something and one that does not. He will not be able to 'understand' the latter, or get definite thoughts out of it, but he will not be able to tell whether it is because of his own stupidity or because there are no definite thoughts there to get; and if he has not this ability to discriminate, he is likely to be the victim of any incoherent writer whose words are

sufficiently high-sounding. This ability to interpret what one reads or hears and discriminate between sense and nonsense is one of the most essential aims of all education, and a person who has not taken the trouble to acquire it ought not to call himself educated.

When we have found the meaning of a word and come to state it two things are essential: to be precise and to be simple. To be precise is to tell exactly what the word means —no more and no less; to state the characteristics of an object, in virtue of which the name is applicable, with perfect definiteness. We must not define a net as something made out of string with holes in it, or as something to catch fish with. For a net is not necessarily made out of string or used to catch fish, and things might be made of string and have holes in them, or be used to catch fish, without being nets. In the same way we should not define man as the animal that laughs, for though man may be the only animal that laughs, it is not laughter that makes him man and entitles him to the name. He would remain man if he never laughed again. So, we should not define virtue as the only thing which makes one truly and permanently happy, or acid as that which turns blue litmus paper red; for virtue might still be virtue if it ceased to make us happy, and the word acid would be quite as applicable to various substances if litmus paper had never existed.

How to frame definitions.

To be simple in a definition is to frame it in such a way that it will immediately mean something to the people for whom it is intended. Dr. Johnson defines a net as something reticulated and decussated, with interstices between the intersections; and Herbert Spencer defines evolution as "an integration of matter and concomitant dissipation of motion, during which the matter passes from an indefinite, incoherent homogeneity to a definite, coherent heterogeneity, and during which the retained motion undergoes a parallel transformation." These definitions are precise and to the expert they may be simple, but to ordinary people they are not.

Simplicity and precision are difficult to unite, but with patience the union can nearly always be made.

There is an old rule that definitions should be by *genus* and *differentiæ*. This means that it is not necessary or desirable to enumerate every single attribute of the things denoted by the name defined. It is sufficient to tell what class they belong to and how they differ from other members of that class. If we are defining the word 'bloodhound' we should say that bloodhounds are a certain specified kind of dog. It is not necessary to say that they are living things, belonging to the animal kingdom, and possessed of a backbone, mouth, eyes, ears, teeth, and paws. These attributes and a hundred others are all implied by the generic term 'dog'.

To define a term by itself or by some other equally obscure word from the same root, *e.g.*, 'a preacher is one who preaches', is practically the same as not to define it at all. It adds neither precision nor simplicity. But if two words from the same root do not indicate precisely the same relations it may be proper to define one by means of the other and some modifying phrase, *e.g.*, 'a preacher is one who preaches by profession', 'a liar is one who tells lies habitually'. Again, we are likely to lose rather than gain in both precision and simplicity when we define in metaphorical language, as when we say that words are barbed arrows, the soul is life's star, truth is the food of the soul, or the camel is the ship of the desert. The metaphors may be useful, but not as definitions.

Definitions are usually supplemented by examples, diagrams, or other illustrations. These do not make the definition any more accurate, for they give only one case out of many that fall within it, and unless some explanation is added, they do not show exactly where the dividing line is. But they do make the definition easier to understand, because they turn one's attention in the right direction. When we know some of the things which a boundary is intended to include we are better prepared to learn precisely

Illustrations.

where that boundary is. Because examples give this kind of preparation it is often wise to put some of them before the formal definition.

The illustration chosen should be typical. If we try to prepare for the definition of virtue by an example, we should choose some act or character that is recognized as virtuous by everybody; something well within the class, and, if possible, something of which virtue is the most striking feature. To prepare for the definition of a fish we should not draw an eel. It looks too much like a snake. After the definition is given, however, and its general purport is understood it is often wise to give an example of something that falls within it and of something else quite like it that falls without, and then explain why it is that the one is included and the other not. When one is making a contrast of this sort one might well use an eel to illustrate the difference between a fish and a snake, or a whale to illustrate the difference between a fish and a mammal. The nearer we get to the boundary on any side the easier it is to understand a description of that particular part of it.

When an example is not so striking as to be unmistakable we should take pains to make clear in precisely what respect it illustrates. It will not do to say, 'The subject of a sentence is the name of the thing spoken about, *e.g.*, John struck James'. We should underline the word 'John', put it in quotation-marks, or indicate in some other way that it is that word that is the subject, and not the real John or the word 'James'. Similarly we cannot explain what a house is by merely handing some one a picture and leaving him to wonder whether the house is the thing in the background with windows or the thing in the foreground with ears.

The reason why the common uneducated person does not recognize the importance of understanding the meaning of words precisely is that it is really not very important for his purposes. To play out of doors, to find one's way along the street and notice what people are doing there, to buy clothes

or groceries: all these are very concrete performances, and any one can carry them on or tell about them afterwards fairly well without a painstaking choice of words. With the work of school and the first year or two of college the need for a careful examination and selection of words becomes more apparent; and yet only a fair amount of care and skill in the use of words is required to tell about the concrete facts of history and geography, describe the processes involved in a chemical experiment, and even translate concrete statements from a foreign language, without confusion. But when we come to the abstract sciences, and have to deal, not with the surface of things, but with their deeper relations, the case is very different. The political economist must make sharp distinctions between wealth, capital, and money; the psychologist, between sensation and perception, conception and imagination, illusion, delusion, and hallucination; the student of ethics, between intention and motive, pleasure and satisfaction; the theologian, between wrong and sin, providence and predestination, substance and personality; the lawyer, between torts and crimes, corporations and partnerships, and so on. Here the things under discussion are not visible and tangible, and we cannot explain what we are talking about by merely pointing the finger. It takes much skill to talk or think about them without confusion, and the only way to be sure of doing so is to make absolutely clear-cut, precise, and rigid definitions, and keep them in mind throughout the whole discussion. Such definitions are not conventional ornaments at the head of a page, they are necessities.

Where precision is needed most.

Although the meaning of every word, and therefore all definition, has reference to things, we do not define things. Names are defined when we tell what they mean; classes may be said to be 'defined' when we point out their boundaries; things may be described in sentences but never defined, for no words can make them any more definite

Caution.

than they are. The question sometimes raised whether we define names or things depends, as Mill has shown, upon the fact that some definitions carry with them the assumption of the thing's existence in the real word (*e.g.*, any definition of cow or horse), while others do not (*e.g.*, a definition of a dragon or of a perfect man). But in each case it is the meaning of the word, not the thing, with which the definition is primarily concerned. To be sure in the former case we can prove a definition to be wrong by showing that it does not agree with the thing; but that is because the name stands for the thing, and if the definition does not agree with the thing it cannot correctly explain the name.

In the following chapters it will be necessary to define a considerable number of logical terms. The definitions given were made with care, but doubtless some of them are incorrect. Every reader is urged to make out as well as he can from the explanations, the illustrations, and the definitions themselves exactly what each one of them was intended to express, and then go back and see whether it really did express it or not, and if not why not. This will help him to understand the book and—what is far more important—it will give him good general practice in the accurate interpretation and use of words. To increase the opportunities for this practice there are among the exercises at the back of the book a large number of definitions of these same logical terms which are incorrect, and the reader is advised to go over as many of them as possible and state clearly and exactly what is wrong with each. This he will find much harder than to merely score them out and put correct ones in their places; but the practice is correspondingly better.

CHAPTER III.

THE MEANING OF STATEMENTS AS A WHOLE.

When we know the literal meaning of every individual word and phrase used by a speaker or writer there is still danger of misinterpreting him; for often people do not intend what they say to be taken seriously and literally.

Not seriously, for he may be using some customary phrase which is hardly intended to mean anything at all. If a letter begins with the phrase 'Dear Sir' and ends with 'Yours faithfully' or 'Your obedient, humble servant', or even if the words are somewhat more effusive, we must not conclude that the writer necessarily meant any more than to be polite. He may be writing to his worst enemy. And of course there are corresponding forms and phrases in every age and country. When a Spaniard tells you that his house is yours he does not expect you to ask for the title-deeds. Even official documents are not free from such polite phrases. The Grand Remonstrance was really an indictment containing over two hundred counts against the King himself, drawn up in 1641 when the Parliament and King were on the verge of civil war, and yet the petition with which it was presented begins as follows: "*Most Gracious* Sovereign, Your Majesty's *most humble and faithful* subjects the Commons in this present Parliament assembled, do with much thankfulness and joy acknowledge the great mercy and favor of God in giving your majesty a safe and peaceable return out of Scotland into

Conventional phrases.

the Kingdom of England, where the pressing dangers and distempers of the State have caused us with much earnestness to desire the *comfort of your royal presence*, and likewise the *unity and justice of your royal authority*", etc.

In the United States such terms as 'liberty' and 'home' have been used so much for all sorts of political purposes that they have ceased to have any very definite meaning when they are used in political and social controversies. The other day, for example, I read in a newspaper editorial that the Boers in the present war (1900) were fighting for 'their liberty and their homes'. Of course this is only a rhetorical way of saying that they are fighting for a certain form of political independence, but a reader who had no other sources of information might very well conclude from it that the British government really wanted to take forcible possession of private property and drag the owners off into slavery.

When a person expects something that he says to be taken seriously, he may still not expect to have it taken literally. "It is possible that he may have used some expressions in an oblique sense; there are several kinds of cases where this occurs: allegory and symbolism, jests and hoaxes, allusion and implication, even the ordinary figures of speech, metaphor, hyperbole, litotes. In all these cases it is necessary to pierce through the literal meaning to the real meaning, which the author has purposely disguised under an inexact form." * The devil quoted scripture accurately enough, but took it altogether too literally when he set Jesus on the pinnacle of the temple and said to him, "If thou art the Son of God cast thyself down: for it is written, He shall give his angels charge concerning thee: and on their hands they shall bear thee up, lest haply thou dash thy foot against a stone".

Oblique senses.

Many other examples of these oblique senses are to be

* Langlois-Seignobos, p. 151. Hoaxes might have been omitted. They can hardly be said to contain a 'real meaning disguised under an inexact form'.

found in the Bible. The Beast and the Scarlet Woman in the Apocalypse and the visions in the second part of Daniel are certainly not intended to be taken literally. And the books of Job and Jonah? Shall they be taken in the direct sense as history or in the oblique sense as allegory? Surely we must ask what a document really means before we can ask whether it is true or demand that any one accept the truth which it contains.

"A parallel difficulty occurs in the interpretation of illustrative monuments; the representations are not always to be taken literally. In the Behistun monument Darius tramples the vanquished chiefs under foot: this is a metaphor. Mediæval miniatures show us persons lying in bed with crowns on their heads: this is to symbolize their royal rank; the painter did not mean that they wore their crowns to sleep in." *

Exaggeration as a form of oblique speech demands special consideration. It is so common with some people that they constantly assume that what they say will be discounted, and do not realize all that is meant by the statements of others unless they are likewise exaggerated. A picture that pleases them is the most beautiful they ever saw, a pleasant evening is the best time they ever had in their lives, and so on. A person who says what he means, no more and no less, may very well be deceived into attaching altogether too much importance to the statements of such people; and they, on the contrary, are likely to attach altogether too little to his. If he says that some men are honest, they assume that he means to say that most are not; if he gives qualified praise they assume that he does not mean to praise at all or that he means to damn with faint praise.

Exaggeration.

* Langlois-Seignobos.—"Only one general principle [for detection of oblique meanings] can be laid down, and that is, that when the literal sense is absurd, incoherent, or obscure, or in contradiction with the ideas of the author or the facts known to him, then we ought to presume an oblique sense." (Op. cit., p. 152.)

Hence there is always danger of misunderstandings when persons accustomed to accurate statements are interpreting those who are not, and *vice versa*.

Exaggeration, however, is not only a matter of personal habit. Under the name hyperbole it is recognized as a correct form of literary expression; and not to make allowance for it is to misinterpret the passage in which it occurs, *e.g.*: "If a man can number the dust of the earth, then shall thy seed also be numbered"; "He was owner of a piece of ground not larger than a Lacedemonian letter"; "He was so gaunt, the case of a flageolet was a mansion for him".

In connection with this interpretation of statements as a whole, it is customary for writers on logic to point out several kinds of blunders, to which they have given characteristic names, as follows.

To commit the 'Fallacy of Amphibology' is to misinterpret a sentence because its construction is ambiguous. The traditional examples are from ambiguous oracular deliverances, *e.g.*, 'Aio te Æacida Romanos vincere posse' (I say that you Æacus the Romans are able to conquer); 'The Duke yet lives that Henry shall depose'. Exquisite care was needed to make such ambiguous constructions as these, but most of those we meet are the result of gross carelessness, *e.g.*: 'Wanted—a piano, by a young lady made of mahogany'; 'He finished his business and returned on Wednesday'; 'Twice two and three'; 'He who necessarily goes or stays (*i.e.*, who necessarily goes or who necessarily stays) is not a free agent, you must necessarily go or stay (*i.e.*, take the alternative), therefore you are not a free agent'.* The confusion can always be remedied by a reconstruction of the sentence; often by a mere change in punctuation or in the position of a word. **Amphibology.**

The name 'Fallacy of Accident' is applied to three different kinds of blunders:

1) When a statement about the mere substance of some

* Whately gives this last as a fallacy of composition.

individual thing is interpreted as referring to its condition (accident) as well, *e.g.*, 'What you bought yesterday you eat to-day; raw meat is what you bought yesterday; therefore you eat raw meat to-day'.

Accident.

2) When an abstract statement about some of the relations or essential characteristics of a certain *kind* of thing is interpreted as a concrete statement about every thing of the kind in all its relations (accidents), * *e.g.*, 'Meat is good for food, this spoiled horse-flesh is meat, therefore this spoiled horse-flesh is good for food'; 'I do not admire tall women, A. B. is a tall woman, therefore I do not admire A. B.'. In these examples the major premises really refer to 'Meat as such' or 'the essential characteristics of meat', 'tall women as such' or 'tallness in women'.

3) When any loose statement really intended to tell only what is true 'under ordinary conditions' or 'other things being equal' or 'usually' (though such phrases are omitted) is interpreted as though it meant to tell what is true always, no matter how unusual the conditions, *e.g.*, 'Strychnine is a deadly poison, therefore this *minute dose* is sure to poison me'; 'The use of medicine is to be avoided (when possible), therefore this sick man must not touch it'; 'Corporal punishment is debasing (as a rule), therefore this bully should not be thrashed'.

In all of these examples the Fallacy of Accident has been Direct; the reasoning has been from a statement concerning the substance, essence or rule, without reference to any accident or special condition, to a case in which such accident is present (*a dicto simpliciter ad dictum secundum quid*). But corresponding to each of the three kinds of direct fallacy there is also the Converse Fallacy of Accident (*a dicto secundum quid ad dictum simpliciter*) where the accident or special condition is implied in the major premise and omitted in the

* In the first case the *accident* is contrasted with Aristotle's 'material' or Locke's 'real' essence; in the latter with Aristotle's 'formal' or Locke's 'nominal' essence.

minor and conclusion, *e.g.*: (1) 'What you liked yesterday you like to-day, you liked this (fresh) bread yesterday, therefore you like this same (stale) bread to-day'; (2) 'I admire A.B. and C.D.; A.B. and C.D. are tall women; therefore I admire tall women' (as such); (3) 'Strychnine is a magnificent remedy (for certain diseases and in certain doses), A.B. needs a remedy, therefore he should take strychnine'.*

Direct or converse fallacies of accident of the first class are comparatively rare and trivial. Those in the two other classes (which are not always easily distinguished from each other) can be avoided by insisting upon accurate statements or explanations if the speaker is present and willing to make them; but when authoritative interpreters are not at hand they may cause interminable discussions and disputes. Everybody admits, for example, that lying is wrong; but does that mean that every act that involves a lie is wrong, or only that lies *as such* are wrong, and acts that involve a lie are wrong *provided that there is no other and more important moral consideration involved?* If we interpret the law in the first sense it is wrong to lie to a madman or a murderer to save the life of a child; if in the second it is right, provided that the obligation to save an innocent life is greater than the obligation to always refrain from lying, and that to tell a lie is the only available way of saving it. Human relations are so complex that we can only discuss one aspect of them at a time; and it may very well be that some moral laws at least have reference not to acts as a whole but to aspects of them, and that in interpreting such laws one aspect must be balanced against another and the one indissoluble concrete act judged by the most important moral consideration involved.

The interpretation of moral laws is a question of ethics, but

* The article 'a' lends itself easily to this kind of confusion. 'I admire *a* tall woman' may mean that I admire some individual woman who happens to be tall or that I admire tallness in women. It is this confusion that gives point to the time-honored conundrum, 'What makes more noise than a pig under a gate?' 'Two pigs.'

if we accept a law in one sense and then apply it in the other we commit the logical fallacy of accident.

The fallacy of 'Accent' is essentially a fallacy of interpretation. It consists in misinterpreting an author (1) by unduly accenting some particular word in a sentence, *e.g.*, 'Thou shalt not bear false witness *against* thy neighbor', or 'Thou shalt not bear false witness against thy *neighbor*';* or (2) by taking passages out of their immediate context, *e.g.*, proving that Dr. Watts believed in dog-fights because he said "Let dogs delight to bark and bite", or proving future punishment by John xv. 6: "And men gather them and cast them into the fire and they are burned", or by Matt. xxii. 13: "Cast him into outer darkness; there shall be weeping and gnashing of teeth"; or (3) by appealing to some particular passage, even a long one, though it may be contrary to the whole spirit of the author quoted. This is a form of the fallacy of which the members of any Christian sect might very well accuse the members of all the others. The controversy as to the whole spirit of the gospels which such an accusation would raise would be much more profitable than any amount of quibbling over a few proof texts.

Accent.

A remarkably clear exposition of this fallacy is given in the preface to Matthew Arnold's "Literature and Dogma", from which I quote a few sentences. It is of course his account of the fallacy in which we are interested, not his views on the Bible.

"The *homo unius libri*, the man of no range in his reading, must almost inevitably misunderstand the Bible, cannot treat it largely enough, must be inclined to treat it all alike, and to press every word. . . . He has not enough experience of the way in which men have thought and spoken, to feel what

* Jevons quotes the passage from the Book of Kings, 'And he spake to his sons, saying, Saddle me the ass. And they saddled *him*'. But this is surely a case of amphibology. The accent on the word 'him' changes the meaning of the passage only because it changes the antecedent to which the pronoun refers.

the Bible-writers are about; to read between the lines, to discern where he ought to rest with his whole weight, and where he ought to pass lightly. . . . And thus we come back to our old remedy of *culture*,—knowing the best that has been thought and known in the world; which turns out to be in another shape, and in particular relation to the Bible: *getting the power, through reading, to estimate the proportion and relation in what we read.* If we read but a very little, we naturally want to press it all; if we read a great deal, we are willing not to press the whole of what we read, and we learn what ought to be pressed and what not. Now this is really the very foundation of any sane criticism. . . . Things are on such a scale, and progress is so gradual, and what one man can do is so bounded, that the moment we press the whole of what any writer says, we fall into error. He touches a great deal: the thing to know is where he is all himself and his best self, where he shows his power, where he goes to the heart of the matter, where he gives us what no other man gives us or gives us so well."

The danger of this fallacy of accent is well recognized by jurists, and by their rules of evidence they try to guard against it. "'I have always', said Lord Tenterden, 'acted most strictly on the rule, that what is in writing shall only be proved by the writing itself. My experience has taught me the extreme danger of relying on the recollection of witnesses, however honest, as to the contents of written instruments; they may be so easily mistaken that I think the purposes of justice require the strict enforcement of the rule'". This is one reason. But then Tenterden goes on to say: "'By applying the rule to such cases the Court acquires a knowledge of the whole contents of the instrument, which may have a different effect from the statement of a part.'" *

So with confessions and other statements against the interest of the person who makes them. The law gives them

* Greenleaf, "Law of Evidence", Vol. I, Sec. 88.

great weight, but it also insists that they shall not be garbled. "In the proof of confessions, as in the case of admissions in civil cases, *the whole of what the person said* on the subject at the time of making the confession should be taken together. . . . It is not reasonable to assume that the entire proposition, with all its limitations, was contained in one sentence. . . . Unless the whole is received and considered. the true meaning and import of the part which is good evidence against him cannot be ascertained." *

On the same principle it is a rule of evidence that if a witness tells about a part of any conversation the lawyer who cross-examines him has a right to ask about any other part of the same conversation.

The difference between the fallacy of Accent and the fallacy of Accident in the broader sense of each is this : the former misinterprets a writer by confusing incidental statements with essential ; the latter confuses aspects of things or situations (or statements about such aspects) with the things or situations (or statements about them) as a whole. †

* Greenleaf, op. cit., Sec. 218.

† Whether we should call the over-emphasizing of some one aspect of the moral code accent or accident would thus depend upon whether we regarded the law as a revelation each part of which should be interpreted with reference to the whole, or as an analysis of conduct into various good and bad aspects, several of which may be combined in the complex whole. An aspect of a law taken for the whole law is accent ; an aspect of an act taken for the whole act is accident.

CHAPTER IV.

DIVISION AND CLASSIFICATION.

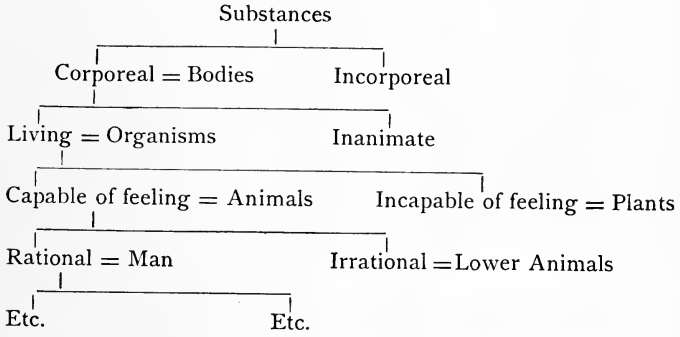
To avoid confusion in the use of names we must define them; but all definition of names involves a classification of objects. If the words 'animal', 'red', 'vertebrate' have any definite meaning at all there must be some things to which they can be properly applied and some things to which they cannot, and the things to which any one of them can be applied must all have the qualities or relations which the name implies, and therefore resemble each other in this respect, while the things to which it cannot be properly applied must all resemble each other in not having these qualities or relations. Hence every time we use a name we imply the existence of two classes of things: those that have the quality or relation which the name implies and those that have not. To define a name is to distinguish between these two classes, and the more clearly we understand this difference between the things the more clearly we can define the word. Hence we shall stop speaking about words for a little and speak about the principles of Division or Classification. At the end of the chapter we shall return to the discussion of words and their interpretation.

Relation to definition.

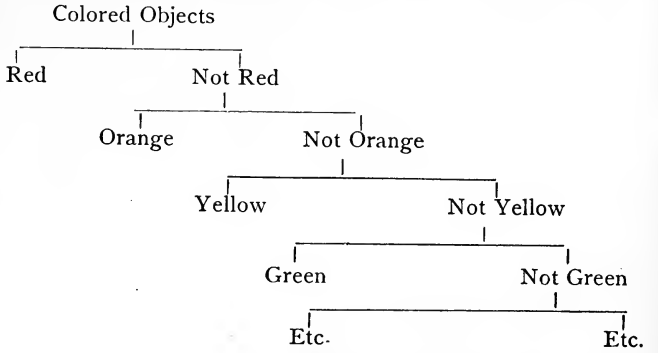
When we have made two classes to one or other of which every object in the world can be assigned according as it has or has not some given quality or relation, each of these classes can be subdivided with reference to some other quality or relation; and this

Principles of division and subdivision.

process can be carried on indefinitely, as in the following table.

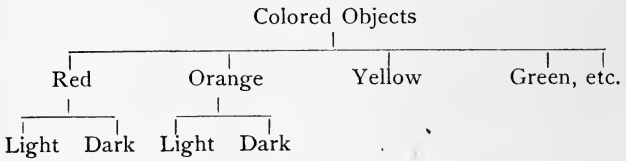


This division and subdivision of everything into precisely two classes according to the presence or absence of some given mark (technically called *Division by Dichotomy*) is often useful, and in the example just given seems very appropriate, but there are cases in which it is cumbersome and rather absurd, *e.g.*:



In this case it would be better for almost every practical purpose to divide directly into Red, Orange, Yellow, Green, and the other colors. Hence instead of dividing according

to the presence or absence of some given quality (*e.g.*, redness), we may divide according to *the determination in some given respect* (*e.g.*, color), making as many co-ordinate classes as there are different determinations in that respect; and then we can proceed to subdivision, with reference to other respects, if such subdivision is necessary, immediately and without so much confusion.



Whether each class should or should not be subdivided in the same respect as every other (*e.g.*, each color into light and dark), depends altogether upon the purpose of the division. Often it is impossible. Creatures with a nervous system, for example, could be divided according to its arrangement or development. Those without one could not.

The one great rule for division is that *each of the objects divided shall have one place in the system, and only one.* Thus, if we should divide all human beings into Americans, Europeans, and uncivilized peoples, we should commit a double blunder, for some peoples, such as the Japanese, would not fall under any head, and some of the American Indians would fall under two.

When any object falls into each of two co-ordinate classes—as in this example—there is said to be a Cross-division. This is always likely to occur *when we classify objects in more than one respect at a time.* If we had first divided peoples according to their geographical distribution and afterwards *subdivided* each group according to their civilization, or *vice versa*, this could not have occurred, *e.g.*,

Cross-division.

PEOPLES.
- I. American
  - A. Civilized
  - B. Uncivilized
    - 1. Savage
      - a.
      - b.
    - 2. Barbarous
      - a.
      - b.
      - c.
      - d.
- II. European
  - A. Civilized
  - B. Uncivilized
- III. Asiatic
  - A. Civilized
  - B. Uncivilized
- IV. African
  - A. Civilized
  - B. Uncivilized
- V. Australian
  - A. Civilized
  - B. Uncivilized
- VI. Other islanders
  - A. Civiliz ed
  - B. Uncivilized

To be sure, the uncivilized Indians still fall into two classes, Americans and uncivilized Americans, but this is now perfectly proper, because the classes are no longer *co-ordinate;* the latter is *subordinate* to the former.

Cross-division is a particularly bad logical blunder, because of the mental confusion which is shown by the indiscriminate use of several principles of classification (technically called Fundamenta Divisionis) at once.

"Overlapping, however, may be unavoidable in practice, owing to the nature of the objects. There may be objects in which the dividing characters are not distinctly marked, objects that possess the differentiæ of more than one group in a greater or less degree. Things are not always marked off from one another by hard and fast lines. They shade into each other by imperceptible gradations. A clear separation of them may be impossible. In that case you may allow a certain indeterminate margin between your classes, and sometimes it may be necessary to put an object into more than one class." *

* Minto's "Logic," p. 95.

Cross-division should be distinguished from cross-references, such as one finds in the subject-catalogues of libraries. If one does not know whether to put a book on the history of English philosophy under 'History' or 'England' or 'Philosophy', he can solve the question practically by putting it under one of them, say 'England', and then saying under each of the other heads 'See also England'.

The blunder of dividing in such a way as to leave out altogether some object that ought to have been included is likely to be the result of haste rather than of confusion. Where one knows all the objects that are to be divided, he must look about carefully to see that none are omitted. Where he does not he must leave room in his scheme of division and subdivision for additions, *e.g.*:

LANGUAGES.
- I. Indo-European
  - A. Slavonic
    1. Russian
    2. Others?
  - B. Græco-Roman
  - C. Celtic
  - D. Teutonic
    1. English
    2. German
    3. Dutch
    4. Others?
  - E. Others?
- II. Semitic
- III. Turanean
- IV. Others?

It need hardly be added that a surbordinate class should not contain any objects not included in the class above it. If we are classifying serious works of history it would not do to include Dumas' "Three Musketeers" in one of the subdivisions, even though it contains some historical matter, for it is not a serious work of history.

Students should make themselves familiar with the following names: *Genus* is the name of a class of objects divided into smaller classes. Each of these smaller classes is called a *Species*. Where there is a system of divisions and subdivisions

any class is called a species with reference to those above it, a genus with reference to those below it, *e.g.*, organisms or living bodies are a 'species' of bodies, but are a 'genus' of which animals are a species. In such a system the highest class of all is called the *Summum Genus;* the lowest, the *Infima Species.* "When a thing is so peculiar and unlike other things that it cannot easily be brought into one class with them, it is said to be *sui generis*, or of its own genus";* *e.g.*, the rings around Saturn amongst heavenly bodies, the ornithorhyncus and amphioxus amongst animals. The qualities peculiar to the members of some one species or genus, and in virtue of which that species or genus is distinguished from other species or genera, are called the *Differentiæ* of the species or genus. Thus the possession of reason, by which men may be distinguished from the lower animals, can be called the differentia of the human species. Qualities peculiar to a species, but not used in defining it or distinguishing it from other species, are called Properties, or *Propria.* Thus laughter is a proprium of human beings. Qualities or states not characteristic of a species are called *Accidents.*

Some technical names.

So far we have taken up the purely formal or negative aspect of division, and pointed out the blunders which any system of division should avoid. But every actual system of division has a purpose, and to attain a purpose it is necessary to move in some definite direction, not merely to avoid the pitfalls on the road. A scientific classification is nothing more than a system of division carried out in such a way as to best serve a given purpose.

Scientific classification and its purposes.

One purpose served by classification is to enable a person to find any given object with the least possible trouble. If I have only two or three dozen books it is not worth while to arrange them on the shelf in any particular order, for I can

*Jevons.

always find the one I want at a single glance. But if I have a library full of books and pamphlets, and I want to be sure of finding one of them at a moment's notice, I must arrange them alphabetically, or topically, or in some other fixed order. When the things themselves cannot be arranged according to any plan, the next best thing is to arrange their *names* in some fixed order—usually alphabetical—and after the name write where the thing is to be found. In this way we construct directories and gazetteers and the indexes of books.

A second purpose served by classification is to give an easy means of identifying an object when it is found. A suspicious looking person is arrested by the police and they wonder who he is; so they turn up their classified list of criminals, looking up (say) the class 'Eyes, blue'; then the subdivision 'Height, five feet eleven inches'; then the further subdivision 'Fingers, tapering'; and so on, till at last they find the photograph or thumb-mark of the individual in question with his name and record. If the descriptions were not classified the work of identifying the man might be endless.

A function similar to these two, yet distinguishable from them, is to enable us to find or identify a given *kind of object.* Often we do not know what book or man we want to find, but the classified list can tell us. We want some book or other about colonial furniture, and the subject-catalogue names several; we want a man who can mend china, and the classified list of tradesmen at the end of the city directory gives a number of names and addresses. In the same way if we find a new kind of plant in the woods and want to know what it is, we use some botanical 'key,' in which kinds of plants are identified by a series of obvious characteristics, and discover the name (not of that individual as such, but) of that *kind* of plant. In the same way also a classified list of symptoms might save a young physician much valuable time in diagnosing a disease.

A fourth object of classification is to make it easy to deal at the same time with things that bear any special relation to each other. To this end we put the things themselves or their names together. A grocer puts in the same basket all the parcels that are to go to the same house ; a lady writes on the same list the names of the people or shops to be visited on the same afternoon. Here the classification is made for an immediate practical purpose—something must be done about each of the people or things in question, and the classified list helps us to do it. But often, and this is the object of classification in science, things or the names of things are put together because it is desirable to think of them not only at the same time but in relation to each other.* Thus chronological tables, maps, and astronomical charts are made not only to show what happened in a given year or the location of a given place, but also to show the general sequence of events at any given period, the general conformation of a country, the general arrangement of a planetary system. In these cases the relations that determine the classification are those of time and space, but they may be anything : degree of scholarship, in arranging a list of students ; degree of strength, in a list of acids ; durability, in a list of fabrics or dyes ; cause and effect, when the writer on medicine puts together all the causes and symptoms of a given disease ; means and end, when he adds a list of remedies.

Scientific classifications are most concerned with relations of resemblance and contrast. When a naturalist, for example,

* It is rather common to speak as though all classification were of one's *ideas* or *concepts* of things, but this is a blunder. A psychologist classifies ideas and feelings when he points out their resemblances and differences ; *e.g.*, the difference between a feeling of terror and a feeling of contempt ; but a naturalist classifies *things*. It is the business of a psychologist to observe the difference between thoughts as such; but every other scientist is concerned with the difference between the objects that we think of, not the thoughts themselves. To think of the difference between things is not the same as to have different thoughts of them.

classifies all animals into vertebrates and non-vertebrates, he merely asserts that each of a certain long list of animals has a backbone, and in this respect and certain others that follow from it resembles each of the others and differs from all of those not on the list.

But what points of resemblance and contrast must be regarded, and in what order, if we are to make a classification scientific? The answer to this is that no basis of classification—no *fundamentum divisionis*—is any better than any other in itself. The only general rule is to choose and arrange *fundamenta divisionis* in the way that best shows the points of resemblance and contrast in which we are interested or likely to be interested. It is just as scientific to classify books by their size, publishers, date, color, type, or language as by their author or subject-matter, and a great deal more so if we are interested in the former and not in the latter.

If we are not interested in any special characteristic of the objects classified the only thing left is to try to get into the same class objects that have a great many things in common and into different classes objects that have very little in common, so "that we shall be enabled to make a maximum amount of aggregate assertion with a minimum number of propositions." * To do this we must choose as our first *fundamentum divisionis* that point on which the greatest number of other noteworthy points of difference depend. It is better to divide all living things into animals and plants than into those which weigh more than a pound and those which weigh less; because the possession of sensation and the power of spontaneous motion † which distinguish animals from plants involve innumerable other points of difference, while the size involves little or nothing more: to say that a

* Venn's "Empirical Logic," Chap. XIII., to which the reader is referred. (Macmillan.)

† This of course is the popular distinction. If we wish to be more scientific we should say 'the fact that they require protoplasmic foodstuff, or cannot decompose carbonic acid gas, which distinguishes,' etc.

thing is a plant tells a great deal about it; to say that it weighs less than a pound tells hardly anything.

It is this principle of trying to get into the same class those objects which on the whole are most alike that prevails in the classification of animals and plants in natural history and of books in the subject-catalogues of libraries.

Any system of classification that regards general resemblance is liable to be upset by an advance in knowledge or a change in scientific interests. The common man calls a whale a fish, the zoölogist says it is not a fish but a mammal, because he has found out that on the whole living in the sea and looking like a fish involves fewer other noteworthy characteristics than suckling the young. In Dewey's Library Index *illusions* are classified with witchcraft and fraud because they involve deception, and for the general reader this is perhaps the best classification; but a psychologist would classify illusions along with ordinary perceptions, because both are interpretations of sensations made in precisely the same way, and he knows that the correctness or incorrectness of the interpretation, which strikes the layman and upon which Dewey's classification depends, is a mere accident and does not involve any further differences in the mental processes. To illustrate the influence of wider knowledge or deeper insight upon a whole system of classification it is only necessary to point out how the old hard and fast lines between genera and species in natural history have been wiped out by the theory of evolution, which shows how new species are being created continually though slowly by the inheritance and consequent accumulation of a vast number of small individual variations.

The fact that a system of classification is likely to be upset by wider knowledge is no reason why it should not be constructed, for a bad classification is better than confusion; and however bad it may be, it is likely to contribute something toward the attainment of the wider knowledge by which it can be corrected.

When a classification has been made, the resemblances and differences which it indicates can be still further marked and more easily remembered and talked about by the use of class names. The words *vertebrate, mammal, radiate, batrachian* are all scientific terms invented for this purpose. But the use of class names was not invented by scientists, for every common name marks out a class of things which it serves to distinguish from all others. Animals and plants, trees and grasses, hawks and doves, were distinguished and contrasted long before science existed. The scientist, as Venn says, usually finds the highest and lowest classes of things already made and named. His business is only to state their most important differences distinctly (which sometimes involves a correction in the classification of an ambiguous kind, such as the whale) and to arrange the intermediate groups.

Naming.

CHAPTER V.

THE USES OF SINGLE WORDS AND PHRASES.

We have seen already how to avoid certain gross blunders which result from the ambiguity or misinterpretation of words and statements; but if we are to acquire any fine discrimination in the interpretation and use of language our study of words and sentences must not end here. Hence several chapters more must be devoted to them.

To understand the exact meaning of a word in any particular sentence it is not enough to know its definition; for however unambiguous the meaning of a word may be as given in a dictionary it may be used for any one of several different purposes, and if we do not understand the differences between these purposes we cannot be sure of interpreting the word aright.

With reference to each one of these different purposes words are divided into different classes—usually two; but since the same word can be classified with reference to different purposes it can belong to as many different classes as there are purposes with reference to which it can be classified. We shall see, however, that not every word can be classified with reference to all these different purposes.

Terms. The first division of words which we shall consider is into those that are 'Terms', and those that are not.

A term is a word or group of words used to indicate or identify the objects about which a person speaks, or the states, qualities, actions or other relations whose possession by

an object is under consideration. It is, in brief, a name in the broadest sense of the word. In the sentence 'The present Emperor of Germany is remarkably energetic' the first five words are regarded as a single term because they are used together to point out the individual under discussion; and because the last two words of the sentence are used together to indicate a single quality, they also are regarded as a single term. In other connections, of course, the words Emperor, Germany, energetic, would be regarded as separate terms, *e.g.*, The *Emperor* is *energetic; Germany* is a beautiful country. Even in the case of the sentence given, if we should be asked to 'define our terms' it would be proper enough to take up the words separately, as they might appear elsewhere, and explain that by 'remarkably' was meant, not 'excessively', but 'noticeably'; by 'energetic', not 'meddlesome' but vigorous'. But if we wished to be absolutely accurate, though tedious, we should add to our explanation of the meaning of the separate words an explanation of the phrase or term as a whole.

Though every term is a word or combination of words, the structure of language is such that some words can never be used as complete terms. The object of thought is indicated y the subject of a sentence; its relations under discussion, by the predicate. Such words as prepositions, conjunctions and adverbs cannot be either subjects or predicates and therefore they cannot be terms, though they can be parts of terms. We cannot say, for example: *Of* is energetic; *nevertheless* is *very;* etc. We can say 'of' is a preposition; but then the word is used as a noun and is not taken in its ordinary sense. Unless a relation is discussed—affirmed, denied, or questioned—the word that indicates it is not a term. The word 'of' in the phrase 'The Emperor of Germany' indicates a relation, but it is not a term, because nothing is said about the relation it indicates. It merely forms a part of the description of the object some other of whose relations is discussed. Prepositions joining nouns to

each other, and possessive cases are always devoted to the expression of relations which are involved in the conception of an object but not discussed. For that reason they can never be terms. On the same principle, adjectives are true terms when they follow the verb 'to be' or its equivalents as predicates, *e.g.*, in the sentence 'The horse is white', but not when they are in their usual position, as in the sentence 'The white horse kicks'.

Words which can be used as complete terms are called Categorematic (Greek, κατηγορέω, to assert); those which can not, and which therefore can not be used except when they are combined into a term with others, are called Syncategorematic (Greek, σύν, together with, and κατηγορέω).

The distinctions which we have to discuss in the rest of this chapter all have reference to terms. We shall therefore have nothing more to say at present about syncategorematic words.

Demonstrative and descriptive.

The first distinction to be made with reference to terms is between those which are used demonstratively and those which are used descriptively. By a Demonstrative term is meant one that points out an object; by a Descriptive term, one that tells something about it. When we say 'John is angry' or 'That is very beautiful', the words 'John' and 'that' are used demonstratively and the remainder of the sentences are intended to describe the objects that they point out. The word 'John' is of course a proper name and the word 'that' a 'demonstrative' pronoun. These parts of speech are devoted so exclusively to pointing objects out, and adjectival phrases like 'angry' and 'very beautiful' are devoted so exclusively to their description that if the order of the sentences were reversed and we said 'Angry is John' or 'Very beautiful is that', it would still be plain which words were used to point out the objects under discussion, and which were used to describe them.

In the examples just given the subject of each sentence

was demonstrative and the predicate descriptive, and by straining matters a little we can say that this is always so. We can say, for example, that in the propositions 'A is larger than B', 'A is bullying B', everything but the subject 'A' is part of a predicate whose function is merely to describe or tell something about A. This, I suppose, would be the grammarian's interpretation of the sentences, and it is the interpretation that we assumed when we defined the subject of a sentence as the name of the object about which we are speaking. But it is not quite fair, for in each of these sentences we tell quite as much about B as about A. The fairer way is to regard the sentence as made up, not of two parts, a subject and a predicate, but of three: two demonstrative terms, and a third term that describes some relation between the things which they point out. In view of this interpretation of propositions like these we cannot say that a predicate is always descriptive (or even that it always consists of a single term). But there is nothing in what we have said to prevent us from saying that the subject is always demonstrative. If we do say so, however, we must make it plain that we are speaking of real subjects and not such nominal subjects as 'it' and 'there' in propositions like these: 'It is a long distance from A to B'; 'It is a long road that knows no turning'; 'There is a lion in the way'. Sometimes, moreover, it is hard to tell what is the real subject. In this last proposition, for example, are we telling about the lion, as the form of the sentence seems to imply, or are we telling about the way and why we cannot travel in it?

Nouns (*i.e.*, nouns substantive) are usually used demonstratively; but they can also be used descriptively. When we say 'A man came to the house', the term 'a man' is demonstrative, for it points out, rather indefinitely to be sure, who it was that came; but when we say 'John is a man', the term is used descriptively, for it is intended to summarize a great many of John's attributes, to indicate his general resemblance to the other creatures we call men, and perhaps

to convey an idea of his biological relations to them and to other animals.

Since it is possible to identify an object more or less definitely by describing it, a term whose primary function in a given sentence is to do the one often serves at the same time incidentally to do the other. When we say that a man came to the house we not only tell the hearer that one or other of a certain large group of things came to the house, but we incidentally describe that thing, implying that it has all the qualities and other relations of a man ; and when we say that John is a man we not only describe him but incidentally we tell that he is one of a certain group. Terms which perform this double function are called Connotative; those which do not, Non-connotative.*

Connotative and non-connotative.

What words really are connotative and what non-connotative, and even the definition of these terms themselves, is a matter about which logicians are not all agreed; but the distinction will be illustrated well enough for our purposes if we say that the ordinary Common Nouns of grammar are connotative (*e.g.*, 'man', 'horse', 'pig'), and that Proper Names and Abstract Nouns are probably not.

Whether a proper name is to some extent descriptive as well as demonstrative depends altogether upon whether there exists any convention in virtue of which any particular name is applied only to certain classes of objects. In English-speaking countries nowadays, for example, surnames usually indicate family connections. Elsewhere and at other times they have indicated something else. A Christian name may be given merely because a parent thinks it pretty ; and yet it is usually chosen with some reference to sex. Nevertheless

* A 'connotative' term, as defined in logic, must not be confused with the somewhat similar terms 'which have connotation' as defined in books of rhetoric. They both do something incidentally; but this incidental function as described in rhetoric is generally some kind of appeal to feeling.

I doubt whether we can say that to that extent it is descriptive, for however unusual and outrageous it might be to name an English boy Mary, one could hardly say that the parent who did it had falsely asserted that the child was a girl. It need hardly be said that when we call some one a Nero or a Socrates or a Napoleon these proper names are used altogether in a descriptive sense.

When a term is used demonstratively—to point out an object of thought—it is said by logicians to be used 'Denotatively' or in its 'Extension'; when used to describe one, it is said to be used 'Connotatively' or in its 'Intension'. From this it follows that when we define a term we tell the qualities it *connotes;* when we give an example we tell one of the things it *denotes.* As Bosanquet puts it: "The denotation of a name consists of the things *to which* it *ap*plies, the connotation consists of the properties which it *im*plies." *

Proper names are applied (more or less arbitrarily) to some individual object for the sake of indicating that particular object as distinguished from all others. But this function can also be discharged by some descriptive phrase which is obviously applicable to only one particular object (*e.g.*, 'the man at my right', 'my black dog'). In either case a term which is intended to discharge this function is called Singular. A General term, on the other hand, is one applied to every object which possesses some given characteristic or characteristics, and used to distinguish any object which possesses such characteristic or characteristics from objects which do not (*e.g.*, 'triangle', 'angular').

Singular and general.

Because a general term is not applicable to an object unless that object possesses a given attribute, it is evident that general terms are all connotative.

Singular terms are intended to distinguish a certain "real essence", as Locke would have said, a certain person or thing that remains that same person or that identical thing throughout all the changes it may undergo. 'The thin black-

* The Essentials of Logic, p. 88.

haired man who passes here every morning'—this phrase is a singular term, because it is designed for the purpose of pointing out a special individual who would remain the same person when the description was no longer applicable. For the purpose in hand any other description might have done as well, for though we may identify the person by means of a description, it is the person, not the described characteristics, that we *mean*. General terms are intended to distinguish what Locke would have called "nominal essences" from each other. The terms 'a thin man', 'travellers', 'good students', 'good', 'gas' are each applicable to any object or set of objects only so long as the objects possess the attributes indicated; they are equally applicable to any objects possessed of those attributes, and are chosen for the express purpose of calling attention to the presence of the attributes.

Sometimes the same term may be singular in one connection (*e.g.*, '*the child* is sick'; '*Cæsar* was killed'), and general in another (*e.g.*, '*the child* is father to the man'; 'he is a regular *Cæsar*'). Thus in logic it is the meaning of a word that counts, not the outward form.

Often a descriptive phrase whose primary purpose is to identify an object is not quite accurate, but if it suggests the required object this does no harm so far as the identification is concerned. If I say that the peddler who was so impudent at our door yesterday morning was afterwards arrested on a serious charge, and if you recognize the man from my description, the description has answered its purpose, even though it was not yesterday, but the day before, that he was at our door. Sometimes, however, a number of descriptive words are added to a demonstrative term when they are wholly unnecessary for the identification of the object, *e.g.*, 'swift-footed' Achilles, 'the beautiful and accomplished' Miss Blank, this 'most dangerous' disease. Such epithets have a rhetorical value, for they convey in a neat way a desired conception of an object about which something is to be

Epithets.

said; but no one is logically justified in using one of them if its applicability is a question at issue or depends in any way upon one. A lawyer, for example, has no right to refer to a person accused of murder as 'this brutal assassin' until after the trial is over and the accused is found guilty, and then he may not wish to. Such descriptive terms used in this way are called Question-begging Epithets. They beg the question or take for granted what is to be proved, because they have the form and position of descriptive epithets used to identify an object; and these nearly always point to qualities that are obvious and admitted. Question-begging epithets are generally intended to appeal to the listeners' emotions and make them partisans instead of unprejudiced judges. Thus they are doubly unfair.

Collective and distributive.

When we consider terms from another standpoint we must distinguish between those that are called Collective (such as 'jury', 'army', 'mob', 'herd', 'crew', 'crowd', 'heap') and those which are called Distributive (such as 'man', 'soldier', 'juryman', 'goat'). A Collective term as distinguished from a Distributive is one used to denote any aggregate of similar and separable things regarded as constituting some sort of temporary unit. When all notion of the separateness of the individuals in an aggregate is lost, so that the combined whole is regarded, at least for the moment, as a true unit, the collective term which names it become distributive. This is most likely to happen when we think of several such aggregates, *e.g.*, 'There were *mobs* in six cities at once', 'He was convicted by three different *juries*'.

It is extremely important to distinguish between statements which are intended to apply to several things considered as a whole and those intended to apply to each of a number of individuals. A jury as a whole cannot get hungry, though each of its members may. The words 'and', 'all', 'many' are singularly ambiguous in this respect. It was this ambiguity that gave point to the newsboy's re-

mark: 'Astor and I are worth millions.' 'Three and five' make eight, not severally but conjointly, while 'three and five' are odd numbers severally and not conjointly. 'They all' lifted a log conjointly (*cuncti*), and 'they all' told about it severally (*omnes*). 'The mosquitoes in Alaska are so large that *many of* them weigh a pound.' The terms *severally* and *conjointly* are often used in legal documents to indicate the distinction we are discussing. The most usual terms in logic are *distributively* and *collectively*.*

The danger.

When we take any of these ambiguous words collectively at one stage of an argument and distributively at another, it is possible to draw conclusions which a fair and unambiguous interpretation of the words would not warrant, *e.g.*, '*All* the angles of a plane triangle are equal to two right angles; this is one of them; therefore this angle is equal to two right angles.' '*All* the feathers in a bed are extremely light; the bed is made up of all the feathers; therefore the bed is extremely light.' In the first of these examples the word 'all' is taken collectively when the premise is admitted, but distributively when it is used to prove the conclusion, and the fallacy is called one of Division. In the second example the word 'all' is taken distributively in the ambiguous premise when it is admitted, but collectively when the premise is used to prove the conclusion, and the fallacy is called one of Composition.†

* Both Jevons and Minto contrast Collective terms with General terms. Jevons says "We must carefully avoid any confusion between general and collective terms" (p. 19); and Minto speaks of "Collective names as distinguished from general names" (Logic, p. 58). In each case, however, it seems to be an oversight, for in the exercise at the end of the chapter Jevons says the reader is to determine whether a term is "collective or distributive", *not* collective or general. So with Minto, most collective names fall under his definition of general names (see p. 44).

A collective term may be either singular or general, *e.g.*: *The Smiths* have a new house (singular); he was attacked by *a mob* (general).

† A good many fallacies, such as the stock argument for protective tariffs, are credited by Whately, Jevons, and others to this confusion,

A more serious danger than that just mentioned is that when we use collective terms we shall forget that the individuals in the group are not a real unit, but can be considered as one for certain purposes only. I give one example of this fallacy here. There will be others in a later chapter.

"During the last ten years I have read a great many books and articles, especially by German writers, in which an attempt has been made to set up 'the State' as an entity having conscience, power, and will, sublimated above human limitations, and as constituting a tutelary genius over us all. I have never been able to find in history or experience anything to fit this conception. . . . My notion of the State has dwindled with growing experience of life. As an abstraction, the State is to me only All-of-us. In practice—that is, when it exercises will or adopts a line of action—it is only a little group of men chosen in a very haphazard way by the majority of us to perform certain services for all of us. The majority do not go about their selection very rationally, and they are almost always disappointed by the results of their own operation. Hence 'the State', instead of offering resources of wisdom, right reason, and pure moral sense beyond what the average of us possess, generally offers much less of all those things." *

From another standpoint terms are divided into Abstract (such as 'redness', 'anger', 'kindness') and Concrete (such as 'red', 'angry man', 'kind'). Abstract terms are nouns used to indicate the qualities, states, acts, or other relations of things. They are 'Abstract' because, unlike adjectives and verbs, they can be used grammatically without any mention of the thing to which the qualities and so forth belong and without

Abstract and concrete.

when, so far as I can see, they have absolutely nothing to do with it. I have treated of them elsewhere.

* Sumner, "What Social Classes Owe to Each Other", pp. 9, 10.

which these latter could not exist. We can say, 'Anger is foolish', or 'A man is foolish to be angry', but not 'Angry is foolish'. Abstract terms thus seem to 'abstract' or *draw away* one's thought from things.

Every term which is not abstract is called Concrete. Concrete terms are therefore (1) the names of things, or (2) adjectives or verbs, that is to say, names of relations which cannot be used grammatically without any mention of the thing to which they belong.*.

The value of abstract terms lies in the fact that through their use attention can be called more briefly and more effectively than in any other way to certain features of things which we wish to discuss without any special reference to the things themselves. It is easier, and on the whole more effective, to say 'Self-sacrifice deserves gratitude' than to say 'When a person acts in such a way as to injure himself because he wishes to benefit some one else, the person whom he meant to benefit ought to feel grateful, no matter who the persons may be'. But in spite of the great value of abstract terms, there is no more important practical rule in logic than that which says Beware of abstractions!

Hypostatising abstractions.

The danger involved in the use of abstract terms is due to the fact that such terms are always nouns and that nouns are usually the names of things. We are therefore very prone to regard the quality or relation referred to by an abstract term as a sort of *thing* with a certain independent existence, possessing attributes and playing an active part in the world as things alone can. In this way 'life', 'natural laws', 'motion', 'force', 'ideas', 'justice', 'evil', 'the Zeitgeist', 'Public Opinion', and a host of other abstractions are liable "to play the part of sham-essences, and cheat their way into recognition as realities." †

* The word 'thing' is intended here and is usually intended elsewhere in this book to refer to whatever possesses substantial reality. It therefore includes persons.

† Jas. Martineau, "Study of Spinoza", p. 12.

Nothing exists in the whole universe but a vast number of persons and things acting in various ways. It has no place for abstractions. To say that a person *has an idea* means that he thinks. To say that a speaker *conveys an idea* to his hearers means that he makes them know what he thinks. To say that *a smile spread through* the company means that several persons smiled at once, or that some smiled and then others smiled because they saw them do it. To say that *energy is stored up* means that a thing is not acting, but is in a condition to act, upon occasion. To say that *motion is transmitted* means that one thing stops moving and another begins. Imagine a smile spreading through a company like water through a sponge, or energy stored up like grain in the inner recesses of a thing, or motion being carried from its resting-place in one thing to a quiet nook in another! *Public opinion* is only what is similar in everybody's way of thinking about a question. A *law* is nothing but a statement of how people must act if they wish not to be punished by the law-maker, or a statement of how things as a matter of fact do act under certain circumstances. It is not some shadowy reality existing before or apart from all things and compelling their obedience by its own strength. A law is never *imposed on* things. To say that two chemicals *have an affinity* for each other does not explain why they combine or act in conjunction; it merely states the fact that they do.—And so of all the rest.

The habit of *defining* abstract nouns rather than their corresponding verbs and adjectives helps to entrap us in this "snare of abstractions". We have been taught to say that attraction is the principle or power in virtue of which one body approaches another, or that beneficence is the trait of character manifested in acts of general kindness. It would be better to say that one thing attracts another when it makes it come nearer, and that a person is beneficent when he is kind to everybody. "The snare of abstractions concealing itself chiefly in common nouns, we shall best guard against it

by admitting to our definition no substantive where an adjective [or verb] ought to serve as well." *

It is often said that the tendency to take abstractions for things is particularly characteristic of philosophers. This is probably not true. And yet the philosopher must take special care to overcome it; for the nature of his work is such that a very few blunders of this kind can spoil it. Hence the advice contained in the following quotation is excellent, and the criticism implied in the last sentence is not altogether unjust: "If the student of philosophy would always, or at least in cases of importance, adopt the rule of throwing the abstract language in which it is so frequently couched into a concrete form, he would find it a powerful aid in dealing with the obscurities and perplexities of metaphysical speculation. He would then see clearly the character of the immense mass of nothings which constitute what passes for philosophy." *

Philosophy, however, is not the only subject that suffers by this tendency to take abstractions for things, as can be shown by the following passages from Langlois and Seignobos' beautiful "Introduction to the Study of History", already quoted.

"The facts of society are of an elusive nature, and for the purpose of seizing and expressing them, fixed and precise language is an indispensable instrument; no historian is complete without good language. It will be well to make the greatest possible use of concrete and descriptive terms: their meaning is always clear. It will be prudent to designate collective groups only by collective, not by abstract names (Royalty, State, Democracy, Reformation, Revolution), and to avoid personifying abstractions. We think we are simply using metaphors, and then we are carried away by the force of the words. Certainly abstract terms have something very

* Martineau, *loc. cit.* p. 124. Lotze somewhere says about the same thing. See also the introduction to Berkeley's "Principles of Human Knowledge".

† Bailey's "Letters on the Mind", vol. II, p. 159, quoted by Bain, "Logic", p. 53.

seductive about them, they give a scientific appearance to a proposition. But it is only an appearance, behind which scholasticism is apt to be concealed; the word, having no concrete meaning, becomes a purely verbal notion (like the soporific virtue of which Molière speaks)." (Pp. 266–7).

"Specialists, influenced by a natural metaphor, and struck by the regularity of these successions, have regarded the evolution of usages (of a word, a rite, a dogma, a rule of law), as if it were an organic development analogous to the growth of a plant; we hear of the 'life of words', of the 'death of dogmas', of the 'growth of myths'. Then, in forgetfulness of the fact that all these things are pure abstractions, it has been tacitly assumed that there is a force inhering in the word, the rite, the rule, which produces its evolution. . . . Just as usages have been treated as if they were existences possessing a separate life of their own, so the succession of individuals composing the various bodies within a society (royalty, church, senate, parliament) has been personified by the attribution to it of a will, which is treated as an active cause. A world of imaginary beings has thus been created behind the historical facts, and has replaced Providence in the explanation of them. For our defence against this deceptive mythology a single rule will suffice: Never seek the causes of an historical fact without having first expressed it concretely in terms of acting and thinking individuals. If abstractions are used, every metaphor must be avoided which would make them play the part of living beings." (Pp. 288–9.)*

* Abstract terms are often demonstrative rather than descriptive. To be sure, they point out qualities and relations which have no independent existence and which are usually best indicated by descriptive terms; but whether the qualities or relations in question have such an existence or not, they can be the objects of thought, and when they are, the words used to point them out are demonstrative. Indeed we may go so far as to use a demonstrative term to indicate a quality, act or relation and a descriptive term to indicate the real thing without which it could not

The term 'Abstract' is often applied, with a somewhat broader meaning than that given in the definition, to words which indicate complex relations that are not easily perceived by the senses. Thus we might say that when the words 'free' and 'equal' are used to mean 'not tied or locked up' and 'of the same size' they are concrete; but when they are used ('in the second intention') to mean what politicians mean when they say that all men are born 'free and equal' they are abstract. The first relations can be easily perceived by the senses and easily defined; the second can not. In the same way the terms 'universe', 'siderial system', &c., might be called abstract.*

Secondary meaning of 'Abstract'.

General terms are used to distinguish objects which possess certain characteristics from those which do not. They are

exist, as when we speak of 'Asiatic duplicity', 'the Turkish atrocities', or 'the Franco-German war'.

* Abstract and Concrete words are often distinguished by their form; but "unfortunately" the two forms "are frequently confused, and it is by no means always easy to distinguish the meanings. Thus 'relation' properly is the abstract name for the position of two people or things to each other, and those people are properly called 'relatives' (Latin *relativus*, one who is related). But we constantly speak now of 'relations', meaning the persons themselves; and when we want to indicate the abstract relation they have to each other we have to invent a new abstract term, 'relationship'. 'Nation' has long been a concrete term, though from its form it was probably abstract at first; but so far does the abuse of language now go, especially in newspaper writing, that we hear of a 'nationality' meaning a nation, although of course if 'nation' is the concrete, 'nationality' ought to be the abstract, meaning the quality of being a nation. Similarly 'action', 'intention', 'extension', 'conception', and a multitude of other properly abstract names, are used confusedly for the corresponding concrete, namely 'act', 'intent', 'extent', 'concept', etc. 'Production' is properly the condition or state of a person who is producing or drawing something forth; but it has now become confused with that which is produced, so that we constantly talk of the 'productions' of a country, meaning the 'products'. . . . Much injury is done to language by this abuse" (Jevons, "Elementary Lessons in Logic", p. 21).

usually applied to the objects which possess the characteristic in question, but they can be so altered as to apply—or wholly different terms can be invented which apply—only to the objects which do not possess the characteristic in question. General terms of the first sort are called Positive, those of the second sort Negative. The words 'Jew', 'Greek', 'citizen', 'clergyman', 'educated', 'Aryan', 'white', are positive terms. Their corresponding negatives are 'Gentile', 'barbarian' (in the earlier sense of the word), 'alien', 'layman', 'uneducated', 'non-Aryan', 'not white'. Where negative terms do not exist they can always be made by attaching some such prefix as 'in-', 'un-', 'non-', or 'not-', or some such suffix as '-less' to a positive term; but we must not take it for granted that terms which have these affixes always correspond in meaning in this way with those that have not. It is said that a college chaplain once became sleepy during a service and implored the irreligious to become religious; the immoral, moral; the intemperate, temperate; the inebriate, ebriate; and the indifferent, different!

Positive and negative.

Sometimes terms which are negative in form have a positive meaning, and *vice versa*. Thus the terms 'uncomfortable', 'unhappiness', 'uneasiness', are all used to indicate that disagreeable feelings are present, and, on the other hand, the terms 'free', 'sober', and 'healthy' are used to indicate that certain undesirable conditions are absent. The terms 'moral' and 'good' are probably more often used in a negative sense than in a positive.

Though negative terms are used to indicate the absence of certain characteristics, they are not properly applicable to everything which does not possess these characteristics. A mutton-chop is neither a Jew nor moral nor able to see, but neither is it a Gentile nor immoral nor blind. A man, on the other hand, must be one or the other. Negative terms are thus only applicable to things that are capable of possessing, or might reasonably be supposed to possess, the char-

acteristics whose absence the negative term denotes. They are used with reference to what De Morgan has called a limited universe of discourse. Within that universe, but not beyond it, everything can be described either by a positive term or by the corresponding negative. In the example just given the universe of discourse was human beings. If we say that everything must be either light or heavy, here or there, we have in mind the universe of tangible objects existing in space. The statement would not be true of a soul or a feeling of remorse. When Euclid says that things which are equal to the same thing are equal to one another, his universe of discourse is the size of lines or figures. If A is equal to B in social position and B is equal to C in intelligence, it does not follow that there is any respect in which A is equal to C.

There are a vast number of words whose meaning is ambiguous until one knows the universe of discourse to which they refer. The term 'irregular', for example, may have reference to the distribution of lines in space, or the succession of events in time, or a person's moral relations.

Which of two mutually exclusive terms, one or other of which is applicable to every individual in a given universe of discourse, is to be regarded as negative is often a matter of indifference, for within a given universe the absence of one characteristic involves the presence of another belonging to the same general class. A substance which is immaterial must be spiritual, a thing which we take the trouble to describe as not white is usually a kind of thing that must have some color or other. There is as much fulness of determination—there are as many attributes—in one case as in the other.

Though 'immaterial' and 'not white' are both terms which imply the presence of some corresponding attribute, there is this difference between them: in the one case we know immediately what the attribute is—the thing must be spiritual; in the other we do not, for the thing may have any one of many colors. Thus when there are only two alterna-

tives, a term which indicates the absence of one of them has a definite positive significance; when there are many the positive significance is indefinite.

A positive term and its negative are called with reference to each other *negative*, *contradictory*, or more properly, *contrapositive* terms.

Some terms (such as 'giant', 'dwarf'; 'immense', 'tiny'; 'courageous', 'cowardly'; 'noble', 'ignoble') are used to name contrasting and mutually exclusive relations; but there are objects in the universe of discourse to which neither term in such a pair is applicable. There are many men who are neither giants nor dwarfs, many acts that are neither noble nor ignoble. In such cases the terms are not Contradictories, but *contraries* or *opposites*.

Whether such terms as 'thin' and 'thick', 'large' and 'small' are contrary or contradictory depends upon the circumstances in which they are used. The frequent controversies to which they give rise in this respect are due to the fact that they are never terms of precision in any respect. When we wish to be accurate we give measurements. Neither speaker nor hearer usually stops to ask what size a thing must be in order to be large or small; they therefore do not ask whether there can be anything of the kind discussed which is neither large nor small. To raise the question is to attempt to render definite the meaning of terms whose value lies in their essential vagueness. Hence the old catch called Sorites: 'How many things does it take to make a heap?'

The last distinction between terms which we shall have to consider is between Relative and Absolute. Unfortunately there are two distinct senses in which a term can be said to be relative. In the first sense of the word a term is called Relative when (as with 'master', 'combatant', 'lover') it is applied to a person or thing to mark a certain active relation to some other person or thing *,—a relation which might have been

Relative and absolute: first sense.

* These relative terms are usually nouns or words used as such, *e.g.*,

expressed by a transitive verb or a phrase involving one. Such names are usually found in pairs called *correlatives*, one for each party to the relation, *e.g.*, 'master', 'servant'; 'landlord', 'tenant'; 'debtor', 'creditor'; 'victor', 'vanquished'; 'lover', 'beloved'.

We might avoid the use of such terms by saying that one person works for another, or has rented his house, or has borrowed money from him and not yet paid it back, and so forth. In a world in which nothing affected anything else this first kind of relative terms would have no place. They express a relation existing only between two active beings. Relative terms of the second kind, on the other hand—of which we are about to speak—express no action of one thing upon another, but merely the fact that some one has compared them in a given respect.

In the second sense of the word terms are called 'Relative' when, like such words as 'larger', they indicate the result of a comparison with some standard which the term itself does not indicate. The most obvious examples of such terms are adjectives of the comparative degree, for with them it is necessary to name the standard in some added words in order to give the term any meaning, *e.g.*, 'A robin is larger *than a sparrow*'. With superlatives we recognize that a standard is involved though we do not always mention it, *e.g.*, 'I saw *the loveliest* picture'. Such expressions as 'rather beautiful', 'very beautiful', 'extremely beautiful', 'most beautiful' (not distinguished in Latin from ordinary comparatives and superlatives) also involve some reference to a standard. By a 'rather beautiful' thing we mean one that is perhaps somewhat more beautiful than the average of its kind; by an 'excellent' piece of work, one that is exceptionally good.

Second sense.

my *beloved*. They are perhaps occasionally adjectives: *e.g.*, *parental*, though I am not sure whether this should really be called a relative term or not. They are almost always applied to persons, though not always: *e.g.*, *reagent*.

It is less obvious, but no less true, that an immense number of terms 'positive' in form are really used in a comparative or relative sense. A large thing is larger than a small thing, a clean thing cleaner than a dirty thing, an intelligent creature more intelligent than a dull one; but a large rat is not necessarily larger than a small elephant, a clean stable cleaner than a dirty table-cloth, or an intelligent horse more intelligent than a dull man. In each of these cases the name of the thing to which the adjective is applied indicates the standard of comparison. A large rat is one larger than most other rats; an intelligent horse, one more intelligent than most other horses; a clean stable, one cleaner than most others, or as clean as one could reasonably hope to keep a stable. A rat is an animal, but we cannot say that a large rat is a large animal, because the standard of largeness changes as we pass from the consideration of rats to that of animals in general. What we have a right to say is that a large rat is an animal larger than most rats. In this way we retain the meaning of the word 'large' with which we started.

Amongst other kinds of inference Jevons mentions *Immediate Inference by added Determinants*, which "consists in joining some adjective or similar qualification both to the subject and predicate of a proposition." **Dangers.** Hyslop says of it that terms expressing quantity, such as 'large', 'long', 'small', 'short', must be used carefully; but that terms expressing quality "can be used with perfect freedom, provided they are not used equivocally".* If more examples are needed to show that this is not the case there are plenty to be had. An Australian Bushman is a man, but an intelligent Bushman is not an intelligent man, a respectable saloon is hardly a respectable place, an energetic snail an energetic animal, nor a fast mule-car a fast means of transportation.

This "inference by added determinants" gives a good

* Elements of Logic, p. 168 (Scribners, 1894. Third Edition).

illustration of the trouble we get into when we substitute rules of verbal manipulation for thought about the things that the words are intended to denote.

The following passage from Schopenhauer shows how even a first-class author is likely to deceive himself and his readers when he makes too much use of relative terms. The passage seems to be full of meaning, but it turns out on analysis to be absolutely empty.

"This human world is the kingdom of chance and error. . . . [1] Everything better only struggles through with difficulty; what is noble and wise seldom attains to expression, becomes effective and claims attention, but [2] the absurd and perverse in the sphere of thought, the dull and tasteless in the sphere of art, the wicked and deceitful in the sphere of action, really assert a supremacy, only disturbed by short interruptions. On the other hand [3] everything that is excellent is always a mere exception, one case in millions, and therefore if it presents itself in a lasting work, this, when it has outlived the enmity of its contemporaries, exists in isolation, is preserved like a meteoric stone, sprung from an order of things different from that which prevails here." *

These words mean something like this: 'I am dissatisfied with the world because [1] the exceptionally good is exceptional, [2] what is no better [and no worse] than the common is common, and [3] anything good that is exceptional enough for me to call it excellent is very exceptional indeed.' If the value of everything in the world were increased a thousandfold a new philosopher who happened to feel dissatisfied could use Schopenhauer's very words, for 'good' and 'bad' would be interpreted then as they are now with reference to the average.

* "The World as Will and Idea", Vol. I, p. 417 (Kegan Paul, Trench, Trübner & Co., 1891).

CHAPTER VI

THE RELATIONS EXPRESSED IN PROPOSITIONS

The meaning of words and statements has been discussed enough in the foregoing chapters to guard against various kinds of rather gross blunders. Our next task is to inquire what fundamental relations of things statements of various kinds imply. This is the work of the present chapter. In the next we shall carry the same subject further by inquiring what any proposition implies about the existence of the things it mentions. After these more general discussions we shall come back and consider propositions from a more formal standpoint, taking up the difference between various forms of statement, the significance of each, and the way in which a statement in one form of words may imply the truth or falsity of a statement in some other form of words.

'This *is* John'; 'John *is* happy'; 'John *is* riding a horse'. It is perfectly evident that in these propositions the relations expressed by the verb 'is' are wholly different and incomparable. In the first case it helps to identify a person; in the second, it helps to tell something about his state of mind; in the third, it helps to tell his relation to something else. How many absolutely different kinds of relation we can think of and express in propositions it is hard to say. The question is one whose full discussion belongs to metaphysics rather than to logic; and it is one about which metaphysicians have not agreed. Nevertheless, something should be said about it here; for the

Five fundamental relations.

very first rule of logic is to understand the meaning of the words we use.

In thinking about the question it is well to remember that in reality nothing exists but things (including persons) with their various attributes and acts. The following list is perhaps as good as any.

1. We are able, first of all, to distinguish between various individual things and to recognize the fact that through all the changes they undergo each one of them remains the same: the Paul who preached Christ was identical with the Saul who was present at Stephen's death, in spite of his change of heart and name in the meantime; the rock on Emerson's grave to-day is the very same rock that was there yesterday and that was carried there years ago. Precisely what it is that makes a thing to be the same throughout all its changes of state and circumstances,—whether the ship of Argos, repaired so often that at last none of the original material remained, was really the same ship or not,—this is a detail of metaphysics which we cannot discuss here. But in a general way every one knows what it is to *recognize* an old friend, or to say that he himself is *the very person* who performed such and such an act, or that the book in his hand *is not the one* he bought yesterday, though the two look precisely alike.

The first kind of proposition, then, is that which asserts or denies this unity or *individual identity* of a thing that then was there or did that, with one that afterwards was here or did this.

It is evident that any proposition of this kind must involve two demonstrative terms connected by the verb *to be*, *e.g.*, This—is—the man who was here yesterday.

2. The second kind of relation affirmed or denied by propositions is that which exists between a thing and its qualities, states, or activities (whether we suppose it to be conscious of these or not); in short, it is a relation of *subject and attribute*, *e.g.*: 'Bulldogs are courageous'; 'this iron is

not cold'; 'he laughed'; 'he is asleep'. The subject of such propositions is always a demonstrative term; the predicate is always descriptive, but its meaning may be expressed either by the verb *to be* or its equivalent (*i.e.*, *to seem*, *to appear*, or some other similar verb) followed by an adjective or its equivalent (*i.e.*, some descriptive noun or phrase), or merely by an intransitive verb (with or without a completing or modifying adverb), *e.g.*: 'He is insane'; 'he is a lunatic'; 'he raves'; 'he acts insanely'.

3. The third kind of relation affirmed or denied by propositions is the action of one thing upon another, or the mutual relations of two things as active beings. Propositions expressing such relations involve two demonstrative terms and a transitive verb or its equivalent, *e.g.*: 'John strikes his horse'; 'A. and B. are quarreling'; 'C. is D.'s landlord', *i.e.*, rents a house to him; 'E. *walks on* the grass', *i.e.*, treads the grass. These relations may be called *causal* or *dynamic*.

Whether a proposition belongs to this third class or to the second is often a mere question of interest. A man cannot walk without treading upon something, but we are not usually interested in the effect upon the object beneath his feet, and so we regard the statement that he walks as merely descriptive. When we are interested enough to tell what he walks upon the dynamic relation becomes prominent. Further, as Sigwart says in his "Logic":* "When a man walks he moves his legs; that which from one point of view is mere action appears from another as an effect upon his limbs, which are relatively independent things." Here, again, the classification of the proposition depends upon the interest which one can reasonably be supposed to take in some particular aspect of the whole fact stated.

Sometimes, but not usually, it is proper to regard a statement of a thing's color, taste, smell, or other perceptible

* Vol. I, p. 37 (Macmillan)

quality as dynamic rather than merely descriptive; for to have a certain color is to reflect light in a certain way, and to have a certain smell is to act chemically in a certain way upon an olfactory organ. But when the ultimate nature of things is not under discussion it is more convenient to ignore these facts and regard such statements as merely descriptive.

4. Propositions of the fourth kind affirm or deny the existence of various non-dynamic or non-causal relations between two or more different things or other objects of thought. They express our comparisons between them rather than the action of one upon another. We may say, for example, that several objects are *similar* or *dissimilar;* that things or events *coexist* or *succeed* each other in *time ;* that they bear to each other various relations in *space ;* that one thing is more or less *beautiful,* one musical note '*higher*' or '*lower*', or one man *morally better* or *worse* than another, without supposing the objects whose relations we discuss to affect each other in any way whatever. The number of relations of this kind which anything bears to anything else is limited only by the ability of beings that know them both to compare them together in various respects; and whether many or few such relations be discovered or exist has not the slightest effect upon the things compared or their activities. Causal relations, on the other hand, engross more or less of a thing's energy, and may thus interfere with each other. Anything, A, might resemble B, coexist with C, be heavier than D, lighter than E and beside F, all at once; but if A were a man he could not at any one time fight with B, dance with C, and discuss philosophy with D. Causal relations exist directly for the things related, and seem to penetrate into and affect their inmost being; non-dynamic relations concern nothing and nobody but the being that discovers them.

The most important of the non-dynamic relations are those of Space and Time, without which the mathematical sciences would be impossible. They are so important that many eminent writers give them a separate place in their

lists of relations. Indeed relations of time and space are often supposed to comprehend causal relations. The question is one of metaphysics which cannot be discussed here.*

When we speak of the color or of any other attribute of an object as *identical* with that which exists elsewhere, the relation asserted is really a relation of resemblance rather than of individual identity. We merely mean that the similarity is complete. We must not be deceived by a merely verbal resemblance between two propositions. If I say that your clothes are the *same as* mine the relation in question is one of resemblance; if I say that they are the *same that* you wore a year ago the relation is one of identity. If I say the house has the same color it had ten years ago the relation indicated by the word 'same' is one of similarity; if I say it has the same paint the relation is probably intended to be one of individual identity. The distinction here pointed out between the same thing and the same kind of thing is very important, though sometimes lost sight of, in metaphysical discussions. In which sense, for example, do we see the same rainbow when the sun comes out again, and have the same idea or make 'the very same' remark as somebody else?

5. So far we have discussed the various relations which exist between the things we know or think about, but we have said nothing about our knowledge of these relations. It is one thing for the frog that exists to-day to be identical with the tadpole that existed six months ago, and quite another thing for you or me to recognize that it is identical; one thing for

* It may occur to the reader that the non-dynamic relations between several objects can be affirmed or denied only because the things compared act upon the person making the comparison, *e.g.*, that we say one thing is prettier than another because it acts upon our senses in such a way as to give us more æsthetic enjoyment. This is often the case, but we must not infer from it that the non-dynamic relations can be resolved into dynamic, for even if we suppose that to be pretty means merely to please the beholder, in the example given there is still a comparison expressed between the amount of enjoyment present in the two cases. We cannot therefore get rid of the element of mere comparison.

a man to be angry, and quite another thing for us or even for the man himself to know that he is angry; one thing for A to be larger than B, or to be near B, or to influence B, and quite another thing for us to know or think about these relations.

We have thus a new kind of relation to deal with, namely, that between a person thinking and the thing he thinks about —between thought and its object. This relation can be called *noetic*.

This noetic relation, or relation between thought and its object, can be affirmed or denied like any other: *e.g.*, I was *thinking of* you; I *love* you or *fear* you; I do not *desire* a certain end; he *knew*, or was *mistaken*, or was *doubtful about* a certain matter; your ideas about it are *consistent*, or *contradictory*, or *absurd*; he is *right* about it; he *thought he knew*; I *mean* you. In all these cases something is said about some aspect of the relation between a thinker or his thought and the object thought about.

In the history of philosophy much has been made of the distinction between *real* and *verbal* propositions (otherwise called Synthetic and Analytic, Ampliative and Explicative, Accidental and Essential). A Real proposition tells something about an object, *e.g.*: 'A thrush is in the tree', 'Tully is dead'. A Verbal proposition tells the meaning or part of the meaning of a word, *e.g.*: 'A thrush is a kind of bird', 'Tully is Cicero'.* Every definition is thus a verbal proposition. Euclid's definitions are supposed to be merely verbal—telling nothing more than the meaning of words; his axioms and postulates to be real—telling something more than the names of various figures strictly imply, though perhaps not more than everybody knows. Real propositions may belong to any one of our five classes. Verbal propositions are always noetic; for to tell the meaning of a word is to tell of what I

* Contrast with these propositions such a one as this: 'That man's name is Washington Jefferson Madison Stokes'. Here the name is regarded as a kind of appendage, and the proposition is as *descriptive* as if it had been said that the man had thirteen fingers.

am thinking and of what I wish my hearer to think when I use it.*

It is probable that we can never speak or think of a relation not included in this list: Individual Identity, Subject and Attribute, Causal relations between several things, Non-dynamic, or Non-causal, relations between several things, and relations between thought and its object or Noetic relations. These then are the so-called *categories;* but it must be remembered that the fourth class includes a great many different relations which agree in only one respect, namely, that they involve at least two objects without involving any action of the one upon the other.†

* The terms Real and Verbal seem to me preferable to Kant's Synthetic and Analytic, partly because as Mansel says, "propositions in which the predicate is a single term synonymous with the subject" cannot possibly involve any *analysis* or splitting up of the meaning of the subject; *e.g.*, Tully is Cicero (see Keynes, "Logic", p. 43, 3d ed.); but mainly because the term Analytic implies that both subject and predicate of verbal propositions are always used in a connotative or descriptive sense and that the function of such propositions is to *analyze ideas* rather than to identify things spoken of. In a word, Kant's terminology does not suggest any clear distinction between thought and its object.

† Aristotle's list of categories is as follows: Substance, Quantity, Quality, Relation, Action, Passion, Place, Time, Posture, Habit.

Hume gives Resemblance, Identity, Space and Time, Quantity or Number, Degree of Quality, Contrariety, Cause and Effect.

Kant gives Space and Time and four sets of 'pure conceptions', viz., Quantity, including Unity, Plurality, Totality; Quality, including Reality, Negation, Limitation; Relation, including Substance and Accident, Cause and Effect, Reciprocity between the active and the passive; Modality, including Possibility and Impossibility, Existence and Non-existence, Necessity and Contingency.

Mill gives Sequence, Coexistence, Simple Existence, Causation, Resemblance.

For an explanation of Aristotle's list see Minto's "Logic", Chap. III. Hume's categories, called by him "philosophical relations", are enumerated and explained in the "Treatise of Human Nature", Bk. I, Pt. I, Sec. V. But his discussion of Space and Time and Causation runs through the first three 'Parts'. Kant's categories, which he makes to correspond with the formal differences between propositions as set forth

Since abstract propositions can always be reduced to concrete, it is evident that no new class of relations need be made on their account.

Though these relations are probably all we can think of, they are often so combined as to give the appearance of something quite different from any of them. The relation of *means and end*, for example, is made up of a *desire* for something (a noetic relation) and an attempt to *get* it (any one of the five relations, depending upon the kind of object desired) by acting on certain things (causal) in such a way that they in turn will do something that will lead (causal) to the attainment of the desire. *Change* again, is an alteration (time) in the states (subject and attribute) or outer relations (causal or non-dynamic) of a thing which remains self-identical throughout them all (individual identity). This case of Change shows how inextricably different categories are sometimes interwoven.

Certain combined relations.

The idea of *whole and part* is, as Sigwart points out, primarily the idea of a relation in space—*i.e.*, of a larger figure or object comprehending or containing a smaller; but in many cases the idea involves also the notion of an influence exerted by the whole upon the parts or by the parts upon the whole. This is most obvious in the case of living beings. It is because I can control the movements of my hand and feel its injuries that it seems so much a part of me—more so than my hair or finger-nails. Thus the idea of whole and part often involves both dynamic and non-dynamic relations.

in the traditional logic, are enumerated near the beginning of the "Critique of Pure Reason"; Mill's are given in Bk. I, Chap. V, of his "Logic".

Of all these lists Kant's is much the worst. As to Mill's Existence and Non-existence, we shall see in the next chapter that it is not a new kind of category comparable with those which I have enumerated; for as Lotze says, "to exist is to stand related". In other words, 'Existence' is a general term applied to whatever has any of the particular relations specified. To exist and to have a place in the world of related things are one and the same.

The unity of an animal body involves also the relation of means and end, since all the parts coöperate.

The relation of Number corresponds closely with that of whole and part. Until a given whole, such as the distance from Cleveland to Buffalo, is split up mentally into a series of parts each of which is regarded as a whole, we cannot express it in numbers, *e.g.*, 61 leagues, 183 miles; until I distinguish between the units in 'a mass of humanity' (as the newspapers sometimes say) or in a flock of sheep or a cloud of dust, I cannot count them. The units, of course, can be chosen perfectly arbitrarily: leagues, miles, kilometers, persons, couples, families, dozens, pounds. But they must be similar as well as homogeneous; I cannot add persons and miles, and if I wish to add leagues and kilometers I must reduce one to the other. Having settled upon our similar units, we do not get the idea of any definite number until we count them, and this is possible only when they are so arranged as to be perceptible separately, that is, only when they are a certain distance apart in space, like dots on a page or the sides or angles of a figure; or in time, like successive strokes of a bell or throbs of pain.

Thus number, like whole and part, is primarily a relation of space or time. We can distinguish a triangle from a square without counting the sides, because they do not look alike. Similarly any single object looks different from a group of two or more, and groups of two, three, and four look different from each other. Similarly also three strokes of a bell sound different from one or two. The higher numbers we understand largely through these lower ones. To count is thus to tell something about the spatial or temporal appearance the objects in question can be made to present.

All propositions are divided by some writers into *pure* and *modal*. Modal propositions contain some word or phrase to intimate "the degree of certainty or probability with which a judgment is made and asserted" (*e.g.*: He will *probably*

come, it will *certainly* rain, *perhaps* he is here). Pure propositions do not. Of modal propositions we shall have something more to say. For the present we may regard the modal element as merely noetic, equivalent to some such sentence as 'I think so', 'I am sure of it', 'I am doubtful about it'.

Some writers who speak of modal propositions include all those which contain an adverb, *e.g.*: 'He acts *clumsily*', 'he goes *quickly*'. Such adverbs, however, are mere completions of the verb, and in many cases they can be avoided altogether by using a verb which already contains their meaning, *e.g.*: 'He blunders', 'he hastens'. Whether one word or a dozen is necessary to tell the precise relations of the object in question is a mere accident of language, and no logical distinction should be based upon it.

CHAPTER VII.

WHAT PROPOSITIONS IMPLY ABOUT EXISTENCE.*

When we say that a thing is green or large or terrible, do we necessarily imply that it *is*, or exists? To put the question in its most general form, does the copula 'is' or does any proposition imply the existence of the things whose relations it affirms or denies?

In discussing this question it is necessary first of all to distinguish between the subject of a relation and what is named in the subject of a sentence, for they are not always identical. It is thus possible to believe that every relation involves the existence of something related without being forced to conclude that every proposition assumes the existence of what is named in its subject. By way of illustration let us quote some sentences from Keynes: "The following may be given as examples of universal propositions, which need not be regarded as implying the existence of their subjects: No unicorns have ever been seen; All candidates arriving five minutes late are fined one shilling; Who steals my purse steals trash; . . . Every body not compelled by impressed forces to change its state, continues in a state of rest or of uniform motion in a straight line. . . . We may make the first of the above assertions without intending to imply that unicorns exist unseen; the second does not commit us to the prophecy that any candi-

The two 'subjects'.

* See the excellent chapter in T. N. Keynes' "Formal Logic". Macmillan & Co.

dates will arrive five minutes late; and similarly for the remaining propositions." *

The grammatical subjects of these propositions are the terms 'Unicorn', 'Candidates arriving five minutes late', 'He who steals my purse', and 'Every body not compelled by impressed forces to change its state', and Keynes is certainly right in saying that the propositions in question do not imply the existence of any of the things named by these terms.

It is true, nevertheless, that the propositions do imply the existence of something; and this something is the subject of the relations expressed. When we say that no unicorns have ever been seen we mean that no human being has ever had the experience which we call the perception of a unicorn, and we take for granted the existence of human beings. The statement about candidates arriving late means that if any one should appear before a certain board as a candidate but should not arrive on time, he would be fined. It assumes the existence of the members of the board and of persons who may wish to be candidates, and points out certain causal relations which may arise between them. Iago's words do not assume the existence of some one who steals his purse; but they do assume the existence of people who have purses and of other people who might steal them if they could. Newton's law of motion does not assume the existence of bodies not acted upon from without; but it does assume the existence of material bodies, and helps to explain their nature and their movements by telling what any of them would do if they were not acted upon from without.

Thus when we distinguish between the subjects of a relation and the things denoted by the subject of a proposition, or, better, when we distinguish between the things we are really talking about and the things that the structure of a sentence sometimes makes us seem to be talking about, it is easy

* Pp. 201–2, third edition.

enough to see not only that the copula 'is', but that every proposition, regardless of the copula, implies the existence of that whose relations it discusses.

The last statement holds true not only of affirmative but of negative propositions and even of those in which there is a downright denial of existence.

Denials of relations and of existence.

Negative propositions as such are easily disposed of. When we deny that a thing has such and such qualities we usually assume that it exists and possesses other qualities incompatible with the first. If any one should assert that John Smith was not good-natured we might assume that he was more or less morose. We should usually assume that he did exist. Sometimes, however, we say that a thing has not certain qualities because it does not exist at all; *e.g.*, 'A snark is not terrible'; 'ghosts are not to be feared'. Here the negative proposition amounts to one in which existence is denied.

But even when we say that something does not exist, our statement is really one concerning what does exist. To illustrate what I mean let us examine a peculiar but important passage in Herbert Spencer's "First Principles" (Pt. 2, Chap. 4). Mr. Spencer thinks that the lately discovered law of the conservation of matter is one which no rational being ever seriously doubted, even though he supposed himself to do so. He bases this paradox on the conviction that nobody can possibly succeed in thinking of *nothing*. Let us see "what happens", says Mr. Spencer, "when the attempt is made to annihilate matter in thought. . . . Conceive the space before you to be cleared of all bodies save one. Now imagine the remaining one not to be removed from its place, but to lapse into nothing while standing in that place. You fail. . . . It is impossible to think of something becoming nothing, for the same reason that it is impossible to think of nothing becoming something—the reason, namely, that nothing cannot become an object of consciousness."

From this argument about the impossibility of thinking of nothing, Mr. Spencer believes he has proved that no one can possibly think of a single atom of matter either beginning or ceasing to exist. We must therefore, according to the argument, think of the world and every atom in it as eternal, uncreated, and indestructible.

Now, I believe that Mr. Spencer is quite right in saying that we cannot think of nothing (a very different thing from not thinking at all). Everything we imagine or think of we tend to think of as existing, and as long as our thought is concerned wholly with any given object we cannot possibly think, though through force of habit we may perhaps speak, of that object as non-existent. But then Mr. Spencer overlooked the fact that when we assert a thing's non-existence our thought, so far as we have any, is a thought, not of the thing, but of the empty background where the thing might have been. He is right enough in saying that we cannot imagine a non-existent body *as* non-existent, but we certainly can think, whether we believe it or not, of God existing in a worldless void at one instant and with worlds about him at the next, or of a universe enriched or impoverished by the addition or loss of some speck of dust or of some whole world. Certainly every child has seen plants grow without thinking of the nourishment they appropriate, and seen them burnt up without thinking of the smoke in which their elements are preserved.

The way we think of non-existence is well illustrated in the nursery rhyme:

> Old Mother Hubbard
> Went to the cupboard
> To get her poor dog a bone.
> But when she got there
> *The cupboard was bare,*
> And so the poor dog got none.

A bare cupboard, a disappointed woman, and a hungry dog! Here is a vivid picture of the bone's absence, but not a word

about the bone itself! Our attention is turned not to the bone, but to the empty background. "As for man, his days are as grass; as a flower of the field, so he flourisheth, for the wind passeth over it and it is gone, and *the place thereof shall know it no more.*" Here again it is the empty place, not the grass and the flower, that we must think of in order to get the full sense of their annihilation.

To say that a thing does not exist, means therefore that the world or whatever other reality there is exists without it and with relations other than those which its presence would have involved.

Thus we think, not of what is not, but of what is; and whatever we think of, we think of as existing in some way or other.

This whole question of existence can be made clear by tracing the origin and growth of the distinction between realities and illusions or fictions. Young children cannot make this distinction. Savages make it very imperfectly, and even adult members of a civilized society often fail in the effort to apply it.

The conception of reality.

It sometimes happens that a person hears his name called, looks around, can see nobody, and finally concludes that the sound was imaginary. At first the sensation of sound carried with it an instinctive belief in the reality of something beyond the hearer which made it. Every sensation does this. If it did not we should never be able to perceive *things* at all. Instead of saying, 'I hear some one speak', 'I see the sun', 'I smell a rose', 'I feel the ground', 'There is a mouse'; we should only be able to say, 'Lo, a feeling like the sound of words!' 'Lo, a vivid sensation of sight!' 'Lo, a sweet smell!' 'Lo, a touch-feeling!' 'Lo, a succession of peculiar visual and auditory feelings!'

Thus every conception of reality which we have depends ultimately upon our instinctive tendency to interpret feelings in terms of things acting upon us—to say, not that such and such a feeling is now going on, but that such and such an

object is now present. From this tendency to objectify our experience we can never wholly escape.

If when we had sensations we did nothing more than to refer them vaguely to something or other acting upon us, we should never be able to detect an illusion. As a matter of fact we do much more, for we build up as soon as we can the conception of a large number of definite objects, acting or disposed to act in definite ways upon each other and upon us. We learn that sounds and smells come from objects that can be seen and handled, and we expect floors to support us, food to taste good, and the people around us to be pleased or annoyed, as the case may be, at a given kind of conduct. We gain, in short, a knowledge of the nature and relations of a great many things. Moreover, we expect each thing to appear and act at one time as it did under similar circumstances at another. If we did not, there would be no meaning in the distinction between various kinds of things.

It is by means of such knowledge of things and the way they act that we are able to correct our first impressions and distinguish between that which we have experienced and that which we have only imagined. If we hear a voice but see no one, we conclude that we were mistaken about the voice, because it is easier to discredit the testimony of a single sense on a few occasions than to discredit our conviction that names are not called in the absence of a visible and tangible person who calls them. What is true of sensation is true of all thought. What we think about, whether it be an ink-bottle or a dragon, is thought about, for the instant at least, as though it were real, and if we afterwards deny its reality, this is because our thought has turned from the object itself to a wider system of things in which we find that it has no place.

We have just seen how we conceive of every object of thought as real until we find that it will not fit into a wider system of things. The ultimate and highest test of individual facts would be, therefore, a thoroughly consistent and well-established conception of the whole material and spirit-

ual universe. But this is something which nobody possesses. As a matter of fact we seldom think of such a universe at all.

Knowledge comes at first in disconnected patches. These gradually grow together and are combined into larger fields. Within each field our conceptions are moderately consistent, but we rarely think of the relations between various fields, or test our conceptions of one by comparing it with another. Sunday-school stories, Greek mythology, German fairy tales, novels, histories, science, theology: these are all more or less consistent within themselves and inconsistent with each other. We live as it were at different times in different worlds, each dominated by its own fundamental laws. We do not expect to find cherubim and archangels on Olympus or muses "on the secret top of Oreb or of Sinai", nor do we usually think of Hamlet, Solomon, and Cinderella meeting together beyond the Styx. As long as we keep our worlds apart, each seems real; the more vividly we picture it the more real it seems; and the impression of reality lasts until we compare two inconsistent worlds together. I think a good illustration of this is to be found in Kipling's story of "Mowgli's Brothers". After telling how a little naked man-cub toddled into a wolf's den and was adopted by Mother Wolf, the author explains what Mother and Father Wolf must do in order to have the adoption legally recognized and ratified by the Pack.

"The Law of the Jungle lays down very clearly that any wolf may, when he marries, withdraw from the Pack he belongs to; but as soon as his cubs are old enough to stand on their feet he must bring them to the Pack Council, which is generally held once a month at full moon, in order that the other wolves may identify them. After that inspection the cubs are free to run where they please, and until they have killed their first buck no excuse is accepted if a grown wolf of the Pack kills one of them. The punishment is death where the murderer can be found, and if you will think for a minute you will see that this must be so."

Now, in the constitution of jungle society as it is pictured in the story, there is absolutely no reason why "this must be so", and so when we "think for a minute" we have to seek for the reason in what we know in other ways of wild beasts and their habits; and, doing this, we suddenly see the gap between the every-day world and the world of Kipling's fancy, and realize how great is the fiction that we have been treating as real.

It is, of course, by our every-day world, the world of greatest coherence and most importance for us, that we test all others, and for most civilized adults nowadays that world is the world of physical and historical science. But it must not be forgotten that the scientific conception of the world is a very new one. In the middle ages the every-day world, the world of greatest coherence and most importance, was the world of heaven and hell, God and the devil. Beside it the earth and mundane affairs were as visions and empty dreams.

By history we test the truth of stories, but history is of more recent origin than physics. It is one great story consistent with itself and with physical science and comprehending and explaining as many shorter stories as possible. Before it was told the truth of self-consistent shorter stories or sets of stories could not be questioned. Any tale seemed as true as any other if it appealed strongly enough to the imagination and emotions of the hearer. It is because they know little science and history that children and savages distinguish between truth and fiction so imperfectly.

To ask whether some object really exists or whether some story is true implies the possession of some accepted system of things by which smaller or less vital systems can be tested. If a smaller system is found to agree with the larger a deliberate conviction based upon this agreement is added to the spontaneous and naïve conception of its objects as real. The objects are now thought of as real in a new and deeper sense. But it must not be forgotten that the distinction

between the two kinds of reality is simply one of degree, and that the conception of the reality of the larger system of things which we use as a test grows itself out of the same spontaneous tendency to objectify our impressions that accounts for the more fragmentary systems which we have tested by means of it.

When we discuss the nature of centaurs and dragons we treat these creatures as real in the sense that we imagine them before us with perceptible qualities and relations. We afterwards deny their reality in the sense that we seek in vain to find a place for them in the wider scheme of consistently related things which we have come to regard as alone of vital interest. Unless we took the wider system of things for granted we could not test the narrower. Thus to deny the existence of one thing is really to say something about the relations of some wider or more certain universe whose existence we assume; and thus every proposition, whether its copula is some part of the verb 'to be' or whether it is something else, implies the existence of something, though not necessarily of the object described in its subject.

CHAPTER VIII.

THE FORMAL CHARACTERISTICS OF PROPOSITIONS.

In every proposition something is either asserted or denied of a given object more or less definitely pointed out. When something is asserted the proposition is called *affirmative* (*e.g.*, 'Dogs like meat', 'Iron is a metal'), when denied, *negative* (*e.g.*, 'Dogs do not like meat', 'Iron is not a metal'). The character of a proposition as affirmative or negative is called its *quality*.

Quality and quantity.

When a proposition states something about some one definitely designated object it is called *singular*, *e.g.*, 'Socrates was flat-faced', 'My dog is not savage', 'The last man in the row is my cousin'.

When it states something about every member of a designated group of objects it is called *universal*, *e.g.*, 'All men are mortal', 'No Spanish-American state has a stable government', 'Every event has a cause'.

When it states something about some undesignated or imperfectly designated member or members of a given group of objects it is called *particular*, *e.g.*, 'Some of the American races were highly civilized', 'One of the men in the row is my cousin', 'Some dogs are not savage', 'A certain man had two sons'.

The term 'particular' is here used in a peculiar sense, quite contrary to its ordinary meaning, for particular propositions are the only ones which do *not* give information about particular or definitely designated objects. If a gen-

eral tells his officers that one of them has blundered, each can ask: Do you mean me? But if he uses a singular proposition and says that Captain Jones has blundered, or a universal and says that they have all blundered, the question is no longer possible. The term 'particular' as used in logic is derived directly from the Latin *particula*, a particle, and has been applied to certain propositions merely because, unlike universals, they refer to only part of the class or group of objects mentioned.

Singular propositions are usually classified with universals because they point out the object spoken of in the same definite way, and because for most logical purposes definiteness of reference is much more important than the number of objects to which we refer.*

The character of a proposition as universal, singular, or particular is called its *quantity*.

If we neglect singular propositions and consider the various combinations of quantity and quality in universals and particulars there are four kinds of propositions, each of which it is customary to denote by one of the following symbols:

A. Universal affirmative, as 'All S's are P'.

E. Universal negative, as 'No S is P'.

I. Particular affirmative, as 'Some S's are P'.

O. Particular negative, as 'Some S's are not P'.

The symbols A and I are respectively the first and the second vowels in the word *affirmo*. E and O belong in the same way to *nego*.

The reader should notice that the proposition 'All S's are

* This is the real reason; but it is not always the reason given. Whately, for example, says that singular propositions are to be treated as universals, because they tell something 'about the whole of the subject'. When, for example, we say that Brutus killed Cæsar, we are speaking about *the whole of Brutus*. This is of course absurd. We should speak just as much about the whole of Brutus if we said that a certain Roman killed Cæsar. It is not a question of *how much?* but of *who?* or *which?*

not P' usually means 'It is not true that all S's are P', *i.e.*, 'Not all S's are P', or 'Some S's are not P'. It is therefore O, not E. When we realize that expressions of this sort are ambiguous we should try hard to avoid using them. When we find them used by others who may not have recognized their ambiguity we should try to interpret them according to the real meaning of the speaker—if he had a definite meaning—and not according to any arbitrary rule. If it is impossible to tell what his real meaning is, we should make it plain that this is the case. The one thing that we should certainly not do is to allow such expressions to pass without question. If we do so they are likely to be taken in one sense at one time and in another at another; and thus to lead us to conclusions which we really have no right to reach or to disputes for which there was really no occasion.

Ambiguities of quantity or quality.

The word 'few', as Jevons has pointed out, must be interpreted with care; "for if I say '*few books* are at once learned and amusing', I may fairly be taken to assert that *a few books* certainly are so, but what I really mean to draw attention to is my belief that '*the greater number of books are not* at once learned and amusing.' A proposition of this class is generally to be classed rather as O than I".*

Whether the word *some* means *some but not all* or *at least some, perhaps all,* depends largely upon the scientific training of the speaker. Like the fish that bites at every wriggling object and the baby that grasps everything within its reach regardless of possible burns or cuts, we all tend to generalize too carelessly. When a confiding boy leaves home he is likely to take it for granted that every one is trustworthy—proposition A; because of his credulity he is soon cheated, and then like David in his wrath he may say that all men are liars, *i.e.*, that no one is trustworthy — proposition E. Soon, however, he gets a letter from home or is befriended

* "Elementary Lessons in Logic", p. 67.

by an old acquaintance and he qualifies his sweeping condemnation. "*Some* people are trustworthy", he now says, implying that all but a chosen few are unreliable. After a while, however, he learns the value of cautious statements, and if he should then go to a new place and be fortunate in his first acquaintances he might say "Some people here are trustworthy", implying nothing whatever about the rest except, perhaps, that he did not know them.

In short, the development of particular propositions is a mark of increasing caution and accuracy. The end which they serve is therefore defeated, at least in part, when they are understood to imply more than they state. Often they do imply more—but merely because a speaker or hearer is not yet sufficiently well trained to realize that a qualified statement can be made on general principles. Here again one must take care to speak in such a way as to be understood aright by his hearers (whoever they may be), and to try and find a definite interpretation for the statements of others which they themselves would accept.

Undesignated quantity or quality.

Instead of being ambiguous, words used to denote quantity and quality are often lacking altogether. Sometimes the meaning is clear enough without them; but sometimes it is not.

The term *indefinite*, *indesignate*, or *preindesignate* is applied to propositions whose form does not show whether they are intended to be universals or particulars. When, for example, we say that dogs delight to bark and bite, or that republics are more free than monarchies, or that crows are black, do we mean the statement to apply to every dog, every republic, every crow, or only to some larger or smaller number of them? Indefinite propositions must not be confused with particulars. The quantity of a *particular* proposition is perfectly clear, but the object to which it refers is not definitely designated. The quantity of an *indefinite* proposition, on the other hand, is not clear, and for that reason it is impossible to say whether it is meant to refer to any

definitely designated objects or not. Particular propositions are indefinite in their reference; indefinite propositions, ambiguous.

When the form of a proposition gives no indication of its quantity it is very easy to accept it or prove it true when interpreted as a particular and then use it as though it were true as a universal. Here is an example: "Improbable events happen almost every day; events which happen almost every day are probable events; therefore improbable events are probable events." When the first premise of this argument is assumed to be true it is evidently understood to mean that some improbable event or other happens almost every day; and from it in conjunction with the other premise we have the right to conclude that the occurrence of some improbable event or other is probable, but nothing more. When the premise is understood in this sense throughout it certainly will not help us to conclude that improbable events are probable events. It is capable, however, of being interpreted to mean that every improbable event happens almost every day; and when taken in this latter sense it certainly will help us to reach the conclusion. Every improbable event happens almost every day; events which happen almost every day are probable events; therefore every improbable event is a probable event. But, then, when the premise is understood in this sense nobody would admit its truth.

We must not assume, however, that the quantity or quality of a proposition has not been indicated merely because it has not been expressed formally by one of the words 'some', 'all', 'none', 'not'. Minto puts the matter as follows:

"The expression of Quantity, that is, of universality or non-universality, is all-important in syllogistic formulæ. In them universality is expressed by *all* or *none*. In ordinary speech universality is expressed in various forms, concrete and abstract, plain and figurative, without the use of 'all' or 'none'.

Uneasy lies the head that wears a crown.
He can't be wrong whose life is in the right.
What cat's averse to fish?
Can the leopard change his spots?
The longest road has an end.
Suspicion ever haunts the guilty mind.
Irresolution is always a sign of weakness.
Treason never prospers.

* * * * * *

"All the above propositions are 'Pre-designate' [*i.e.*, definite] universals, and reducible to the form All S is P, or No S is P.

"The following propositions are no less definitely particular, reducible to the form I or O [*i.e.*, Some S is P, or Some S is not P]. In them, as in the preceding, quantity is formally expressed, though the forms used are not the artificial syllogistic forms:

Afflictions are often salutary.
Not every advice is a safe one.
All that glitters is not gold.
Rivers generally run into the sea.

"Often, however, it is really uncertain from the form of common speech whether it is intended to express a universal or a particular. The quantity is not formally expressed. This is especially the case with proverbs and loose floating sayings of a general tendency. For example:

Haste makes waste.
Knowledge is power.
Light come, light go.
Left-handed men are awkward antagonists.
Veteran soldiers are the steadiest in fight.

"Such sayings are in actual speech for the most part delivered as universals. It is a useful exercise of the Socratic kind to decide whether they are really so. This can only

be determined by a survey of facts. The best method of conducting such a survey is probably (1) to pick out the concrete subject, 'hasty actions', 'men possessed of knowledge', 'things lightly acquired'; (2) to fix the attribute or attributes predicated; (3) to run over the individuals of the subject class and settle whether the attribute is as a matter of fact meant to be predicated of each and every one.

"This is the operation of Induction. If one individual can be found of whom the attribute is not meant to be predicated, the proposition is not intended as universal.

"Mark the difference between settling what is intended and settling what is true. . . .

"The bare forms of Syllogistic are a useless item of knowledge, unless they are applied to concrete thought. And determining the quantity of a common aphorism or saw, the limits within which it is meant to hold good, is a valuable discipline in exactness of understanding." *

Double quantity.

When the function of a proposition is, not to describe some one object or set of objects, but to tell of a causal or other relation which exists between several (*e.g.*, John strikes James; David defeated the Philistines), it is rather arbitrary to determine the quantity of the proposition with reference merely to what happens to be named in the subject. It would be fairer to recognize both parties to the relation, and to determine the quantity of the proposition with reference to each. 'Each of these hunters shot a bird' is a universal proposition with reference to the hunters, but particular with reference to the birds. 'Almost any Turk hates a Greek' is particular with reference to the Turks, universal with reference to the Greeks. 'All Turks and Greeks hate each other' is universal with reference to both. 'There are many thieves in the land' is particular with reference to the thieves, singular with reference to the land.

* William Minto, "Logic, Inductive and Deductive", Scribners, 1895, pp. 70–73.

Let it be remembered, therefore, that though it may sometimes be convenient to describe the quantity of such a proposition with reference to only one of the related parties, such a description is both arbitrary and incomplete.

There are two closely allied kinds of propositions, much harder to define than to deal with in practice, called respectively *exceptive* and *exclusive*. The subject of each kind contains some such limiting phrase as *none but, only, alone, except;* and on this account they are often confused, in spite of a real contrast between them.

Exclusives and exceptives.

Exceptive propositions state that something is true of all the members of a given group of objects except those specified: *Exclusive* propositions, on the other hand, state that something is true of certain specified members of a group only.

Here are some examples of the two:

Exceptive affirmative: All but the Germans departed.
Exclusive " : The Germans alone departed.
Exceptive negative: No one but the Germans departed.
Exclusive " : The Germans alone did not depart.

Exceptive affirmative: All but the brave deserve the fair.
Exclusive " : The brave alone deserve the fair.
Exceptive negative: None but the brave deserve the fair.
Exclusive " : The brave are the only ones who do not deserve the fair.

It will be noticed that each kind of proposition can be either affirmative or negative,* and that the exclusive affir-

* The fact that each kind of proposition can be either affirmative or negative is overlooked in some of the text-books. Jevons, for example, assumes in the following definitions that both kinds must be affirmative: "Exceptive propositions affirm a predicate of all the subject with the exception of certain defined cases, to which, as is implied, the predicate does not belong." "Exclusive propositions contain some words, such as *only, alone, none but*, which limit the predicate to the subject." Where a predicate is 'limited to a subject', it is certainly affirmed and not

mative has the same meaning as the exceptive negative, and *vice versa*.

In the first set of examples above given there can be no doubt about the meaning: in each case we are told that the Germans did one thing and that the others did the other. In the second set, however, the meaning is not so clear. When we say that all but the brave deserve the fair, or, to make the example less unnatural, all but the brave deserve to die, or that the brave are the only ones who do not deserve to die, do we mean that every brave man deserves to live, or merely that so far as courage is concerned, brave men do not deserve death? In the latter case a brave man might deserve it on other grounds. So when we say that

denied of the subject. Both definitions thus assume that the propositions are affirmative. As examples of exclusive propositions Jevons gives "Elements alone are metals" and "None but elements are metals". He states that they are equivalent and assumes that they are both affirmative. As an example of exceptive propositions he gives "All the planets except Venus and Mercury are beyond the earth's orbit". But suppose that instead of affirming this we should deny all the facts asserted, the proposition would then read: None of the planets except Venus and Mercury are beyond the earth's orbit. This form is precisely identical with "None but elements are metals", which Jevons regards as an affirmative exclusive proposition. The form is clearly exceptive and clearly negative, and there is no reason why the proposition should be regarded as either exclusive or affirmative, unless the distinction between the two kinds of proposition is abolished altogether. This is actually done by Minto, as follows: "The formula for EXCLUSIVE PROPOSITIONS. 'None but the brave deserve the fair'; 'No admittance except on business'; 'Only Protestants can sit on the throne of England'. These propositions exemplify different ways in common speech of naming a subject *exclusively*, the predication being made of all outside a certain term." (P. 76.) The trouble with this description is that where the subject is 'named exclusively', as in the example about Protestants, the predication as it stands is not made about 'all outside the term', but about those inside it. On the other hand, when the predication is made about 'all outside the term' as in the two other examples, the subject is not 'named exclusively'; for that which is named is not the subject.

the brave alone deserve the fair or that none but the brave deserve the fair, do we mean that every brave man deserves a fair wife no matter what he may be in other respects, or merely that to deserve one he must at least be brave?

The first set of propositions are unambiguous because they are purely historical statements about certain individuals as such. The second are ambiguous because they express conditions about kinds of objects and they do not make it plain whether the condition mentioned is or is not the only one upon which the case depends.*

* Exclusive and exceptive propositions can be varied a good deal in quantity. When we say The Germans alone remained we (1) specify clearly the smaller group (of Germans as distinguished from the rest of the persons involved, and (2*a*) say something about each member of the specified smaller group (they remained) and (2*b*) about each of the rest (they did not remain). The proposition is thus in every possible respect universal.

When we say The brave alone deserve the fair, we (1) distinguish clearly enough between our groups, (2*b*) we say something about all who are not brave, and (2*a*) if we are interpreted as saying anything at all about those who are brave,—namely, that they have complied with one condition—we say it about all of them. This proposition is thus also universal in every respect.

When we say Some of the Germans were the only persons who remained, we still (1) specify the smaller group clearly and (2*b*) still say something about all the persons outside of it; but (2*a*) the individuals within it of whom we speak are no longer definitely designated. The proposition is thus in one respect particular. When we say The Germans and some others alone remained, 1 is still definite, 2*a* is universal, and 2*b* particular. When we say Some of the Germans and some of the others alone remained, the groups are still clearly distinguished, but the original proposition is broken up into two exclusives, each particular in so far as it fails to specify the distinction between those who did and those who did not stay. Each of these exclusives is equivalent to both I and O: some stayed and some did not.

When we say The soldiers of one nation alone remained, a statement is made (2) about each member of each group. To this extent the proposition is universal. But as (1) the smaller group is no longer definitely specified, the proposition is in this respect particular.

The precise meaning as to quantity of an exclusive or exceptive proposition, like that of any other, may be indefinite.

So far we have been dealing with what are called *categorical* propositions; those in which something is, or at least seems from the form of the proposition to be, stated without alternative and without condition. Propositions in which it is affirmed that one or other of several alternative states of affairs exists are called *disjunctive* or *alternative*, *e.g.*, Every man is either married or single; He is either a fool or a knave; Either he is a knave or I have been grossly deceived; Either A or B did it; He is either not here or not there.* Propositions in which it is affirmed that if some specified state of affairs exists another specified state of affairs also exists are called *hypothetical*,† *e.g.*, If he is not a fool he is a knave; If he is a knave I have been grossly deceived; If he is not in the room he is not in the house.

Disjunctives and Hypotheticals.

The part of a hypothetical proposition which specifies the condition, either of something being so, or of our knowing it, is called the *antecedent*, the part which specifies what follows from that condition is called the *consequent*.

Disjunctive propositions state that one of two things must be true; but do they imply that both cannot be true? This question has been discussed at much length. If a man is married he cannot possibly be single. We know this from the nature of things, but there is no reason in the nature of

* A negative proposition asserts the existence of a state of affairs just as much as an affirmative.

† "This is the familiar form of the disjunctive judgment. . . . It is usual to mention along with it the *copulative* judgment ('S is both p and q and r'), and the *remotive* judgment ('S is neither p nor q nor r'); but in spite of the external analogy of form, neither of these has the same logical value as the disjunctive; the first is only a collection of positive, the second of negative, judgments with the same subject and different predicates, which latter are not placed in any logically important relation to each other. The disjunctive judgment alone expresses a special relation between its members: it gives its subject no predicate at all, but prescribes to it the alternative between a definite number of different predicates." Lotze, "Logic", § 69. (Clarendon Press.)

things why a person cannot be both fool and knave. When it is asserted that he is either one or the other, is it necessarily implied by the form of the statement that he is not both? Fowler says: "It seems to me that in the expression 'either — or —' we distinctly exclude the possibility of both alternatives being true, as well as of both being false. In fact, when we do not wish to exclude the possibility of both being true, we add the words 'or both', thus: 'He is either a fool or a knave, or both'; 'I shall come either to-day or to-morrow, or perhaps both days'." * With this view Thomas Aquinas, Kant, Hamilton, Boole, Bradley, and others agree. Whately, Mansel, Mill, Jevons, Keynes,† and others maintain on the other hand that such propositions merely mean that both alternatives cannot be false, though both may be true. Says Keynes: "Suppose it laid down as a condition of eligibility for some appointment that every candidate must be a member either of the University of Oxford, or of the University of Cambridge, or of the University of London. Would any one regard this as implying the ineligibility of persons who happened to be members of more than one of these universities?"

The question is, of course, one of the interpretation of language, not of logical processes. So far as logic is concerned any one is at liberty to use language in any sense he pleases, provided that he explains beforehand the sense in which he means to use it; but since there is a real difference in usage it seems to me better in this case, as in the case of the word 'some', to assume that the words are used with the greatest caution and imply nothing but what is stated. Let us, therefore, agree, at least for the purposes of this book, that when we say that one or other of several alternatives is true we do not necessarily imply that both cannot be true, though of course we do imply that both cannot be false.

* "Deductive Logic", p. 118, Ninth Ed. (Clarendon Press).

† See Jevons, "Principles of Science", p. 68, and Keynes, "Formal Logic", § 140.

Like exclusive and exceptive propositions, hypothetical and disjunctive propositions are different in form and must be distinguished from each other, though they can be made to express the same meaning. The following table shows the relations between them:

Disjunctive.	*Hypothetical.*
A is either B or C	= If A is not B, it is C.
	= If A is not C, it is B.
A is either not B or not C	= If A is B, it is not C.
	= If A is C, it is not B.
A is either B or not C	= If A is not B, it is not C.
	= If A is C, it is B.

In each case there are two hypothetical propositions, either of which is equivalent to the disjunctive, and each of which is exactly equivalent to the other. To say If A is not B it is C means precisely the same thing as to say If A is not C it is B; and so with the rest.

If any one were asked the use of disjunctive and hypothetical propositions, the first answer that occurred to him would probably be: To express knowledge combined with doubt. To use Venn's illustration, if I say that A.B. is either a barrister or a solicitor, I express my knowledge that he is a lawyer and my doubt as to his precise standing at the bar. The same thought would be expressed in the hypothetical proposition, 'If he is not a barrister he is a solicitor'.

But disjunctive and hypothetical propositions are not always used to express doubt. When, for example, we say that in the United States every person is either married or single, the statement does not express the slightest doubt as to the condition of any given individual in this respect. Its real force is to explain the laws or social customs of the country, under which a person is regarded as single until some prescribed condition has been fulfilled, and then as married. The statement would hardly hold of an oriental

society in which concubinage was recognized. Such propositions, therefore, express knowledge, not ignorance; but it is a knowledge of the laws or general conditions of existence prevailing in any sphere or 'universe', not of the precise state of some particular individual in that universe.

In the example just discussed the proposition affirming the existence of a general law happened to be disjunctive. It is more common to affirm such laws in hypothetical, *or even in universal categorical propositions, e.g.*, If a man is insulted he becomes angry, or Insulted men become angry; When it rains hard the streets are wet, or Hard rains wet the streets; The nearer bodies get together the more they attract each other, or Contiguous bodies attract each other more than those that are farther apart.*

* On p. 87 there are examples taken from Keynes of several other universal propositions of this kind. Such propositions, as we there saw, do not necessarily imply the existence of things as they are described in the grammatical subjects of the propositions; but they do imply the existence of a universe whose laws they more or less accurately express. There doubtless are universal propositions founded upon direct observation of the things named in them and intended to imply the existence of those things as well as to describe them; *e.g.*, None of the Stuarts were good sovereigns; Each of the United States contains colored citizens. Such propositions cannot be put into hypothetical form. But universals arrived at by deductive reasoning, or reasoning from general considerations, are probably always capable of being put into hypothetical form and seldom or never necessarily imply the existence of the things described by their subjects, though they probably do imply the existence of the things named in the equivalent hypothetical propositions, *e.g.*, Seniors are wiser than Sophomores; Every husband has a wife. Turned into hypothetical form these propositions would run: If a person belongs to the Senior class he is wiser than if he belonged only to the Sophomore class. If a man is married, he has a wife. The general consideration in the first of these examples lies in the supposed law of the college universe, that two more years of college life must add something to one's wisdom. It is a statement which will be as valid as it is now as long as colleges and human nature remain what they are. It is not concerned specially with the present, the past, or the future existence of Seniors and Sophomores and colleges, the present term of the verb *to be*, like the phrase *must be*, being used in a perfectly

timeless sense. The proposition merely states the effects supposed to result from certain causal agencies whenever and wherever they may be supposed to exist. The statement that every husband has a wife is based upon a similar consideration of the nature of things. We mean by a husband a man who is married, and we know perfectly well that as the world is constituted men can marry only women, that is, wives. It is this general fact which the proposition expresses; it does not necessarily imply that any one is married.

Particular propositions, unlike universals, are not usually deduced from general considerations; though sometimes they may be: *e.g.*, 'Some Sophomores *must be* wiser than the average Senior'. As a rule, however, particular propositions are based upon the direct observation of individuals to which we are forced to resort when general considerations are inapplicable, and they naturally imply the existence of the individuals observed. The propositions Some Sophomores are wiser than some Seniors, Some husbands are not happy, do not lay down general laws of the universe or state the effects that certain causes necessarily produce. For this reason they would hardly ever be put into disjunctive form, for though such a form is possible in this case it is not very clear, and has no special value when the implication of general law is omitted; *e.g.*, A student either is not a Sophomore or is a member of a group of persons some of whom are wiser than some Seniors. Put into hypothetical form particular propositions have considerable significance, for they serve to deny the existence of the kind of law that universals of opposite quality assert; *e.g.*, If a person is a Sophomore he *may be* wiser than a Senior. This is the hypothetical form of the particular proposition 'Some Sophomores are wiser than some Seniors'. They both *serve to deny the universal law* expressed in the universal categorical proposition, 'No Sophomore is wiser than a Senior', or in the hypothetical proposition, 'If a student is a Sophomore he is not as wise as a Senior'.

Universal laws expressible in the above forms can also be expressed by the phrases *must be*, *are necessarily*, etc., and denied by the phrases *need not be*, *are not necessarily*, etc.

CHAPTER IX.

THE OPPOSITION OF PROPOSITIONS.

In the last two chapters we dealt with the deeper interpretation of propositions. We must now discuss another question which is practically one of interpretation, but which is not at all deep. Such a discussion is important enough to be found in all the text-books of logic; and yet the only end which it serves is to force the reader to think about the obvious meaning of his words and to show him how easy it is to make foolish blunders when we rattle off words without thinking.

The 'Opposition of Propositions', as the phrase is used in logic, means merely the mutual implications of propositions which differ in quantity or quality or both. To be 'opposed' in this sense it is therefore not necessary that two propositions should be inconsistent. This use of the term opposition is not happy, but since it is common we must understand it.

With common propositions.

Anybody who will exercise a little patience ought to be able to work out an answer to this question: Assuming the truth or the falsity of one of the propositions A, E, I, and O, what can we know about the truth or falsity of the others? If it is true that all the members of the present senior class are in good health (proposition A), is it true or false that some of them are in good health (proposition I), that none of them are in good health (proposition E), that some of them are not in good health (proposition O)?

For the sake of helping the reader to verify his reasoning I shall give a table showing the relations of the various cate-

gorical propositions. This table, like any other that may be found in a text-book of logic, ought to be understood, but not learned by heart. The only thing in logic that ever ought to be learned by heart is a definition, for we must depend upon memory for a precise meaning of words, but even a definition ought not to be learned in this way, if there is any other way in which a person can remember and restate its precise meaning. With a logical table the case is entirely different. It is valuable only because clear thinking is required to construct it. It is not worth remembering; and to commit it to heart like a multiplication table is a pure waste of time.

If A is true,	E is false,	I true,	O false.
" E " "	A " false,	I false,	O true.
" I " "	A " doubtful,	E false,	O doubtful.
" O " "	A " false,	E doubtful,	I doubtful.
" A is false,	E is doubtful,	I doubtful,	O true.
" E " "	A " doubtful,	I true,	O doubtful.
" I " "	A " false,	E true,	O true.
" O " "	A " true,	E false,	I true.

This table is concerned only with the relations of universal propositions and particulars. It tells us nothing about the relation of either universals or particulars to propositions which tell something about some *designated* individual or class of individuals within the larger group. If we designate all propositions dealing with a designated individual or class within the larger group by the letter S we get the followng:

If A is true,	S	affirmative	is true	and	S	negative	is false.
" E " "	S	"	" false	"	S	"	" true.
" I " "	S	"	" doubtful	"	S	"	" doubtful.
" O " "	S	"	" doubtful	"	S	"	" doubtful.
" A is false,	S	"	" doubtful	"	S	"	" doubtful.
" E " "	S	"	" doubtful	"	S	"	" doubtful.
" I " "	S	"	" false	"	S	"	" true.
" O " "	S	"	" true	"	S	"	" false.

If S aff. is true, A is doubtful, E false, I true, O doubtful.
If S neg. is true, A is false, E doubtful, I doubtful, O true.*

It should be noticed that the truth of a universal proposition involves the truth of the corresponding singulars and particulars, and the falsity of a singular or particular involves the falsity of the corresponding universal, but not *vice versa.*

Of propositions which differ only in quality, if both are particular one must be true and both may be; if both are universal one must be false and both may be. Of the propositions A, E, I, O, having the same subject and predicate, it is only when they differ in both quantity and quality (*i.e.*, A and O, E and I), that one must necessarily be true and the other false. Such propositions are called *contradictories.* Universal propositions of different quality (*i.e.*, A and E), are called *contraries.*

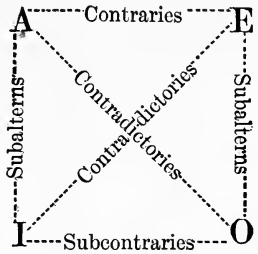
To 'contradict' a statement is to deny its truth. If you make any statement whatever (*e.g.*, that the moon is made of green cheese, that every Englishman likes roast beef), and if I say 'That is not true', I contradict you, and the important thing to notice is that if either of us is right the other is wrong, and, *vice versa*, if either of us is wrong the other is right. We cannot both be right, and we cannot both be wrong. To contradict a statement it is not necessary to say 'That is not true' in these exact words. Any statement contradicts another if the two are so related that when either of them is true the other must be false and *vice versa.* In formal logic, however, contradictory propositions are supposed also to have the same terms in the subject and in the predicate, thus: 'The moon is made of green cheese' and 'The moon is not made of green cheese', 'All Englishmen like roast beef' and 'Some Englishmen do not like roast beef'. In connection with this last example it must be

* It is often said that for logical purposes singular propositions can be treated as universals. In the present case they must be treated rather as particulars, though not precisely. In actual experience the matter presents no difficulties.

noticed that the contradictory of a universal proposition is always either a particular or a singular, and that of a particular a universal. Universal propositions of opposite quality ('All Englishmen like roast beef', 'No Englishmen like roast beef') cannot be contradictories; for while they cannot possibly both be true, they may both be false. Such propositions are always 'Contraries'.

The lesson to be learned from these facts is that guarded statements are often quite as useful as sweeping statements and much safer. If two opponents make guarded statements both may be right; if they make sweeping statements they cannot both be right, but both may be wrong; and if either of them makes a sweeping statement, the other need not make a statement equally sweeping in order to prove him wrong, for a universal proposition can be disproved by a single exception. Cautious statements may not always be very interesting, but they are not likely to be ridiculous.

The terms contrary and contradictory are the only ones in this connection which for ordinary purposes are worth remembering. There are others, however, whose meaning is made clear enough in the following traditional 'square of opposition':



A and E are each called a subalternans (active).
I and O are each called a subalternate (passive).*

* The table which I have given on p. 112 (not the square) shows the relations of the various propositions when it is taken for granted that objects of the kind described in their subjects exist. We never talk

What the logical opposites of exclusive and exceptive propositions are depends upon their interpretation. We have seen that when propositions in this form relate concrete facts about individual objects they usually imply something about the objects specifically mentioned as well as about the other members of the class in question, but that when they express some

With exclusives and exceptives.

about things that we do not assume for the moment at least to exist in some universe or other. But when a universal proposition is used as the equivalent of a hypothetical to express a general law, then the thing as it is described in the subject of the universal categorical proposition is not what we are really talking about and the proposition does not necessarily imply its existence. This has been already explained. (See p. 87.) It has also been pointed out that particular propositions usually do imply the existence of things as they are described in the subject. Particular propositions therefore imply something that one kind of universal propositions do not imply. In this case therefore the truth of the universal does not necessarily imply the truth of the particular. By way of example let us suppose that the universal proposition 'Candidates arriving late are fined' is equivalent to the hypothetical 'If any candidate arrives late he will be fined'. This statement may be true as stating a rule of the board whether any candidate happens to be late or not. But if the particular proposition 'Some candidates, or some of the candidates, arriving late are fined' means that there are candidates arriving late and some of them are fined; then this statement is not true unless there are candidates arriving late. It may thus be possible that the universal proposition is true when the particular is false.

Similarly if particular propositions imply the existence of candidates arriving late and there are not any, the propositions I and O will both be false at the same time, as they cannot be when the existence of the objects named is taken for granted throughout.

Keynes thinks that it can be shown in a similar way that "the ordinary doctrine of contrariety does not hold good". If universal propositions do not imply the existence of the kind of things described in their subjects, "*All S is P* and *no S is P* are not inconsistent with one another, but the force of asserting both of them is to deny that there are any S's". (p. 195.) To take his example, if there is no such thing as an honest miller it is true that an honest miller has a golden thumb, and it is true that he has not.

I think that here Dr. Keynes is mistaken. The statement that an honest miller has a golden thumb is equivalent to the hypothetical: 'If

general law or condition it is only the unspecified members of the class about which something is necessarily implied. 'Only the red pills are to be taken' means that the red pills are to be taken, and the others left; 'Only the good are happy' means that goodness is essential to happiness, so that the bad are never happy; but it does not mean that every good man is happy.

To take the last case first; apart from the implication of a general law which may be contradicted by saying that virtue is not at all essential to happiness, the proposition is merely equivalent to the statement that no bad people are happy, or that all happy people are good; and the logical opposites of these propositions are obvious.

In the other case (Only the red pills are to be taken), where an exceptive or exclusive proposition is equivalent to two ordinary propositions (The red are to be taken and the others are not to be taken), it would be false if any of the following were true:

a miller is honest his thumb turns into gold'. Of course this is intended to imply that honest millers are not to be found, but it also implies the existence of some occult causal relation between honesty in a miller and a golden thumb, and a person cannot deny the statement without implying that this conception of the universe is fictitious.

On the whole subject see Keynes, pp. 186–210.

Since hypothetical propositions and universals equivalent to them are usually intended to imply the existence of some general law, they are sufficiently contradicted by any proposition denying the existence of such a law. The equivalent statements 'If a man is rich he is stingy' and 'All rich men are stingy' can be always contradicted by the statement 'It is not *always* so'; but if the supposed causal relation between riches and stinginess were the real subject of interest it would be sufficient to say 'It is not *necessarily* so'; and this might be proved even though no concrete exception to the universal categorical proposition could be found. To put the matter otherwise: Hypothetical propositions tell what under certain circumstances *must* be. To contradict them it is sufficient to say that it *need not* be. This can be proved by showing that it sometimes *is not*, but if a concrete exception could not be found it might be proved in some other way.

(1) Neither the red are to be taken nor the others not to be taken = The others alone are to be taken.

(2) The red or some of the red are not to be taken.

(3) The others or some of them are to be taken.

(4) Either the red or some of them are not to be taken or the others or some of them are to be taken.

The last of these four propositions—the disjunctive—is the only one that must necessarily be true when the original proposition is false. It therefore is its *contradictory*. The first of the four is the most extreme statement in the other direction. It therefore is the *contrary* of the original. The other two are contraries or contradictories of the parts into which the original is resolved.

From all this it can be seen that a logical opposite of an exclusive or exceptive proposition is itself rarely exclusive or exceptive.

To give concreteness to what has been said about the opposition of propositions the following system of symbols is suggested.

Symbols.

Let a small circle represent any object S of the kind discussed, and a plain stroke through it ϕ indicate the presence of a given attribute P, while a stroke with a small bar or tick across it ϕ̶ indicates the absence of that attribute. It is evident that the stroke cannot be both plain and crossed.

Suppose all the objects of the kind discussed to be represented by a number of small circles. When anything is said about all the objects of the kind under discussion draw a plain or ticked stroke as the case may be through each of the little circles. When something is said about only some of the objects leave some of the circles unmarked. The result is as follows:

A: All S is P ϕ ϕ ϕ
E: No S is P ϕ̶ ϕ̶ ϕ̶
I: Some S is P ϕ ϕ ○
O: Some S is not P ϕ̶ ○ ϕ̶

I and O: Some S is P and some is not ϕ ϕ ϕ ϕ o

To show how these symbols help. Let us represent the supposed fact that All S is P—that every circle has a plain stroke through it—thus: ϕ ϕ ϕ ϕ It does not matter how many of these circles we draw so long as the plain stroke is drawn through each one of them, to show that there are no exceptions.

Now suppose the question to arise: How many of these S's have a crossed stroke? We need only glance at the circles to see that there are none marked that way and no unmarked circles that *might* be marked that way. Hence we say: None of the circles can be marked with a crossed stroke; none of the S's can be non-P; no S is non-P. Thus the symbols enable us to *see* that this follows from the supposed fact that each of the S's is P. In the same way if each of the S's is P—if each circle has a plain stroke—we need only look at the above figures to *see* that it is also true that at least some of the circles have plain strokes—that some S's are P (Proposition I); false that some of the circles have not plain strokes—that some S's are not P (Proposition O); and still more false that none of the circles have plain strokes —that no S's are P (Proposition E).

Again, let us suppose that some S's are not P (Proposition O) and represent it by drawing a crossed stroke through some but not all of the circles—it does not matter how many: ϕ ϕ o o We leave some of the circles unmarked because there are some that the proposition does not say anything about. We know that in reality each of these must have one character or the other, but we do not attempt to represent it until we know which character it is.

What now can we say about the truth or falsity of the statement that no S's are P (Proposition E)—that none of the circles should really be marked with a plain stroke? All we can say is that the marks already there will not tell us. In other words, if we know that Proposition O is true

and if that is all we know, we must remain in doubt about the truth or falsity of E. So likewise with I; so long as we do not know whether those unmarked circles should really be marked with a plain stroke or with a crossed stroke we cannot say whether it is true or false that some S's are not P. We *can* tell, however, about the truth or falsity of A; for if A were true and all S's were P, all the circles would have to be marked with a plain stroke, and that is not possible so long as at least some of them are marked with a crossed stroke. Hence we can *see* from the symbols that represent the truth of O that A must be false. And so of the rest.

So far no particular S has been definitely and individually designated. To indicate some particular individual or subgroup of individuals use a small black dot or blacken the circle. All the remaining categorical propositions can then be symbolized.

Singular A: Socrates is P ϕ

Singular E: Plato is not P ϕ

Exceptive A: All the S's but B are P ϕ ϕ ϕ or ϕ ϕ •

The first of these figures indicates 'All the S's but B are P (and B is not)', the second indicates 'All the S's but B are P (and it is not said whether B is P or not)'.

Exceptive E: No S but B is P ϕ ϕ ϕ or ϕ ϕ •

Exclusive A: B is the only S which is P ϕ • ϕ

Exclusive E: B is the only S which is not P ϕ ϕ ϕ

It is to be noticed that a person interpreting these diagrams could not distinguish between a proposition and its 'obverse'; for example, between the affirmative proposition All S is P and the negative No S is non-P, or between the negative No S is P and the affirmative All S is non-P. This is an advantage rather than a defect; indeed the whole value of the symbols rests upon such facts, for the difference between a proposition and its obverse expresses a difference of shading or accent in the thought, but not a difference in the objects thought about. The relations of the objects remain the

same whether they are told about in one way or in another, and the diagrams symbolize these relations as they are supposed to exist in the objects. They point to the reality with which thought is concerned and to which it must always conform whatever its shading, rather than to the particular shading which the thought may happen to take or the words in which it happens to be expressed, and they can be used to test the thought no matter what its shading or form of expression.

The fact that these diagrams express no difference between a proposition and its obverse suggests the question that is sometimes discussed whether proposition A is not after all negative rather than affirmative. When we say that every nation prefers its own interests to the good of humanity (All S is P), do we have in mind all the nations that do this, or the fact that none can be found which does not? Certainly we cannot be sure that the statement is true until we find that there is no nation which does not (No S is non-P). Perhaps we can say that when proposition A expresses a hasty and unverified generalization it is affirmative, when it is derived deductively from general considerations it may also be affirmative, but when it is reached cautiously in the absence of general considerations it is usually negative. When we seek to verify a general statement, we do not count the cases in which it holds, but we look for exceptions.*

* This system of diagrams seems to me to indicate the opposition of propositions better than Euler's (explained elsewhere), partly because it provides a diagram for every proposition, while his only provides for the first four, partly because the same diagram represents a proposition and its obverse, partly because the diagrams for all the propositions that express different facts are distinctly different, but mainly because it preserves the distinction between things and attributes, and represents the presence or absence of the latter in the former rather than the partial or complete inclusion or exclusion of one class by another.

CHAPTER X.

INFERENCE AND THE SO-CALLED LAWS OF THOUGHT.

In previous chapters we have given examples of good and bad inference; we have said that all inference involves judgments about real or supposed objects of thought different from the judgments themselves; we have said that these judgments can be expressed in propositions, and in the chapter on the Opposition of Propositions we have had practical examples of the relation between the facts and the propositions about them. We must now inquire more fully what inference really is. In doing so we turn, though never altogether, from the question of words and their meanings, and fix our attention more fully upon things and their relations.

We infer when we suppose that because one state of affairs exists another exists also. The real or supposed facts that we reason from are called *premises;* those that we reason to, *conclusions ;* and we may say that the conclusion of any argument is true *because* the premises are true, or that the premises are true and *therefore* the conclusion is true. **Inference—what.**

Clear as this matter seems it is not fully understood until we distinguish the relation of premise and conclusion from two other relations each of which may likewise be indicated by the words 'because' and 'therefore', namely, the relation of cause and effect and that of motive and act. A man may say, for example, that he believes in Christianity because he was born and bred in a Christian community, or

because he wants to go to heaven, or because the four gospels and the sacraments of the Church must have had some cause. The first 'because' indicates a cause, the second a motive, the third a premise. The knowledge that such and such causes or motives exist may enable us to infer the existence of the corresponding effects or acts. If it is raining we know that people will put up their umbrellas. Similarly the knowledge that the effects or acts exist may enable us to infer the existence of the causes or motives. If people have up their umbrellas we know that it is raining. But with causes and motives as such, inference has no more to do than with any other relations.

Inferences are usually divided into two classes: Deductive, or those in which conclusions follow so necessarily from their premises that their truth is as certain as that of the premises themselves; and Inductive, or those in which the conclusion follows from the premises with more or less probability, but by no means so inevitably—so that the premises might sometimes be true and yet the conclusion be false. To illustrate the latter first: from the presence of dark clouds and a moist atmosphere we can infer that it will rain, but we cannot be certain of it. On the other hand, if we know that on cloudy days it always rains and that to-day it is cloudy, we can be quite certain that there has been or will be rain to-day.

Deduction.

Deductive inference is the only kind that logicians discussed for two thousand years. All that we shall have to say in the next seven chapters has direct reference to it. Induction will be discussed afterwards; something also will be said about the relation between the two kinds of inference. In the meantime we need only say that there is not nearly so much difference between them as has often been supposed.

Deduction, or the absolutely indisputable kind of inference, does not depend, as most logicians have assumed, upon any special relation between our thoughts, but—like

the other kind—it depends upon the nature and inner relations of the objects thought about. It can be drawn only *when the state of affairs asserted by the premise or premises could not possibly exist without the state of affairs asserted by the conclusion,* or in other words, only when what is asserted by the premise or premises and what is asserted by the conclusion are different aspects of some wider system in which the former could not exist without the latter. If the line A cuts the line B we can infer with absolute certainty that the line B cuts the line A, because one cannot cut the other without making some such figure as this +, in which the other also cuts the one.

To take another example: If any one is told that three athletic teams, A, B, and C, each played three games with each of the others, that there were no drawn games, and that A won twice and twice only from B and twice and twice only from C, while B won only once from C, he has been told enough to enable him to construct a general scheme of things that includes also the number of games won by C and the relative standing of the teams: facts about which he was told nothing, and which even now some reader may not take the trouble to work out. That it can be worked out is not due primarily to any relation between the ideas merely as ideas of the person working it out; but it is due to the fact which he knows, and which would exist whether he knew it or not, that when a contestant plays a game which is not drawn he must either win it or lose it, and if he does not win it his opponent does, and to the known relations of number. If the rules of the game made it possible for both sides to win or for both to lose, or if two from three left two instead of one, the reasoning to be correct would have to take these facts into account and the conclusion would be different.

A curved surface cannot be concave on one side without being convex on the other; and in general the nature of things is such that every variation in one aspect of a complex

state of affairs involves a corresponding variation in some other aspect. To infer we must know what some one aspect of the situation really is and the rule according to which it involves another aspect. Our knowledge of the first aspect is the premise of our reasoning, and from our knowledge of this aspect and of the rule we can reach the conclusion, or a knowledge of the other aspect. But if various aspects of a situation did not involve each other whether we reasoned about them or not, we should not be able to reason at all.

It is thus not the business of logic—or of any part of it—as most writers have said that it is, to describe 'necessary forms or laws of *thought*' connecting one idea with another, but rather to direct attention to the most fundamental laws or relations of *things* which all reasoning takes for granted and which alone make it possible for any one state of affairs to involve any other.

If this is a correct account of the nature of inference in general, the only way to test the validity of any specific case of inference is to ask: Is there any possible way in which the relations asserted in the premises could exist in the absence of those asserted in the conclusion? Is it possible for one and the same object or general state of affairs to have the one set of relations without the other also? If it is possible, then the denial of the conclusion may be *consistent* with the affirmation of the premises; if it is not possible, this denial is not consistent, and the conclusion *follows*.

Logical Consistency is thus a matter of possibility and impossibility in the objects under discussion. This will become clearer as we proceed.

Though the ultimate justification of every act of inference must be found in the nature of the things about which the inference is drawn, it would not be possible for us to draw an inference unless we were able to think of these things in some coherent and rational way. The fundamental elements involved in all such coherent and rational thinking are known as the 'Three

The three 'Laws of Thought'.

Laws of Thought'; and we must now explain what these so-called laws of thought really mean.

The first of these is called the Law of Identity; it is usually stated in some such form as this: 'What is, is', 'A is A', 'Everything is what it is'; and this law with the two others are treated as axioms or first principles to which doubtful arguments should be referred, and by which alone they can always be tested.

This first law of thought—the law of identity—does not mean that objects cannot change or cease to exist, that A cannot become B or be wiped out of existence altogether, that what is true at one moment is always true. It merely expresses the fact that we know what we are thinking about and what we are thinking about it; that we can recognize an old object of thought as the same even when what we think about it is not the same, and that in a similar way we can recognize whether a new statement about it is or is not the same as an old one. We can consider an object or a situation in as many aspects as we please and still recognize that we are concerned with the same object or situation. A person can say, for example, that a certain house is red, that it is four stories high, that it is old, and that it was once inhabited by George Washington; and the speaker and his hearers can both recognize that he is talking about the same house all the time. But if he should add something about its pale-green color, its snowy peaks, its delicious flavor, its angry billows, its flushed and anxious countenance and its relation to the square root of the difference, we should then say that his mind was wandering, his thought was not consecutive, his various '*its*' did not refer to the same object. We, the hearers, could say that the objects referred to must all be different, but if the speaker's mind were really wandering, if he were utterly incapable of holding fast to an old object of thought and identifying it, he would not know that his various sentences referred to different objects, for without the power to identify an old object of thought he

would have no more idea of difference than of identity. To him no pronoun could have an antecedent and the words 'same' and 'different' would be absolutely meaningless.

This illustration has particular reference to the object thought about, but the power of remembering and recognizing statements made about it is just as essential to sanity. The law of identity thus expresses the fact that thought points to objects and that we can know or recognize what objects we are thinking about and what we are thinking about them.

The second 'law of thought' is called the Law of Contradiction, and is expressed in such formulæ as these: 'Nothing can both be and not be', 'A is not not-A', 'A cannot be both B and not-B'. While the law of identity rests upon our power of identifying an object of thought, the law of contradiction rests upon our power of distinguishing between an affirmation and a denial, between the meaning of 'is' and the meaning of 'is not', of 'yes' and of 'no'. The law, in its primary sense, at least, simply means,—what everybody knows,—that we cannot both affirm and deny the same thing about the same object. Understanding by S whatever can be named in the subject of a sentence, and by P whatever can be named in the predicate, the law means: If it is true that S is P, then it is false that S is not P and if it is true that S is *not* P, then it is false that S is P. The law as thus stated does not depend upon any particular knowledge about things; it follows inevitably from the nature of thought. Why thought should take the form of a judgment, and why affirmative and negative judgments should exclude each other we do not know, but as soon as we know anything about ourselves we know that such is the case.

There is a secondary meaning often attached to the law of contradiction, namely: that we must not ascribe incompatible qualities or relations to the same object. According to the primary sense of the law we contradict ourselves if we say that a certain thing is white and that it isn't white; accord-

ing to the secondary sense we contradict ourselves if we say it is white and that it is black. In the first case we had in mind the same quality—white, and the law said we could not both affirm and deny it; in the second case we had in mind two different qualities, and the law said we could not affirm them both. In the first case the sense of contradiction is due to the fact that it is mentally impossible to affirm and to deny the same thing at the same time, just as it is physically impossible to say yes and no at the same time, to nod the head and to shake it, to approach or draw a thing toward you and to recede or push it away; in the second case it rests upon our knowledge that the same object cannot have two different colors at once. In the first case the contradiction rests upon the nature of a judgment and could be recognized by any one who could distinguish between the meanings of 'is' and 'is not', however limited his experience; in the second case it rests upon the nature of things as people gifted with sight believe them to be.

If we did not believe in the existence of a world so constituted that the presence of any quality in a thing excludes certain other qualities, we should not recognize any contradiction in saying that a thing is both white and black, three feet long and one inch long, round and square, before and after.

The qualities which do not, or cannot, exist together in the same thing happen to be those which appeal to the same sense, and which we are therefore able to compare together, and for which we usually have some general name, such as 'color', 'size', 'shape', 'time', 'place'. Incomparable qualities, such as red and square, may or may not coexist in the same object, and we have no difficulty in imagining any combination of them. Either because of the nature of our faculties or because of the limitations of our experience, we cannot imagine an object which has at the same time two different qualities of the same general kind.

Because we cannot imagine a thing to have at once two

qualities of the same general kind, we assume that such qualities cannot really coexist, and when we assume this, it follows that to affirm one quality of an object is equivalent to denying another, so that when any one says that a thing is large and small, white and black, we take it for granted he means that it is large and isn't large, that it is white and isn't white, and that he is therefore contradicting himself. Thus through the limitations of our imagination the law of contradiction, which in its primary sense is concerned only with the impossibility of both affirming and denying the same qualities or relations, comes to take on a secondary meaning concerned with the inconsistency of affirming certain different qualities or relations.* It is this secondary sense of the law that is expressed in the last two formulæ I gave for its expression.

The third general principle or law of thought is called the Law of Excluded Middle. The usual formula is 'Everything must either be or not be', 'A must be either B or not B'. This law is the complement of the Law of Contradiction and means that every statement must be either true or false. If it is false that S is P, then it is true that S is not P, and if it is false that S is not P, then it is true that S is P. This law, like the law of contradiction in its primary and proper sense, is not derived from an examination of things, but follows inevitably from the nature of thought in judging; for not only do affirmative and negative judgments necessarily exclude each other, as the law of contradiction says, but every positive or active judgment must either affirm something or deny it; there is no middle ground.

Like the law of contradiction, the law of excluded middle has also acquired a secondary meaning, concerned not with the difference between affirmation and negation, but with the mutual implications of various qualities and relations in the objects judged about.

* If we distinguish between the copula and the predicate we can say that in its primary sense the law is a mere matter of the copula; but in its secondary sense is made a matter of the predicate.

Experience teaches that every real thing, indeed every object of which it is possible to think at all, has qualities and relations of some kind. If it is not large it is small, if it is not here it is elsewhere; or else it is a spirit or some other kind of immaterial object existing without space-relations, but not without the moral or other relations which that kind of immaterial objects possess. In general, if any object, S, has not a certain quality or relation, P, it must have some other quality or relation incompatible with P, and which we may therefore call not-P or non-P. To put it more briefly: If S isn't P it is non-P; every S must be either P or non-P; and to say that S isn't P is equivalent to saying that it is non-P.

This is the secondary sense of the law; and it does not depend, like the primary, wholly upon the mere nature of judgment, but partly also upon the fact that we have never found and cannot think of an object that does not possess some definite relation or other.

But there is often still a third implication which the law seems to cover.

From earliest childhood we find those about us dividing objects into various kinds, and in the course of experience we learn to take it for granted that however much they may differ from each other all objects of the same kind are determined in the same respects; have the same kind of qualities and relations. Every man has some moral standing. Hence if he is not good he must be bad. He has some color. Hence if he is not white he must be black, brown, yellow, or red. And so of size, weight, and all the other general qualities which every man possesses.

Now, as it happens, it is just as easy to find out that some object, say the soul or the square root of 33, has no color or weight or shape at all as to find out that it is not red or heavy or round, and in ninety-nine cases out of a hundred it is a great deal more serviceable to say so. Consequently when any one says that some object, X, is not red his hearer takes

it for granted that he is talking about an object that has some color or other, since otherwise he would have said it was colorless.* And so of every other quality and relation: we interpret the statement 'S isn't P' to mean that S has some quality P′ or P″, which, though different from P, belongs to the same general class.

The third matter which the law of excluded middle seems to cover is thus a rule for the interpretation of language. But such a rule is by no means infallible. It is quite possible to say that a triangle is not virtuous without the slightest intention of implying that it is vicious.

Assuming the truth of the law in the secondary sense, and understanding that the phrase non-P is used to indicate some quality or relation incompatible with P, we can turn every affirmative proposition into an equivalent negative, and *vice versa*: 'All S is P' (A) into 'No S is non-P' (E); 'Some S isn't P' (O) into 'Some S is non-P' (I); 'All men are mortal' into 'No men are immortal'; and *vice versa*.

Obversion.

The manipulation of propositions in this way is called *Obversion* or *Immediate Inference by Privitive Conception.* It is less commonly known as *Permutation* or *Infinitation.* If we wish a formal definition we can say:

To OBVERT a proposition is to deny or affirm the absence or presence of a relation whose presence or absence the original proposition affirmed or denied.

* "No one would be so foolish as to deny what no one could have the slightest temptation to affirm. If I say, then, that X is not Y, I imply that there are certain elements in X, by which, if they were taken alone, it might be confounded with Y. Of course the elements of resemblance may be comparatively few, but something in this case must have occurred to bring it into prominence." C. C. Everett, "Fichte's Science of Knowledge", p. 103.

CHAPTER XI.

IMMEDIATE INFERENCE, OR INFERENCE FROM A SINGLE PREMISE.

When an inference is drawn from a single premise it is called *immediate*, when from several premises taken together, *mediate*. The terms immediate and mediate as thus used have only a secondary reference to time. Their main object is to indicate the absence or presence of some *intermediate* process. Where an inference rests upon several premises and cannot be drawn from one of them alone, the intermediate process consists in constructing a notion of a total state of affairs according to specifications part of which are laid down by one premise and part by another. It is this total state of affairs in which alone both or all the premises can be realized that implicates the conclusion. A state of affairs in which none or only a part of the premises were realized might implicate it, but we know that this other must.

We are at present concerned with immediate inference, or the cases in which the state of affairs described by a single premise necessarily implicates that described in the conclusion.

The most interesting and important kind of immediate inference is called 'Conversion'. In discussing obversion we saw how it was possible to pass from one statement to another about the same object; in conversion we feel warranted in passing from a statement

Conversion.

about one object or kind of object to a statement about another object or kind of object about which something has been implied, though not directly said in the statement about the first object. If, for example, we should happen to know that some white things are square we should be able to infer that some square things are white. To be white and to be square are two very different matters; but yet the world and our minds are so constituted that a statement about the one class of objects may serve as the basis for a statement about the other.* In like manner, if a person interested in the city of Cleveland should be told that it is 183 miles west of Buffalo and he should afterwards have occasion to tell all that he knew about Buffalo, he would be able to say that it is 183 miles east of Cleveland. If he were interested in places 183 miles west of Buffalo he would be able to say that at least one of them was Cleveland. And if he were interested in the distance of 183 miles he could say that it is as far as from Buffalo to Cleveland.—The first statement was about Cleveland; but the fact asserted was of such a nature that it could not be true unless the other statements, not about Cleveland but about Buffalo and places 183 miles west of Buffalo and the distance of 183 miles, were also true.

The traditional treatment.

The doctrine of conversion found in most text-books on logic provides for only one of the three inferences which is here drawn, namely: a place 183 miles west of Buffalo is Cleveland. The reason for this lies in the traditional way of dividing every proposition into subject, predicate, and copula, and of regarding the copula as a perfectly colorless sign of affirmative or negative predication. Every proposition was regarded as a kind of

* The constitution of the world involved in this particular case is the fact that several attributes (*e.g.*, whiteness and squareness) may be possessed with equal intimacy by the same object; in the case about to be mentioned it is the fact that relations are reciprocal—facts of so familiar kind that we forget the debt our logic owes them.

equation of which the copula supplied only the idea of equality or non-equality. Our original proposition, divided in this way, would read

Subject.	Copula.	Predicate.
Cleveland	is	(a place) 183 miles west of Buffalo,

and it would be regarded as meaning nothing more than that Cleveland was identical with a-place-183-miles-west-of-Buffalo. Regarded in this way the proposition says nothing whatever about Buffalo or about 183 miles, but only about Cleveland and a place 183 miles west of Buffalo. These are the terms, and they cannot be broken up, consequently the only inference to be drawn, if inference it is, except by obversion or by way of opposition is found by reversing the equation, and saying a place 183 miles west of Buffalo is Cleveland.

According to the same way of regarding things the proposition 'John is riding a horse' can be converted into 'a person riding a horse is John', but the traditional rules of logic make no provision for any inference about the horse.*

* This limited view of the meaning of propositions can be easily explained. In an age like that of Plato and Aristotle when scientific knowledge was thought to consist merely in description and classification, it was natural enough to overlook all the relations asserted in propositions except those of substance and attribute and individual identity, for the causal and other outer relations between things had no scientific significance except as indicating qualities *of* the things, by which they could be identified and classified. From this point of view the statement that John is riding a horse is of value only as it tells one of John's *accidents* or occasional states by which he might perhaps be identified or distinguished from other people who never ride or at least did not ride at the time referred to. From the same point of view the converse statement that a horse is being ridden by John has scientific significance only as indicating that horses, or at least this particular horse, can be described as capable of being ridden or as having been ridden. This standpoint lent itself only too easily to the purely mechanical and verbal treatment of propositions which is still common. Certainly with modern writers this purely verbal treatment of logical processes is only a pedagogic device. Yet it seems to me that it limits the usefulness of logic and that the subject is capable of being treated more directly.

If one were interested in describing John the facts would naturally be stated in one way; if he were interested in describing the horse they would be stated in the other way. But the mode of statement was settled before the logical process began. It was with the description and classification of things, not with their outer relations, that logic was concerned. It therefore never occurred to the early logicians that it was a part of their business to show how the state or relations of one thing at any particular time involved corresponding conditions in something else. Except as they contained data for classification the mere spatial and temporal and causal relations of things had no logical import and were not worth analyzing. It was with descriptions of things, not with events, that the logical process began.

Let us assume for a time, with the old logicians, that our only logical interest lies in the description and classification of things, and that every proposition with which logic deals must contain a subject, a predicate, and a copula whose sole function is to affirm or deny an equality or identity between the subject and predicate. If a proposition has not such a form it must be given one before it is dealt with logically, so that instead of saying 'John runs', and 'Ducks like water', we must say 'John is running', or more properly 'John is a creature who is running', and 'Ducks are creatures who like water'.

The first thing to be noticed from this standpoint is that description merges insensibly into classification. When we say that ducks like water we undoubtedly describe one of their characteristics; but when we say that they are 'creatures who like water' we may be regarded as *classing* them with other creatures who like it (if such exist) as distinguished from those who do not. Thus the many relations really expressed in propositions are reduced for logical purposes to one; when the traditional logician says that any object or class of objects, S, is P, all he means is that the object or class S is contained in the class P.

What does this statement about S enable us to say about the class P?—This is the question of conversion. "A proposition is said to be converted when its terms are transposed, so that the subject becomes the predicate and the predicate the subject" (Fowler, p. 80). Converting the proposition 'S is P' in this way we get 'P is S'. But how many of the P's are S? From the fact that all ducks like water it does not necessarily follow that every creature that likes water is a duck.

> Sir, I admit your general rule,
> That every poet is a fool;
> But you yourself will serve to show it
> That every fool is not a poet.

The mediæval logician sought mechanical rules for manipulating words, and so he asked 'Is there any rule by which we can tell the quantity and quality of propositions that have been converted?', and he found two which could always be followed with safety:

"1. The quality of the proposition (affirmative or negative) must be preserved, and

"2. No term must be distributed in the Converse, unless it was distributed in the Convertend" (Jevons, p. 82).

The *convertend* means, of course, the proposition that is to be converted; and the *converse* that obtained by converting it. A term is said to be *distributed* when used in such a way as to necessarily include all and not merely some of the members of the class it denotes. The subjects of the propositions A and E are thus said to be distributed; the subjects of I and O to be undistributed. But how about the predicates?

When anybody who has not studied logic says that ducks like water, he uses the term 'ducks' demonstratively, and he can tell fairly well whether he means to speak of *all*, or of only *some* or *most*, of the creatures that the name denotes. The other two words in the sentence—'like water'—he uses descriptively to tell something about the ducks. If he is

now told that he was really talking not only about ducks but also about a class of 'creatures that like water', and is asked whether he refers to all or only some of this class of creatures, he cannot help being puzzled, for the thought of such a class probably never entered his mind. That is the objection to what Sir William Hamilton and others have called the Quantification of the Predicate. But if he is compelled to answer, the only safe thing to say is that he means that ducks are *at least some* of the creatures that like water. Then his statement about the ducks will be true whether other things, such as gulls and frogs and fishes, happen to like water or not. To put the matter generally: *Affirmative propositions*, whether universal or particular, *do not distribute their predicates.*

To convert an affirmative proposition we must therefore reverse the subject and predicate, retain the affirmative copula (rule 1), and see that the subject is undistributed, *i.e.*, that the proposition is particular (rule 2). In other words, the converse of A or I is always I.

When a universal proposition is converted into a universal or a particular into a particular it is said to be converted *simply;* but when a universal is converted into a particular it is said to be converted *per accidens* or *by limitation.* It will be noticed that when A is converted into I it cannot be converted back again into A, but only into I. From the fact that 'all Rhode Islanders are Americans', it follows that 'Some Americans are Rhode Islanders'; but all that can be inferred from the latter proposition is that 'Some Rhode Islanders are Americans'.

In contrast with affirmatives, *negative propositions always distribute their predicates.* The statement that no cats like water means that cats and creatures that like water form two wholly distinct classes, and that no individual belongs to them both. We can therefore be as sure that no single creature that likes water is a cat as that no single cat is a creature that likes water. Thus the converse of E (*e.g.*, No

cats like water) is always E (No creatures that like water are cats).

The other negative proposition, O, is harder to deal with. "In attempting to convert the proposition O we encounter a peculiar difficulty, because its subject is undistributed; and yet the subject should become by conversion the predicate of a negative proposition, which distributes its predicate" (Jevons, p. 83). If certain boys, A, B, and C, do not like water we can be quite sure that no creature that likes water is one of these boys A, B, and C. Here it is easy enough to convert; but the statement that A, B, and C do not like water is equivalent to three singular propositions, not to a particular. If it had been forgotten who these boys were, one might still be sure that some boys do not like water (proposition O); but all that could be said on the strength of this about creatures that like water is that none of them are some boys or other. We could not say that none of them are boys. We might say, to be sure, that some of them are not boys, but this would be on the strength of what we know about cats and monkeys, not on the strength of the statement that some boys do not like water. Since the statement that no P is some S or other conveys practically no information whatever about P, we must conclude that *the proposition O cannot be converted:* nothing definite can be said about either some or all of the objects denotable * by its predicate.

Many logicians say in substance that though O cannot be converted in the usual way we can "apply a new process, which may be called *conversion by negation,* and which consists in first changing the convertend into an affirmative proposition, and then converting it simply" (Jevons, p. 83). In this way 'Some boys do not like water' becomes 'Some boys dislike water', and converting this we get

* I say denotable rather than denoted, because the predicate of a proposition is usually used to describe the things pointed out by the subject, not to point out new ones.

'Some creatures that dislike water are boys'. (Some S isn't P = Some S is non-P = Some non-P is S.)

This so-called *conversion by negation* consists simply in converting the obverse; and it is a process which can be applied just as well to A and E (though not to I, whose obverse is O and inconvertible) as to O.

The process of alternate obversion and conversion can be carried through various stages as follows; but it is valid only if the existence of all the objects named is presupposed:

PROPOSITION A.

Beginning with Obversion.	Beginning with Conversion.
A. All S is P =	A. All S is P =
E. No S is non-P =	I. Some P is S =
E. No non-P is S =	O. Some P isn't non-S.
A. All non-P is non-S =	
I. Some non-S is non-P =	
O. { Some non-S isn't non-non-P, or Some non-S isn't P.	

PROPOSITION E.

Beginning with Obversion.	Beginning with Conversion.
E. No S is P.	E. No S is P =
A. All S is non-P.	E. No P is S =
I. Some non-P is S.	A. All P is non-S =
O. Some non-P isn't non-S.	I. Some non-S is P =
	O. Some non-S isn't non-P.

PROPOSITION I.

Beginning with Obversion.	Beginning with Conversion.
I. Some S is P.	I. Some S is P.
O. Some S isn't non-P.	I. Some P is S.
	O. Some P isn't non-S.

PROPOSITION O.

Beginning with Obversion.	Cannot be Converted.
O. Some S isn't P =	
I. Some S is non-P =	
I. Some non-P is S =	
O. Some non-P isn't non-S.	

Attention is directed to the fact that by this process of alternate conversion and obversion we are able on the strength of a given proposition to make assertions not only about objects to which the subject and predicate terms are applicable, but about objects—if we assume them to exist—to which one or both are wholly inapplicable (*i.e.*, to non-A's and non-P's). We must, however, be very careful not to jump at conclusions of this sort. From the proposition All S is P (All men are mortal) we can infer No non-P is S (No immortals are men), or All non-P is non-S (All immortals are non-human); but we cannot infer All non-S is non-P (All non-humans are immortals).

The principal difficulty in conversion is due, as we have seen, to the fact that a descriptive predicate has to be turned into a demonstrative subject with the proper quantity. From this difficulty exclusive and exceptive propositions are free, since they always distribute their predicates. If Europeans alone are capable of self-government it must be that all races capable of self-government are Europeans. Indeed it is a rather curious fact that exclusive and exceptive propositions imply something about all the objects mentioned in the predicate and about all those not specially mentioned in the subject, but not necessarily about all those specially mentioned in the subject; so that the objects to which they seem to call special attention are those about which they say the least. From the supposed fact that Europeans alone are capable of self-government it follows, as we have seen, that all races capable of self-government are European; it also follows that no non-European races are capable of self-government; but it does not necessarily follow that all European races are capable of self-government. Because of this I am inclined to think that to transpose the subject and predicate of an exclusive or exceptive proposition is an analysis of meaning rather than a conversion. We cannot be sure that European races alone are capable of self-government unless we already know that all the races capable of

self-government are European. The value of the exclusive form seems to lie in the contrast it brings out between the objects specially mentioned (to which the attribute in question at least may belong), and the rest of the class to which they belong—the kind of contrast which serves as the basis for all classification.

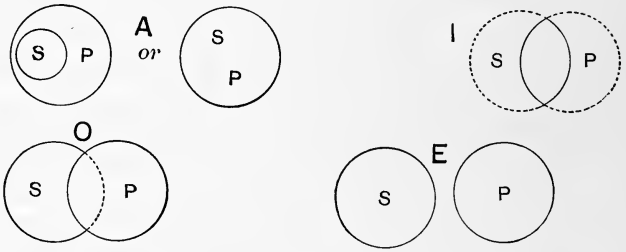
The treatment by diagrams.

The subject of conversion has been discussed so far from as mechanical a standpoint as possible. Following the older logicians we have given rules for the manipulation of words which can be followed blindly.

In the latter half of the eighteenth century Leonard Euler invented, or rather revived, a set of simple diagrams by which the relations between classes of objects could be so easily and so well symbolized that it was no longer necessary to follow rules mechanically, or even to remember them at all. If the members of each class of objects are supposed to be enclosed in a circle, the visible relations of the circles can be relied upon to indicate the relations of the classes. When one class is included in (or is identical with) another (Proposition A), the circle S, supposed to enclose the members of the first, must be drawn inside of (or coincident with) the circle P, supposed to enclose the members of the second; when some members of the first class are also members of the second (Proposition I), at least a part of the circle S must lie inside of the circle P; when there are some members of the first class which are not members of the second (Proposition O), at least a part of the circle S must lie outside of the circle P; and when no member of the first class is a member of the second (Proposition E), all of the circle S must lie outside of the circle P.

So far it has been the circle S we have discussed, as wholly or partly within or without the circle P. But we can neither draw nor conceive of figures so constructed that a circle S lies wholly or partly within another circle P without part of P's area lying within S. From 'All S is P' (Proposition A)

or 'Some S is P' (Proposition I) we can therefore infer 'Some P is S' (Proposition I). Similar grounds can be found in the space relations of the figures for the conversion of E (No S is P) into E (No P is S); while the figure only allows us to convert O (Some S isn't P) into the worthless Proposition E, already referred to (No P is some S or other).



Thus when we draw the diagrams we can convert without reference to the formal rules, merely by observing what the relations of the circles *must* be under the given conditions. This is a much more natural and rational process than to blindly follow mechanical rules. The only rule involved in the construction of diagrams in conversion or syllogism is this: *Try to make them represent the premise or premises without at the same time representing any conclusion you have in mind.* If this cannot be done the conclusion follows. If it can be done it does not.

Euler's diagrams have rendered great service to logic; but it must not be forgotten that in using them or any other diagrams constructed on the same principle *we assume that spatial relations can be relied upon to represent relations which are not spatial.* Diagrams in logic are metaphors, and to reason in metaphors is usually extremely dangerous. Experience happens to show that in the case of Euler's diagrams the metaphor is not misleading, but we must not forget on that account that it is usually better and safer when we can do so to reason about the relations of things themselves directly than through the mutual relations of their symbols.

The reason that Euler's diagrams seem to make logical relations so clear is that they appeal directly to the senses, and that of all the relations perceived by sense those of space are the most constant, the most universal, and the most easily represented. Almost every conceivable relation thus comes to be symbolized in terms of space and seems to be better understood when it is expressed in spatial language. It is said that every preposition once expressed a spatial relation, and the same is true of very many words and phrases used with reference to the mental life (*e.g.*, 'apprehend', 'movement of thought', 'idea *in*, or *before* the mind', '*convey* an idea', '*express* an emotion', 'impression', etc.).

The great objection to Euler's diagrams is that, like the rules which they were intended to supplement, they apply only to relations of inclusion or exclusion between classes. Both are wholly inapplicable to either dynamic or non-dynamic relations between different individuals. Both, therefore, are of service within but a small portion of the whole sphere of thought.

A broader treatment.

There is no reason why the term conversion should not be broadened so as to include the transposition of subject and predicate when the copula is understood to express something other than mere identity or non-identity of things or classes. There are many propositions in which the subject and predicate name two different objects while the copula affirms or denies a dynamic or non-dynamic relation between them. The transposition of the subject and predicate of such propositions might fairly be called conversion. The difficulty connected with the traditional conversion is to settle the distribution of the new subject; and it arises from the fact that a predicate used descriptively is turned into a subject used demonstratively. With the kind of conversion just mentioned there is no such difficulty, for in dynamic and non-dynamic propositions the predicate is already used demonstratively. Whatever mechanical difficulty presented itself would come

from the copula. Sometimes it could remain unchanged and sometimes it would have to be altered so as to express a reversed relation. If John (subject) is-a-relative-of (copula) James (predicate), James is-a-relative-of John. Here the relation, so far at least as it is expressed, is the same for both parties and might be represented by an arrow pointed at both ends: John $\longleftrightarrow$ James; and the proposition can be converted by a mere transposition of subject and predicate. But if John is-the-father-of James we cannot infer that James is-the-father-of John. Here the relation expressed is different for the two parties and should be represented by an arrow pointing in one direction only: John $\rightarrow$ James; and when we convert, the copula must be changed, so as to express the relation from the other side: James is-the-child-of John, James $\leftarrow$ John.

When to reverse the relation expressed in the copula and when to leave it alone is a question that might be seriously considered if it were necessary or desirable to pay attention merely to our words and not to what they mean. But this is not necessary or desirable; and the question needs no serious consideration, for when we pay attention to the real object of discourse and understand the meaning of the words used there is no difficulty.

Whether we use the term conversion in this broad sense or in a still broader sense to include statements about any objects on the strength of statements about other objects in which the first objects were mentioned, there is no general rule for conversion which can be followed blindly and no set of symbols which is always applicable. The only thing to do is to turn from mechanical rules and from symbols to the things themselves, find out exactly what relations are asserted of the object spoken about, and then ask ourselves whether there are not corresponding relations of other objects mentioned or implied without which the relations asserted could not possibly or conceivably exist. To do this we must imagine not only a single state of affairs in which

the asserted relations exist, but many; to find out whether there is not at least one (conceivable, or possible, or actual, as the case may be) in which the other relations that seem to be involved are not really involved. This is thinking, and no mechanical rules can save us the trouble.

CHAPTER XII.

MEDIATE INFERENCE AND SYLLOGISM.

It has already been explained that mediate inference takes place when we recognize some new aspect of the total state of affairs in which alone all the relations asserted by two or more premises can exist together. To put the matter more concretely: *Mediate inference takes place when we conclude anything about the relations of two or more objects to each other from the relations of each to some third object*, the word 'object' being used in the broadest possible sense to include qualities and relations as well as things. From the fact that A is larger than B and that C is smaller than B we can conclude that A is larger than C; and this is mediate inference.

No inference can be drawn about the relations of two objects to each other, unless the object with which each of them is compared is in both cases the same. From the fact that A is larger than B and that C is smaller than D, nothing can be inferred about the relations of A and C. **Limitations of deduction.**

Moreover no inference can usually be drawn unless each of the two objects is compared with the third in the same respect; unless the relations discussed are homogeneous, or at least unless they belong to the same unitary system. From the fact that A is larger than B and that C is lighter than B, no inference can be drawn about the relations of A and C. Where Euclid says "Things which are equal to the same thing are equal to one another", we must understand

him to mean: "Things which are equal to the same thing in any given respect are equal to each other in that respect". A may be equal to C in physical strength, and B equal to C in intelligence without A and B being equal to each other in any respect whatever. In the same way the line AB may be equal in length and the line EF equal in color to the line GH, without their being equal to each other in either one or the other.

While no inference is usually possible when the relations dealt with are heterogeneous, the inferences drawn when all the relations under discussion are homogeneous do not belong to formal logic. If A is B's landlord or creditor or agent, and B is C's, it is a lawyer's business, not that of the formal logician, to say whether A is in any sense C's landlord or creditor or agent; if a substance D has a chemical affinity for E, and E for F, nobody but a chemist can tell whether D necessarily has or has not an affinity for F; if G is four times as large as H, and H is seven times as large as K, the relative sizes of G and K is a question of mathematics, and the traditional field of deduction is so limited that the formal logician as such is debarred from drawing a conclusion.

In syllogism or the kind of mediate inference discussed in formal logic, two of the three relations usually involved in the premises and the conclusion are homogeneous, and the third is (or may reasonably be treated as) a relation of identity.* If we say 'G is four times as large as H and H

* I say 'two of the three relations *usually* involved' because in a sorites there may be an indefinite number of premises. In this case the relations stated in all the premises except the last must be relations of identity.

The rule that two of the three relations must be homogeneous and the third a relation of identity does not exclude the case where all three are relations of identity, *e.g.*, 'A is identical with B, and B is identical with C, therefore A is identical with C'. The most serious objection that I can think of to the rule as I have stated it seems to come from such cases as this: 'A is larger, taller, sweeter, heavier, better, prettier than B, B is larger, etc., than C, therefore A is larger, etc., than C'. Here the rela-

is seven times as large as K; therefore G is twenty-eight times as large as K', the reasoning belongs to mathematics; but when we say 'G is four times as large as H, and H and K are one and the same thing; therefore G is four times as large as K', the reasoning is syllogistic and belongs to deductive logic. Similarly if we say 'A is four miles due west of B and C is three miles due north of B, therefore A is five miles southwest of C', the reasoning is geometrical; but if we say 'A is four miles due west of B, and C is three miles due north of B (*i.e.*, not at all west of B), therefore A and C are not the same', the reasoning is syllogistic. Or to put it somewhat differently, if we say 'A is two miles west of B and three miles east of C, therefore B is five miles east of C', the reasoning is geometrical or arithmetical and beyond the sphere of formal logic (not of course 'illogical'); but if we say 'A is two miles west of B and three miles east of C, therefore something two miles west of B is three miles east of C (*i.e.*, one and the same thing is both two miles west of B and three miles east of C)', the reasoning is syllogistic. In the examples of arithmetical and geometrical reasoning here given all the relations affirmed or denied were relations of size, number, or direction; but in the examples of syllogistic reasoning this was not the case. In the first of these three examples of syllogistic reasoning the second premise

tions are all homogeneous and no one of them is a relation of identity; and yet the conclusion seems to follow in each case from pure logic, and without reference to any special science. I suppose the answer to such an objection would be that before we speak of a thing as having more of a given quality than anything else we recognize that things can be arranged in a series with reference to that quality, so that whatever goes beyond something else goes still more beyond the things which that something else goes beyond. All that is implied by the use of the comparative —er or more —. So that when we say 'A is taller than B, and B is taller than C, therefore A is taller than C', we might have said 'A is taller than B, and whatever B is taller than, C is a thing that B is taller than, therefore A is taller than C'. In the syllogism thus stated the second premise asserts a relation of identity and the rule holds good.

affirmed a relation of identity and thus made the conclusion possible; in the second example the conclusion denied that two objects were identical, because they possessed incompatible relations; in the third example the conclusion called attention to the fact that the objects described in the two premises were identical, so that the relations which they affirmed coexisted, or both belonged to the same object.

'Figures'.

The three examples of syllogistic reasoning which we have just given to illustrate the difference between such reasoning and that which is not syllogistic can also be used to illustrate the difference between three different kinds of syllogism, for there are certain respects in which they are quite different from each other. The examples were these:

1) G is four times as large as H;
H and K are one and the same;
Therefore G is four times as large as K.

2) A is four miles due west of B;
C is three miles due north of B;
Therefore A and C are not the same.

3) A is two miles west of B;
A is three miles east of C;
Therefore something two miles west of B is also three miles east of C.

In each of the syllogisms there is a term (called the Middle Term) which occurs in each of the premises but not in the conclusion; but in the different syllogisms this middle term does not occur in the same place. In the first syllogism the middle term H is the predicate of the first premise and the subject of the second; in the second the middle term B is the predicate of both premises; and in the third the middle term A is subject of both. This difference of order is accompanied by a corresponding difference of thought; and each one of the three syllogisms may be regarded as an example

of one of the three 'Figures' of the syllogism, which we are about to discuss. Most logicians say that there are four of these figures; but Aristotle gave only three, and as the fourth is easily derived from the others by a purely mechanical process, has no special function as distinguished from the others, and is seldom or never used in ordinary reasoning, it may easily be omitted.

CHAPTER XIII.

THE FIRST FIGURE OF THE SYLLOGISM.

In the first figure the reasoning is of this sort. One premise, called the Major, asserts something about a certain object or certain objects; the other premise, called the Minor, points out that one or more specified individuals are identical with some or all of these objects; and on the strength of this the Conclusion makes the statement contained in the major premise with direct reference to the individuals specified in the minor. Examples:

General function.

None of the apostles were Gentiles;
Peter was an apostle;
∴ Peter was not a Gentile.

Every one who has consumption has tubercular bacilli;
This patient has consumption;
∴ This patient has tubercular bacilli.

No Anglo-Saxon likes mob rule;
Most Americans are Anglo-Saxons;
∴ Most Americans do not like mob rule.

It should be noticed that the major premise (which in each of these examples is written first) can affirm or deny any sort of relation whatever, while the minor always keeps saying: 'This is he', 'This is one of them', 'These are some of them'. We may say if we like that in all typical

examples of the first figure the major gives a rule, and the minor points out that a certain case comes under it.

In this figure the conclusion merely makes a specific or more specific application of what was said in the major premise to the objects specially mentioned in the minor. To do this, it substitutes the more specific term which occurs in the minor premise for the less specific term which occurred in the major; but in the conclusion the general sense of the major premise and its general arrangement of terms is preserved. This is not true of any other figure.

In the two other figures the distinction between major and minor premise is purely arbitrary, for both premises deal with the same kind of relations; the conclusion does not preserve the general sense of either; and one arrangement of terms in the conclusion is just as natural as the other.

It should not be overlooked that when the major premise points out a relation of any sort between two objects or sets of objects the minor can specify any or all of the objects in question, and the specification is carried into the conclusion regardless of whether these objects were denoted by the subject or by the predicate of the major. The following, for example, are perfectly valid syllogisms in the first figure:

All Slavs hate all Semites;
The Russians are Slavs and the Jews are Semites;
∴ The Russians hate the Jews.

Oil and water will never mix;
This is oil and that is water;
∴ This and that will not mix.

John is beating Thomas;
Thomas is John's son;
∴ John is beating his son.

John and Thomas are quarreling;
Thomas is John's son;
∴ John and his son are quarreling.

If we use an arrow to indicate any dynamic or non-dynamic relation between two different objects and three horizontal lines to indicate identity and a bar across the symbol to indicate the absence of the relation, these four syllogisms would be represented in this way:

Major:	All Slavs $\rightarrow$ all Semites.
Minors:	Every Russian $\equiv$ a Slav.
	Every Jew $\equiv$ a Semite.
Conclusion:	$\therefore$ All Russians $\rightarrow$ all Jews.
Major:	Oil $\longleftrightarrow$ water.
Minors:	This $\equiv$ oil.
	That $\equiv$ water.
Conclusion:	This $\longleftrightarrow$ that.
Major:	John $\rightarrow$ Thomas.
Minor:	Thomas $\equiv$ John's son.
Conclusion:	John $\rightarrow$ his son.
Major:	John $\longleftrightarrow$ James.
Minor:	Thomas $\equiv$ John's son.
Conclusion:	John $\longleftrightarrow$ his son.*

Is there real inference?

Inference in the first figure amounts, as has been said, merely to this: Some or all of the individuals about which a statement has been made in the major premise are pointed out more specifically in the minor, and then in the conclusion the statement is made over again with specific reference to these individuals. The interesting question about it is whether in such a process

* When we recognize that propositions expressing relations between different objects can be treated by the syllogism we must abandon, for such cases at least, the old rule that the major premise contains the predicate of the conclusion. In the third of these examples this rule would make what I have called the minor premise—Thomas is John's son—the major, though it is perfectly obvious that it is the other premise—John is beating Thomas—whose general sense is preserved in the conclusion. It is infinitely better to judge by the meaning than by the outward form.

there is any inference at all. Does not the conclusion merely repeat in other words a part of what has been already stated in the major premise? And if so can this be called inference? Mill and others have maintained that it cannot. To get a fair view of the subject we must consider three slightly different sets of cases. Let us take an example of each.

Peter, James, and John were all Jews;
Peter is one of these;
∴ Peter was a Jew.

All of the apostles were Jews;
Peter was an apostle;
∴ Peter was a Jew.

Whoever has consumption has tubercular bacilli;
This patient has consumption;
∴ This patient has tubercular bacilli.

In the first of these examples no one would maintain that there is any inference. The conclusion merely repeats what has been said just as explicitly in the major premise.

In the second example the case is somewhat different. No one who investigated the matter could be sure that all the apostles were Jews unless he were first sure about Peter and each of the others individually, but it would be possible, nevertheless, to make a statement about all of the apostles without *thinking* about Peter and each of the others individually. If there is inference in this case it rests upon the curious fact that, by using such words as 'all' and 'every', we can speak of each of a large number of individuals, though we do not, and cannot, have at once separate mental images of more than a very few of them. Such inference as there is consists in pointing out that the statement made applies to certain individuals that we may never have thought of when the statement was made—in realizing to some extent whom or what it was that was spoken about. If we adhere to our definition of inference as the recognition of a new relation of things without which the relations asserted

in the premises could not exist, this process is not inference, for the relation realized in the conclusion is not a new one; there is no new *fact.* But if we broaden our definition so as to include this realization of what has been said, though perhaps not realized, then of course there is inference; there is a new *thought.* Logicians who are mainly interested in their own mental processes are likely to admit this broader definition; those who are mainly interested in the relations of things are likely not to.

In the third example the case is still different. When any one says that all of the apostles were Jews he means to include each member of a certain definite number of individuals determined beforehand. He speaks demonstratively of certain individuals as such. The only question is whether or not he realizes as he should the identity of all the individuals that he speaks about. But when any one says that whoever has consumption is suffering from the presence of tubercular bacilli, he is not pointing to certain definite individuals determined beforehand. Rather he is speaking descriptively of any individual who happens to have consumption, no matter who he may be or how many there may be of them. To know that whoever has consumption has tubercular bacilli, one does not have to know first about each individual patient as such; he needs only to know that these bacilli are the sole cause of the disease. When, therefore, he puts the two premises together and concludes that some particular patient has tubercular bacilli he has gained some knowledge that could not possibly have been derived from a mere analysis of the major premise. He has reached a new aspect of things—found a relation not previously mentioned—and has undoubtedly made an inference.

Looking back at the three kinds of cases, we can see that in the first, where objects are individually specified in the major premise, the minor is superfluous, and in the conclusion there is no real inference. In the second, where a certain definite number of objects are mentioned in general

terms in the major, the minor would be superfluous if we realized all that is said, and such inference as there is consists in realizing what is said. In the third, where the major expresses a general law applicable to every individual of a certain kind, the minor is not superfluous, and in the conclusion there is real inference.

From all that has been said it is evident that the principle upon which we reason in the first figure is as follows: *What is true of an object specified in one way is true of the same object specified in any other way.* If we omit the case in which the major premise is a singular proposition, the principle amounts to this: *What is stated in a universal proposition is stated of every object to which the subject term is applicable;* or, less technically, *What is said to be true of every member of a group* (or of every object which possesses a given relation) *is said about each one of them, even though each is not separately thought of when the statement is made.*

Principle and cautions.

So much for the principle. In applying it we must observe certain cautions. In the first place, if the major premise speaks only of certain unspecified members of a group —*i.e.*, if the major is particular—we cannot be sure that any of the objects named in the minor, though members of the group, are objects spoken of in the major; and consequently no conclusion can be drawn.

It is true that some animals are fierce, and it is also true that all mice are animals; but it is not true that mice are fierce. If such a conclusion did happen to be true in any one case, that would not make it follow from the premises; for a conclusion does not *follow* unless we can be absolutely certain that *whenever premises of that kind* are true that kind of conclusion must be true also.

If we use a number of small circles to represent animals of various kinds, and let a plain stroke drawn through a circle, ϕ, indicate that the animal is fierce, and a stroke with a bar across it, $\not\phi$, indicate that it is not fierce, the major

premise gives this picture: ○ ϕ ○ ϕ ϕ ○ Some are said to be fierce, and of some nothing is said; so some of the circles are marked with a plain stroke and some are left unmarked.

If now we indicate mice by thickened dots, we must put all the dots within the circles to indicate what is stated in the minor premise, that each mouse is identical with some animal; but as there may be animals which are not mice, we must leave some circles without dots, thus: ⊙ ○ ○ ○ If it afterwards turns out that every animal is a mouse, we can fill in the remaining circles.

Now if we indicate in a single set of figures everything which has been asserted and remember this rule: *Do not put more marks than you have to in any one circle,* we get such a diagram as the following: ○ ϕ ϕ ⊙ ⊙ ○

Here dots and plain strokes do not coincide, but there is nothing in the diagram to indicate that they cannot. That would be indicated by drawing a crossed stroke (meaning not-fierce) through every dot, thus: ○ ϕ ϕ ϕ ϕ ○ The figure as it stands merely means that there is no evidence to show that any dot (mouse) must possess a plain stroke (fierceness). Whenever the major premise is particular it is possible to construct such a diagram.

If in the example given the major premise had been universal, every circle would have been marked, so that it would have been impossible to avoid putting the dots where they would also be marked; and so the diagrams would have indicated the conclusion that all mice are fierce: ϕ ϕ ϕ ϕ

If the major premise is not particular, but the minor is, that is, if the minor says that some members of a second group belong to the group spoken of in the major but does not say which members of the second group these are, then we can conclude that what was said in the major premise was said of some members of the group named in the minor, but we cannot possibly say which members they are. The objects cannot be designated any more definitely in the conclusion than they were designated in the premise. If all

young animals like play and some mice are young animals, we can conclude that some mice like play, ϕ ϕ ϕ · , but we cannot conclude that all mice like play or that any particular mouse likes it. In other words, if the minor premise is particular, the conclusion must be particular also. If we have no definite information to begin with, syllogistic manipulation will not supply it.

Putting together what has been said about the two premises, our FIRST CAUTION is this: In the first figure if the major premise is particular, no conclusion can be drawn; if the minor is particular, the conclusion must be particular. This caution can be stated less mechanically and without regard to figure or distinction of premises as follows:

1) *A relation can belong to some members of a group without belonging to all the members, to any given member, or to any one of a given group of members.*

The phrase 'to all the members' is really superfluous; for no relation could belong to each member without belonging to a given member. What is true of all mice must be true of this mouse. The last words of the caution—"or to any one of a given group of members"—are necessary in order to exclude a particular conclusion as well as a singular or universal when the major premise is particular.

The principle on which we reason in the first figure is quite as applicable when the major premise is negative as when it is affirmative. The proposition 'No men are perfect' states something about every man, not about no man. It means that every man is without perfection, and if we know Socrates to be a man, it is applicable to Socrates. If we represent the proposition by a diagram, we may cross our strokes to indicate that the objects represented have not the quality in question; but the strokes must be drawn through every circle, to indicate that every man is described, ϕ ϕ ϕ, and the circle that stands for Socrates must be marked with the rest.

But *when the minor premise in the first figure is negative* it

does not come under the principle, and *no conclusion can be drawn*. If we know that all the apostles were Jews, it will not tell us anything about Job's race to say that he was not an apostle. The function of a minor premise in this figure is to point out particularly some of the things spoken about in the major, and if an object is not one of those spoken about in the major, then nothing has been said about it one way or the other.

Clear as this is, there is a strong tendency in such cases to draw a negative conclusion, *e.g.*, The apostles were Jews; Job was not an apostle; therefore Job was not a Jew. The caution against yielding to this tendency might run as follows:

2) *To say that something is true of certain objects does not imply that it is false of others.*

The same tendency to draw negative conclusions where no conclusion at all should be drawn is found—though it is not so strong—where the minor premise is particular. If we are told that no men are perfect and that some rational beings are men, we have a right to conclude that some rational beings—namely, those that are men—are not perfect; but we have no right to conclude that some rational beings—namely, those that are not men—*are* perfect. To do so is to ignore the caution just given. It also involves the other blunder of interpreting 'some are' to mean 'some are and some are not'. The statement that some rational beings are men gives us no valid reason for believing that they are not all men. Both blunders are covered by the caution.*

Here is a set of diagrams for the benefit of any one who may wish to compare them with his own. If we wish to include negative minors, we may draw a short stroke through one side of the circles to indicate something outside of the

* The first caution covers fallacies of *illicit minor* and *undistributed middle* as they occur in the first figure; the second caution covers fallacies of *illicit major* in the same figure.

group that the circles stand for, or something that does not have their qualities.*

Major		and minor				Conclusion	
"	A	"	"	A	ϕ ϕ ϕ	"	A
"	E	"	"	A	ϕ ϕ ϕ	"	E
"	I	"	"	A	ϕ ϕ ⊙ ⊙	"	none
"	O	"	"	A	ϕ ϕ ⊙ ⊙	"	"
"	A	"	"	I	ϕ ϕ ϕ ·	"	I
"	E	"	"	I	ϕ ϕ ϕ ·	"	O
"	I	"	"	I	ϕ ⊙ ·	"	none
"	O	"	"	I	ϕ ⊙ ·	"	"
"	A	"	"	E	ϕ ϕ · ·	"	"
"	E	"	"	E	ϕ ϕ · ·	"	"

et cetera.

* If we wished to distinguish in our diagrams between syllogisms in which the major premise describes objects as they are in themselves and those in which it tells of some causal or other relation between objects, we could indicate the latter in some such way as the following, using the arrow to indicate the relation:

Every Slav hates every Semite ○ ○ ○ ⟫→ ○ ○ ○

Every Russian is a Slav ⊙ ⊙ ○

Every Jew is a Semite ⊙ ⊙ ○

Therefore every Russian hates every Jew, ⊙ ⊙ ○ ⟫→ ⊙ ⊙ ○

That is to say, The Slavs, including all the Russians, hate the Semites, including all the Jews.

Every Slav hates every Semite, and some Russians are Slavs and some Jews are Semites ○ ⊙ ⊙ ⟫→ ○ ○ ⊙

In the text I have treated universal propositions as though they were nothing more nor less than statements about every member of a certain

group of objects, and have omitted any reference to the causal relation which they so often imply. (See above, p. 109.)

The principle and cautions can be easily restated so as to take account of this causal relation, as follows:

Any object or relation (S) which possesses or involves a given relation (M) possesses or involves every other relation (P) which that relation (M) involves, provided that M really (*i.e.*, necessarily or always) involves it.

The fact that an object or relation (S) does not possess or involve a relation (M) which involves another relation (P) is no evidence that the object or relation (S) does not possess or involve this other relation (P).

The first paragraph covers the rule and the more important part of the first caution; the second covers the second caution. To cover the rest of the first caution we must add that a statement about some undesignated member or members of a group of objects (S) will not enable us to say anything about any particular one of them.

Here is an example of the sort of causal connections I speak of:

Going to war involves danger for the soldier.

Danger for the soldier involves distress for his family.

∴ Going to war involves distress for the soldier's family.

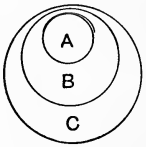
Each of the diagrams in the text is intended to represent at least one sample of every object mentioned in either premise, and to preserve the distinction between those that are described in a given respect and those that are not. I have tried to make the illustrations as concrete as possible by using closed figures—circles or dots—to represent things, and lines drawn through them to indicate attributes. When the sense is once understood these circles are not so good as a matter of practical convenience as simple letters written one after the other with the understanding that the first letter stands for a thing and those that come after it for attributes. AB would thus mean that all A's have the attribute B (*i.e.*, are B's). A, AB would mean that some A's have the attribute B. A, ABC, BC, would mean that some A's are B and all B's are C, and that therefore some A's are C. When these letters are used, a stroke over a letter indicates the absence of the attribute in question. $A\bar{B}C$, $\bar{B}C$ means that all A's are non-B, and all non-B's are C, and therefore all A's are C.

When we use these letters it is easy to get from pictorial representations to algebraic. A = AB means that every A has the attribute B, *i.e.*, that every A is also B.

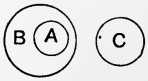
The pictorial representations most used are Euler's. These do not attempt to represent individual objects or to preserve the distinction between things and attributes; but deal with the mutual relations of classes. As the diagrams are drawn on precisely the same principle

for all the figures (including the fourth) there is no denying their convenience for any one who has to work out a set of problems. Here are some of the diagrams for the first figure.

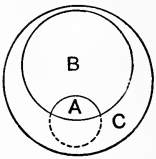
Major A
Minor A
Conclusion A



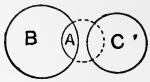
Major E
Minor A
Conclusion E



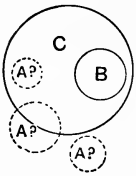
Major A
Minor I
Conclusion I



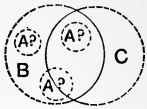
Major E
Minor I
Conclusion O



Major A
Minor E
Conclusion —



Major I
Minor A
Conclusion —



The dotted lines are used to indicate doubt as to where the circle or the part of the circle in question should fall.

CHAPTER XIV.

THE SECOND FIGURE OF THE SYLLOGISM.

Function and general cautions.

In this figure each premise describes an object or set of objects, and from the nature of the two descriptions the conclusion tells whether or not the objects are identical with each other.

The man that came to my house was tall and thin;
The man that went to your house was short and fat (*i.e.*, not tall and thin);
∴ The man that came to my house is not the man that went to your house.

Crows do not sing;
This bird sings;
∴ This bird is not a crow.

Whales suckle their young;
Fishes do not;
∴ Whales are not fishes.

The second and third of these examples differ from the first in this respect: In the first, two given objects are compared, and we conclude that they are not identical; in the second a given object is compared with a class of objects, and we conclude that it does not belong to the class—that it is not the kind of thing to which the class-name applies; in the third two classes of objects are compared. In the first case we are concerned with identification in the narrowest sense of the term; in the others with classification. In the general description of the figure I have mentioned only the identification because it is the more fundamental

and the classification really depends upon it. If we could not distinguish between individual objects, we could not distinguish between classes. To say that whales are not fishes is to say that there is not any whale which is identical with any fish. To classify is thus merely to distinguish between individuals in groups, and the principles by which we distinguish classes are nothing more than those by which we distinguish individuals.

In this figure, therefore, both premises are concerned with descriptive relations, and the conclusion with a relation of identity.*

The next thing to be noticed about the figure is that the two premises must describe their objects in the same respect. If I describe the man I saw as tall, and you describe the one you saw as agreeable, the descriptions indicate absolutely nothing about the identity of the men.

But even when both premises describe their objects in the same respect a conclusion is not always possible. If each of two persons had met a tall man named Smith, they would not necessarily have met the same man. Two Dromios or two atoms of hydrogen might have innumerable points of

* For the purposes of this figure propositions which in themselves are not strictly descriptive are treated as such. When we conclude, for example, that Newhaven and New Haven are different cities because one is on the road from Paris to London, and the other on the road from New York to Boston, the geographical or spatial relations of each of them to the adjacent cities are practically regarded as a part of the city itself. The distance between Newhaven and London or Paris is a spatial relation and belongs as much to London or Paris as to Newhaven. But when it serves to distinguish Newhaven from New Haven it is treated as though it belonged, like its size or its history, to Newhaven itself. For this reason we make the word Newhaven the subject of the sentence in which the facts are expressed. We do not say "London is a usual terminus for persons traveling from Paris and Newhaven", or "A good way to reach London from Paris is through Newhaven", or "London is so far from Newhaven and so much farther from Paris by way of Newhaven". What the syllogism involves is not the distance of London or Paris as such, but all of them in so far as they characterize Newhaven.

resemblance and yet be two. As the points of resemblance between two complex things increase, the probability that the things are really identical also increases; but no amount of resemblance can supply theoretically absolute proof of such identity. The prisoner in the dock might bear every resemblance to the man who was seen reeling on the street the night before and yet possibly, though not probably, be a different man. We could be absolutely certain of their identity only if the reeling man had been arrested at the time and never lost sight of for a moment until he was placed in the dock.

The fact is that the identity of two things involves a great deal more than mere resemblance, no matter how complete the resemblance may be. Consequently, though we can often prove that things are not identical from the fact that they are dissimilar, we can never prove that they are identical from the fact that they are similar. If men are mortal and angels are not mortal, it follows that men are not angels; but if men are mortal and horses are mortal, it does not follow that men are horses. In this figure negative conclusions alone are valid.

There is no logical blunder more frequent than to conclude that because things are alike they are necessarily the same. Flour is white, says the child; what I see all over the ground is white; therefore what I see all over the ground is flour.

Good dollars are silvery-looking discs bearing a certain stamp;
This is a silvery-looking disc bearing that stamp;
∴ This is a good dollar.

Benevolent people smile affably;
This man smiles affably;
∴ This man is benevolent.

All P is M;
All S is M;
∴ All S is P.

But what the child sees on the ground is snow, not flour, and sometimes our silver disc is counterfeit, and the smiling stranger a brute. S is not always P.

The logical trouble comes when we mistake probabilities for certainties. In practical life it is usually better to take an occasional counterfeit coin than to insist upon testing them all, better to be deceived in a character occasionally than to refuse all intercourse with one's fellows until they prove their right to be trusted, better to bow to a stranger than to cut a friend. But a good rule of conduct when we must act in a hurry is not necessarily a good rule of conduct or thought when we have time to be careful. The bank teller must be on the watch for counterfeit money, the employer of a confidential clerk must look behind his face, and the sheriff should be sure of his man. In the same way, as students of deductive logic we must reject all conclusions that do not follow with absolute necessity from the premises.*

* The significance of this fallacious reasoning A A A in the second figure may become clearer if we show its relations to the first figure.

In the second figure we say

All Y is Z
All X is Z
∴ All X is Y

Now if we could convert the first premise simply, *i.e.*, without altering the quantity, we should get a perfectly valid syllogism in the first figure:

All Z is Y ϕ ϕ ϕ
All X is Z
∴ All X is Y

But we cannot convert the premise simply. All we can say is that some Z is Y, and from this major premise no conclusion can be drawn. ϕ ○ ⊙

If we happened to know not only that some Z is Y, but that most Z is Y, we might conclude that X is probably Y.

Most silvery looking discs bearing a certain stamp are good dollars.
This is a silvery looking disc bearing that stamp.
∴ This is *probably* a good dollar.

Even as a rule for hurried action it is not wise to draw affirmative

We have seen that in this figure no affirmative conclusion can be drawn—similar things are not necessarily identical. But how about the negative conclusion? Can we say with any more certainty that dissimilar things are not identical? Is not the tadpole of last summer identical with this summer's frog, the bright-winged bird of the spring with the sober-looking one of the summer, the grub of one month with the butterfly of the next, Saul the persecutor with Paul the apostle? On the other hand, no one can suppose that the tadpole is identical with the later bird, or the grub with Paul, or even that a tadpole seen this morning is identical with a perfect frog seen this noon. The fact is that objects can be distinguished from each other by their qualities or relations only when these are different at one and the same instant; so that if the objects are not observed simultaneously we cannot distinguish them by their qualities or relations unless we believe these latter to be so permanent that they cannot be wholly changed in the time which has elapsed between the two observations. What qualities or relations are relatively permanent and what are not we can learn only through experience, without a constant appeal to which logic is perfectly helpless.*

To sum up, the PRINCIPLE on which we reason in the second figure is that *Dissimilar objects are not identical;* and these are the CAUTIONS:

3) *Similarity does not prove identity.*†

4) *Dissimilarity does not prove non-identity if the object might have changed.*

conclusions in the second figure unless we have reason to believe that the converse of the major premise is *usually* true. In other words, the only possible justification for such reasoning in the second figure is found in the fact that it sometimes represents a fairly good *induction* in the first figure.

* Of course it is only *things* that change and still retain their identity. Blue is not like yellow now, and never will be.

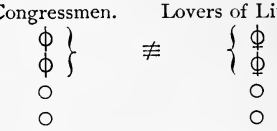
† This caution covers the fallacy of *undistributed middle* as it occurs in the second figure.

5) *Different descriptions do not imply dissimilarity unless the relations described are incompatible.*

So far our examples have all dealt with universal or singular propositions, and no difficulties have arisen from questions of quantity. It is clear that from universal or singular premises a universal or singular conclusion can be drawn; but can we draw any conclusion at all from particular premises?

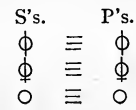
Quantity in the second figure.

First, when both premises are particular. If some members of Congress have blue eyes and some lovers of literature have brown eyes, *i.e.*, have not blue eyes, what inference can be drawn? It is clear enough that certain members of Congress, namely, those with blue eyes, are not identical with certain lovers of literature, namely, those with brown eyes. Using the sign $\equiv$ to indicate identity and $\not\equiv$ to indicate non-identity, we can make such a diagram as this:



meaning that no one of the described congressmen is identical with any one of the described lovers of literature. But for all we know, each one of the described congressmen may be identical with one of the undescribed lovers of literature, and *vice versa*, *e.g.*, the first in one column with the third in the other, and so on. It *might* be that every congressman was a lover of literature and every lover of literature a congressman, in spite of the fact that a blue-eyed congressman is not a brown-eyed lover of literature.

To put the matter somewhat differently. From the fact that certain S's are not identical with certain P's, it by no means follows that certain S's are not P's at all. Suppose every S to be a P as indicated below:



The first S is identical with the first P, but for that very reason it cannot possibly be identical with the second or the third. If there are in all one hundred different S's and one hundred different P's, and if each of the S's is identical with one of the P's, then there are ninety-nine different P's with which that particular S is not identical. Jones the congressman is identical with Jones the literary man, but not with Smith or Brown.

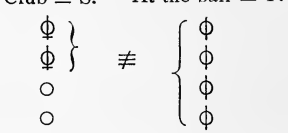
As long as there are two or more S's it must necessarily be true that certain S's and certain P's are not identical, and it does not make any difference whether every S is a P, or no S is a P, or some S's are P and some are not. Hence the inference we seem to draw *from two particular premises in the second figure*—that certain S's are not identical with certain P's—does not follow from these premises any more than from any others in which several S's are mentioned. It is equivalent to the mere truism that no one S can be identical with more than one P. It is something we might have known long before we knew anything about the special facts stated in the premises. It is practically no inference whatever from these premises and we may as well say that from such premises *no inference can be drawn.*

When one premise is particular and the other singular the case is much the same. If we have been told that some of the masqueraders at a ball were tall and if we know that John is short, we can be sure that John was not one of these tall masqueraders. We can say if we like that there were certain persons there with whom John was not identical; but we know that already if we happen to know that there were more persons than one present. It doesn't depend at all upon the question of height. This inference is thus worthless; and no other can be drawn.

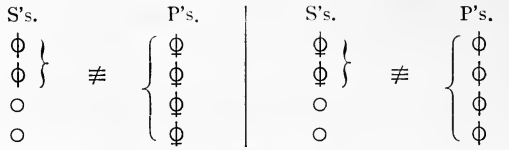
When one premise is universal the case is different. If everybody at the ball was tall and John is short, we know

$$\phi \neq \left\{ \begin{matrix} \phi \\ \phi \\ \phi \end{matrix} \right.$$

that he is not identical with any one of them, *i.e.*, he was not there. Similarly if we had been told that every one at the ball was tall and that there are some members of the club who are not tall, we could be sure that there are some members of the club who are not to be identified with any one who was at the ball, *i.e.*, some members of the club were not at the ball.

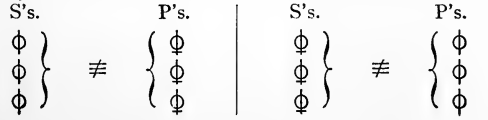


On the other hand we could not be sure that some of those at the ball were not members of the club. If some S's are M and no P's are M, or if some S's are not M and all P's are M, it follows that some S's are not P's; but it does NOT follow that some P's are not S's, for it may be that each of the P's is identical with one of the undescribed S's. The



reasoning is valid only if you arrange your conclusion so as to have for its predicate the term which occurred in the universal premise. In the technical language of the syllogism: *The major premise* (*i.e.*, the one containing the predicate of the conclusion) *must be universal.* The difference between concluding that some S's are not P's and that some P's are not S's may become clearer if we remember that Proposition O cannot be converted.

When both premises are universal it is clear enough that a universal conclusion can be drawn; and of course it makes



no difference which of the premises is affirmative or negative, so long as the quality of the two is different.

Putting all this together we can add another CAUTION:

6) *We cannot say that any S's are not P's unless each of the S's in question is different from every P.** To put it somewhat differently, *Evidence sufficient to prove that some S's are not P's may not be sufficient to prove that some P's are not S's, and vice versa.*

The general principle of the figure and the caution respecting quantity are worked out together in the following formula which a student may use if he likes instead of the separate statements:

If all the members of one group differ in a given respect from all the members of another, then no member of either group is a member of the other. If some members of one group differ from all the members of another, then there are some members of the first group which are not members of the second; but it does not follow that there are members of the second which are not members of the first. The mere fact that some members of one group differ from some members of another proves that those particular individuals are not identical, but it does not prove that any member of either group is not also a member of the other.

* This caution covers the fallacy of *illicit major* in the second figure. *Illicit minor* is covered in the second figure as in the first by the first caution.

CHAPTER XV.

THE THIRD FIGURE OF THE SYLLOGISM.

In this figure one premise asserts that a certain object possesses (or does not possess) a given relation, and the other premise asserts that this same object possesses (or does not possess) another given relation, and on the strength of these premises the conclusion asserts that the presence (or absence) of one of the relations sometimes coincides with the presence (or absence) of the other—*e.g.:*

Purpose, principle, and general caution.

Shakspeare was perfectly sane;
Shakspeare was a genius;
∴ Some geniuses (one at least) are perfectly sane,
or Some perfectly sane persons are geniuses.

Sin is evil;
Sin exists;
∴ Something evil exists.

The ancient Stoics were not enlightened by the Scriptures;
These Stoics believed in God;
∴ Some persons not enlightened by the Scriptures have believed in God.

This figure is used mainly to disprove sweeping statements or alleged general laws, by displaying cases to which they will not apply. If any one maintains that every genius is a morbid degenerate, we can disprove the statement by calling

his attention to the fact that Shakspeare or Goethe or Plato was a man of undoubted genius yet perfectly free from every trace of morbid degeneracy. If he maintains that in God's world no evil can exist, we need only point to sin. If he maintains that through the Scriptures alone can God be known, it is only necessary to remind him of the Stoics.

In this figure more than in any other the machinery of the syllogism seems very cumbersome and unnecessary. In ordinary speech and thought we consolidate the two premises into one statement: Shakspeare was a genius ϕ and yet not morbid ϕ, Sin is an evil ϕ and yet exists ϕ, The Stoics believed in God ϕ ϕ ϕ, though not enlightened by the Scriptures ϕ ϕ ϕ.

The PRINCIPLE on which we reason is evidently this:

A single actual case in which two positive or negative relations coincide proves that they are not incompatible.

In the examples here given Shakspeare's freedom from morbidness and the Stoics' ignorance of the Scriptures may be regarded as negative relations. As applied to these two cases the principle means that freedom from morbidness is not inconsistent with genius, and *vice versa;* that ignorance of the Scriptures is not inconsistent with a knowledge of God, and *vice versa.*

When both relations are negative a conclusion can be drawn quite as well as when one or both are positive. From the fact that stones are neither virtuous nor vicious ϕ ϕ ϕ we can prove that the absence of one of these qualities does not necessarily preclude the absence of the other, and thus disprove the statement that everything in the world must be one or the other.*

* The old syllogistic rule says: *From two negative premises no conclusion can be drawn;* but in the third figure the rule is evaded by obverting one or both premises. So that if we say 'Stones aren't virtuous, and stones aren't vicious' we cannot draw a conclusion, but if we say stones aren't virtuous and stones are not-vicious we can! Conclusions do not depend upon the form of words in which the premises are stated, but upon the real state of affairs to which they point; yet when we consider

Our conclusion in the example given does not depend upon the mere fact that there are no such things as virtuous or vicious stones, for if there were no stones at all this would still be a fact, though the conclusion would not follow; but upon the fact that there are stones which are neither virtuous nor vicious. To state the case more generally: The conclusion does not depend upon the fact that objects with the relation in question do not exist, but upon the fact that objects do exist without the relation.

This last statement suggests what is involved in the principle of the figure as I have stated it, but what cannot be too much emphasized, that the cases from which our conclusions are drawn must actually exist. We cannot prove that a good man may come to grief by Colonel Newcome, that a brave man may murder his wife by Othello, that good nature will not save us from cruelty by Arthur Donnithorne, that wounds will not destroy existence by the heroes of Valhalla, or that a pumpkin-shell may be transformed into a chariot by the adventure of Cinderella. From particular cases in one universe we cannot prove the compatibility of relations in another.

The first CAUTION to be observed in using this figure is —put technically—that *its conclusion is always particular*. If all men are mortal and all men are bipeds, we can be sure that so far as men are concerned these two attributes coincide, but this does not prove that every mortal creature has two legs or that angels and all other bipeds are sure to die. In other words, *the fact that certain objects possess each of several positive or negative relations does not prove that other objects may not possess one without the other or exist without either*. Or more briefly:

7) *Any number of coincidences between relations will not prove that they coincide always.*

this state of affairs there is a sense in which we can say that premises in this figure from which a conclusion can be drawn must both be affirmative in meaning, no matter what their form. See the next paragraph.

The briefer statement is less comprehensive, but it will cover any case that is likely to arise.*

In this figure as in the second we must be careful not to be confused by negative relations. From the fact that all M's are P, and that no M's are S, we can infer that some P's are not S; but we cannot infer that some S's are not P.† From the Pope we may perhaps prove that there are infallible mortals, but not that there are fallible immortals. It takes the Devil for that. The sixth caution or its corollary—"Evidence sufficient to prove that some S's are not P's may not be sufficient to prove that some P's are not S's"—is one which we tend to ignore or misunderstand continually. Altogether the best way to observe it without confusion, whether we are reasoning in the third figure or in one of the others, is to put our premises affirmatively, with the negative element, when there is one, in the predicate (*i.e.*, to obvert negative premises and conclusions). When we say that there are infallible mortals or that there are fallible immortals, our meaning is much clearer and the distinction between the two statements is much more obvious ‡

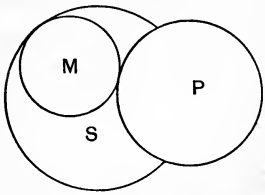
Negative relations.

* This caution covers *illicit minors* in the third figure. Put in terms of causal relations the caution is this:

A single coincidence proves the compatibility of relations, but no number of coincidences can prove their necessary connection.

† This caution covers *illicit majors* in the third figure as well as in the second.

‡ The statement in this form has moreover the advantage of directing attention to the fact that we are talking about real things. (See top of p. 173.) The diagrams in the text seem to me to accent the affirmative element which reasoning in the third figure particularly involves, as well as to guard against the confusion referred to in the text better than Euler's. Students always find it difficult to see why this figure does not mean that some P's are not S as well as that some S's are not P. But if we represent S by a vertical stroke and P by a horizontal the distinction between M which is S but not P ⊕ and M which is P but not S ⊕ is obvious, and with it the distinction between S not-P ┼ and P not-S ┼.



than when we say that some mortals are not fallible or that some fallible beings are not mortal.

By the coincidence of two relations we mean that they both belong to the same individual. Whether they do or not is primarily, of course, a matter of observation in each particular case; but when the coincidence of the relations must be inferred by putting together statements about the existence of each we must remember one more CAUTION:

Quantity of the premises.

8) *Two different relations can belong to individuals of the same class without belonging to the same individual, unless at least one of them belongs to every individual in the class.**

If we know that this particular X is both Y and Z, we know of course that Y and Z coexist. If we know that every X is Y and every X is Z, we can be sure that each X is both Y and Z; if we know that every X is Y and that some X is Z, we can be sure that some X or other is both Y and Z; but if we only know that some X's or other are Y and that some X's or other are Z, we cannot be sure that Y and Z ever belong to the same X. This is what is meant in this figure by the technical rule that *from two particular premises no conclusion can be drawn.* The technical rule should have added that from a particular premise and a singular premise in the third figure no conclusion can be drawn; for it does no good to know which particular X's are Y so long as we do not know which are Z.

The principle and all the cautions can be put together in such a general statement as this:

The coincidence of relations—whether positive or negative—proves that they are compatible, but it does not prove that either of them involves the other, or that the absence of one is compatible either with the presence or with the absence of the other. Moreover the fact that two relations belong to objects of the same class will not prove that they belong to the same objects unless at least one of them belongs to all the objects in the class.

* This covers undistributed middles in the third figure.

CHAPTER XVI.

THE ALLEGED FOURTH FIGURE.*

So long as the various figures of the syllogism were distinguished by the mere arrangement of terms rather than by the relations involved in the reasoning, it seemed reasonable that there should be a figure to represent every possible arrangement. Consequently to the three figures which we have discussed, and which were all that Aristotle recognized, Galen (131–200 A.D.) added a fourth. The arrangement of terms in each is as follows:

Origin of the figure.

First Figure.	Second Figure.	Third Figure.	Fourth Figure.
MP	PM	MP	PM
SM	SM	MS	MS
SP	SP	SP	SP

The four figures cover every possible permutation of the terms.

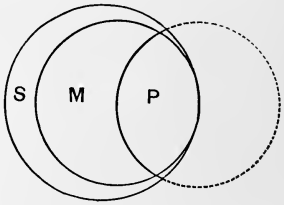
Reasoning in the fourth figure outside of exercises in formal logic is extremely rare. Beyond mere questions of whether one class includes or excludes members of another, it has no significance; and though it is easy to arrange problems in such a way that they will fall within the figure, they lose most of their meaning when so arranged and seem strained and unnatural. Nevertheless one ought to know how to deal with the problems when they arise. There are three ways of doing this. The first is to disregard their meaning and solve them by means of a set of purely mechanical rules. These rules are equally applicable to all four of the figures; but inasmuch as we have tried

Three ways of dealing with it.

* This chapter is not essential.

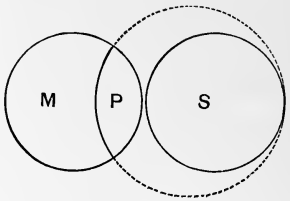
to get along without them—or at least to interpret them—in the other figures, it seems a pity to take refuge in them now.*

The second way of dealing with syllogisms in the fourth figure is to assume that they are concerned merely with relations of inclusion or exclusion between a number of classes, all of which are assumed to exist, and then to test them by Euler's diagrams. For example,



Some P's are M's;
All M's are S's;
∴ Some S's are P's.

The conclusion follows; for we cannot possibly put the circle M within the circle S, and part of the circle P within the circle M, without some part of the area of S falling within the circle P.



Some P's are M's;
No M's are S's;
∴ Some S's are not P's.

* The rules, as stated by Jevons, are as follows:

1. Every syllogism has three and only three terms. These terms are called the major term, the minor term, and the middle term.
2. Every syllogism contains three and only three propositions. These propositions are called the major premise, the minor premise, and the conclusion.
3. The middle term must be distributed once at least, and must not be ambiguous.
4. No term must be distributed in the conclusion which was not distributed in one of the premises.
5. From negative premises nothing can be inferred.
6. If one premise be negative, the conclusion must be negative; and *vice versa*, to prove a negative conclusion one of the premises must be negative.

From the above rules may be deduced two subordinate rules, which it will nevertheless be convenient to state at once.

7. From two particular premises no conclusion can be drawn.
8. If one premise be particular the conclusion must be particular.

These rules are not absolutely reliable unless we assume that the objects denoted by each term in the syllogism exist.

The conclusion does not follow, for we can construct a diagram which represents the premises without representing the supposed conclusion.

The third, and of course the best, way of dealing with syllogisms in the fourth figure which we are called upon to test is to try to give them a rational interpretation and thus work in the light. When we come to interpret such syllogisms we shall find that we must regard them as concerned either with the relations between classes which we have just discussed or with the relations peculiar to some one of the three other figures, to which the syllogism in question can be 'reduced' by converting the conclusion or one or both of the premises.

If we turn back to the table which shows the arrangement of terms in each figure, it is easy to see that by converting the major premise of a syllogism in the fourth figure we get the arrangement of terms found in the third; by converting the minor we get that found in the second; and by converting the conclusion or by converting both premises we get that found in the first. It is thus *formally* possible to interpret the fourth figure by any one of the three others; and so long as we do not attempt to test a universal conclusion by the third figure (which is itself incapable of giving such a conclusion) or an affirmative conclusion by the second (which never gives it), that is to say, so long as we do not ask a figure to test a kind of conclusion that it is itself unable to draw, it does not make the slightest difference, as far as the formal process is concerned, which of the first three figures we use to test a syllogism in the fourth. If it is valid, there are always at least two of the first three figures in which the conclusion can be proved. But since the second and the third figures both have the limitations just referred to, any one who merely wants an easy formal test will save himself some thinking by making it a rule to test every argument in the fourth figure by reducing it to the first. In doing this he may assume that

Formal reduction.

no syllogism in the fourth figure is valid unless the conclusion can be obtained either by converting a conclusion which can be drawn in the first figure from the same premises, or by reasoning in the first figure from the converse of the premises. In other words, if you have to test a syllogism in the fourth figure, ask first whether it is not merely a syllogism in the first figure with the conclusion converted (or converted and weakened, *i.e.*, O from E as well as I from A). If it is, the reasoning is usually assumed to be valid. If it is not, then convert the premises, if they can be converted (remembering that A must be converted into I and that O cannot be converted at all), and see whether the conclusion in question will not follow from them according to the principle of the first figure without the violation of any caution. If it will, the syllogism in the fourth figure is assumed to be valid; but if the syllogism will not stand either of these tests, it certainly is not valid.

Here are some examples:

(1) All P's are M's;
All M's are S's;
∴ Some S's are P's.

According to the rule just laid down this syllogism is valid, because from the premises as they stand we can reason in the first figure to the conclusion 'All P's are S's', which by conversion gives the conclusion in question 'Some S's are P's'.

The following syllogism is not valid:

(2) All P's are M's;
All M's are S's;
∴ All S's are P's;

because (1) the universal conclusion is more than we can get by converting "All P's are S's", and (2) if we convert the premises we get

Some M's are P's;
Some S's are M's;

from which we cannot draw any conclusion whatever without disregarding the caution which says that from a statement about some undesignated members of a class we cannot infer anything about any designated member or any one of a designated set of members.

(3) No gods are Americans ;
All Americans are mortal ;
∴ Some mortals are not gods.

To reduce this syllogism to the first figure we must convert the premises, *e.g.*,

No Americans are gods ;
Some mortals are Americans ;
∴ Some mortals are not gods.

This reasoning is perfectly valid, and thus our original syllogism is vindicated.

One more example :

(4) All students are human ;
No human beings like torture ;
∴ Some beings that like torture are not students.

This syllogism can be proved valid like the others by means of the first figure, though when we try to 'reduce' it a difficulty immediately confronts us, for the conclusion of the syllogism is a particular negative proposition which cannot be converted, and when we convert the premises the conclusion will not follow, viz.:

Some human beings are students ;
No beings that like torture are human ;
∴ Some beings that like torture are not students.

To reason in this way violates this caution : 'To say that something is true of certain objects does not imply that it is false of others', *i.e.*, to say that some (or all) human beings are students does not imply that beings who like torture and are therefore not human are not students.

We cannot convert the conclusion, we cannot prove it by converting the premises, and yet the reasoning is valid ! I

gave this example in order to bring out the difference between converting a conclusion and obtaining that conclusion by converting something else. From the premises in question, 'No human beings like torture' and 'All students are human', we *can* reason in the first figure to the conclusion that 'No students like torture'. Converting this we get 'No beings that like torture are students', and if this is true it is necessarily true also that 'Some beings that like torture are not students'. The fact that this is *less* (if it really is less) than we might have inferred does not interfere with the validity of the inference. That is to say, the conclusion given in the example can be obtained by converting the conclusion in the first figure, though the conclusion in the first figure cannot be obtained by converting the conclusion given in the example. We were testing the fourth figure by the first, not the first by the fourth!

I have spoken at some length about this reduction to the first figure because it is the traditional method of testing syllogisms not only in the fourth figure but in the second and third as well. But an undiscriminating reduction to the first figure has no more value for thought than a mechanical use of cut-and-dried 'rules of the syllogism', and probably has much less value than the use of the diagrams. Our thought grows mechanical all too soon, and it is a pity for logic of all studies to hasten the process. If we are really to work in the light in testing syllogisms of the fourth figure, their 'reduction' must be accompanied by their interpretation, and the figure to which we reduce them must be determined by the interpretation—not by mere convenience for formal manipulation.*

* Looking back at example No. 1, let us fill it out as follows:

All persons a hundred miles above the surface of the earth (P's) are organic beings beyond the pressure of the atmosphere (M's).

All organic beings beyond the pressure of the atmosphere (M's) become greatly swollen (S's).

From these premises it is easy enough to draw the conclusion in the first figure that all persons a hundred miles above the surface of the earth

(P's) become greatly swollen (S); meaning that if a person should reach such an altitude the intra-organic pressures, not counteracted by pressure from without, would cause him to swell up. But the conclusion in the fourth figure, that some things which become greatly swollen (S's) are persons a hundred miles above the surface of the earth (P), looks more like a description of some actually existing swollen objects than like an account of what would happen under certain purely hypothetical circumstances. It cannot be turned into hypothetical form, and the implication that such things as swollen persons a hundred miles above the surface of the earth exist is certainly much stronger, to say the least, than in the conclusion drawn according to the first figure. In so far as it involves such an implication the conclusion in the fourth figure is of course misleading and fallacious, for we have no right to confuse hypothetical and real conditions. (See page 109, note.)

Example No. 3 can be reduced as we have seen to the first figure, but if the two universal propositions which compose the premises are interpreted as stating causal relations no conclusion is possible; that is to say, from the fact that deity involves not being an American and that being an American involves mortality we cannot draw any conclusion. The conclusion follows only if we assume that Americans exist. It is in the third figure that the syllogism is most natural and most significant, *e.g.*: All Americans are mortal and none of them are gods; therefore 'Some mortals are not gods'. The Americans, not gods or mortals as such, are evidently the concrete individuals from a knowledge of whom the conclusion is drawn.

In the case of example No. 4 the conclusion drawn, "Some beings that like torture are not students", stands the traditional test of reduction to—or rather deduction from—the first figure; but so far as it implies that beings who like torture exist, it does not follow from the premises. Even the universal conclusion, "No beings that like torture are students", is likely to be misleading, because the causal relation between being a student and disliking torture is so remote that the statement looks a good deal like a description of actually existing beings that like torture. It would have been better to say that if any being likes torture it is not a student. It is this which follows from the premises.

The sum and substance then of what I have said in criticism of this figure is this: It obscures the real relations under discussion, and in doing so is likely to lead to erroneous conclusions besides tempting us to work in the dark by a rule of thumb.

CHAPTER XVII.

OTHER DEDUCTIVE ARGUMENTS.

We have already distinguished between categorical propositions and those which are hypothetical or disjunctive. The syllogisms discussed so far involved only categorical propositions, but there are also syllogisms in which hypotheticals and disjunctives have a place. **Hypothetical syllogisms.**

Hypothetical syllogisms run as follows:

If A is B, C is D;	If a man is a Christian, he forgives;
A is B;	J. S. is a Christian;
∴ C is D.	∴ J. S. forgives.

If A is B, C is D;	If a man is a Christian, he forgives;
C is not D;	J. S. does not forgive;
∴ A is not B.	∴ J. S. is not a Christian.

A Hypothetical Syllogism is thus one in which the major premise is a hypothetical proposition and the minor a categorical.

The first pair of examples, in which the state of affairs mentioned in the consequent part of the major premise is proved to exist, are said to be *constructive* or of the *modus ponens;* the second pair, in which the state of affairs mentioned in the antecedent part of the major is proved not to exist, are said to be *destructive* or of the *modus tollens.* According to this a syllogism might be constructive though

its conclusion were negative, and destructive though affirmative, *e.g.:*

If A is B, C is not D;
A is B;
∴ C is not D (Constructive).

If A is not B, C is not D;
C is D;
∴ A is B (Destructive).

Hypothetical syllogisms look like an entirely new sort; but the novelty lies altogether in the verbal form, not in the relations expressed; for we have seen (page 109) that the relations expressed by hypothetical propositions can be expressed about as well in universal categorical propositions, and when the hypothetical major premise of a hypothetical syllogism is put into categorical form only a slight change is required in the minor to make the syllogism also categorical. Making these changes, we get

A state of affairs in which A is B is a state of affairs in which C is D;
This is a state of affairs in which A is B;
∴ This is a state of affairs in which C is D.

A Christian forgives;
J. S. is a Christian;
∴ J. S. forgives.

A state of affairs in which A is B is a state of affairs in which C is D;
This is a state of affairs in which C is not D;
∴ This is a state of affairs in which A is not B.

A Christian forgives;
J. S. does not forgive;
∴ J. S. is not a Christian.

The constructive hypothetical syllogism thus resolves itself into an ordinary syllogism in the first figure; the destructive into one in the second.

The formal rule for hypothetical syllogisms is that *the*

minor premise must either affirm the antecedent of the hypothetical proposition in the major (as in the first two of these last examples) *or deny the consequent* (as in the second two). In the first of the following examples we commit the *Fallacy of Denying the Antecedent;* in the second, the *Fallacy of Affirming the Consequent.*

If a man is a Christian, he forgives;
J. S. is not a Christian;
∴ J. S. does not forgive.

If a man is a Christian, he forgives;
J. S. forgives;
∴ J. S. is a Christian.

Turning from words to things, the meaning of the formal rule is as follows:

If the presence of one state of affairs (AB) always involves the presence of another (CD), and if the first state of affairs (AB) is present, the second state of affairs (CD) must also be present; if the second state of affairs (CD) is absent, the first state of affairs (AB) cannot be there to involve its presence; but the first state of affairs (AB) can be absent without involving the absence of the second (CD); and the second (CD) can be present without involving the presence of the first (AB).

These fallacies of ' denying the antecedent ' and ' affirming the consequent ' would not be fallacies at all if the world were so constituted that there was only one cause capable of producing a given effect or one premise capable of involving a given conclusion. If a person had to be drowned in order to be killed, we could not only say, ' He is drowned, therefore he is killed ', ' He is not killed, therefore he is not drowned '; but we could also say, ' He is not drowned, therefore he is not killed ', and ' He is killed, therefore he is drowned '. To avoid the fallacy we should think of what we are saying and remember that the world is not constituted in this way, but that any one of several causes may pro-

duce essentially the same result and any one of several premises involve the same conclusion.

Disjunctive syllogisms. Disjunctive Syllogisms are those which contain a disjunctive major premise and a categorical minor, *e.g.:*

A is either B or C;	Either J is not K or L is M;
A is not B;	J is K;
∴ A is C.	∴ L is M.

In order that any conclusion should be justified it is necessary that the minor premise deny the existence of one of the alternatives mentioned in the major. We cannot say 'A is either B or C; it is B; therefore it is not C'.*

What we must be most careful about in the case of these syllogisms is to see that the major premise is really true; that there is no alternative which it does not mention. We should not say 'This man must be either wise or foolish, he is not wise, therefore he is foolish'; for there are many persons of medium intelligence who cannot fairly be called either wise or foolish.

A Dilemma is 'a syllogism having a hypothetical major premise with more than one antecedent and a disjunctive minor'. "In common speech . . . we are said to be in a dilemma when we have only two courses open to us and both of them are attended by unpleasant consequences. In arguments we are in this position when we are shut into a choice between two admissions and either admission leads to a conclusion which we do not like." †

Dilemmas.

According to Jevons the Dilemma takes at least three different forms. "The first form is called the *Simple Constructive Dilemma:*

If A is B, C is D; and if E is F, C is D;
But either A is B, or E is F;
Therefore C is D.

* See p. 106 on meaning of '*either*'. † Minto, p. 222.

"Thus 'if a science furnishes useful facts, it is worthy of being cultivated; and if the study of it exercises the reasoning powers, it is worthy of being cultivated; but either a science furnishes useful facts, or its study exercises the reasoning powers; therefore it is worthy of being cultivated.'

"The second form of dilemma is the *Complex Constructive Dilemma*, which is as follows:

If A is B, C is D; and if E is F, G is H;
But either A is B, or E is F;
Therefore either C is D, or G is H.

"It is called complex because the conclusion is in the disjunctive form. As an instance we may take the argument, 'If a statesman who sees his former opinions to be wrong does not alter his course, he is guilty of deceit; and if he does alter his course, he is open to a charge of inconsistency; but either he does not alter his course or he does; therefore he is either guilty of deceit or open to a charge of inconsistency.' In this case as in the greater number of dilemmas the terms A, B, C, D, etc., are not different."

The third form—the *Destructive Dilemma*—"is always complex, because it could otherwise be resolved into two unconnected destructive hypothetical syllogisms. It is in the following form:

If A is B, C is D; and if E is F, G is H;
But either C is not D, or G is not H;
Therefore either A is not B, or E is not F.

"For instance, 'If this man were wise, he would not speak irreverently of Scripture in jest; and if he were good, he would not do so in earnest; but he does it either in jest or in earnest; therefore he is either not wise or not good' (Whately).

"Dilemmatic arguments are, however, more often fallacious than not, because it is seldom possible to find instances where two alternatives exhaust all the possible

cases, unless indeed one of them be the simple negative of the other in accordance with the law of excluded middle. Thus if we were to argue that 'if a pupil is fond of learning he needs no stimulus, and that if he dislikes learning no stimulus will be of any avail, but that, as he is either fond of learning or dislikes it, a stimulus is either needless or of no avail', we evidently assume improperly the disjunctive minor premise. Fondness and dislike are not the only two possible alternatives, for there may be some who are neither fond of learning nor dislike it, and to these a stimulus in the shape of rewards may be desirable." *

Three forms of abbreviated argument.

Principles of logic have reference to the relations of objects, not to the words in which those relations are expressed. From this it follows that variations can be introduced into the wording of an argument without affecting its validity. One variation that has seemed important enough to be discussed in almost every text-book since Aristotle consists in taking for granted certain of the relations involved, without any explicit mention of them. 'Enthymemes', 'Epicheiremata' and 'Sorites' are names for arguments abridged in different ways.

An *Enthymeme* (from ἐν, in, and θυμος, the mind) is a syllogism—usually categorical—in which one of the premises or the conclusion is not expressed.

Supposing the syllogism in question to be this:

All men are mortal;
Socrates is a man;
Therefore Socrates is mortal;

our reasoning would have been almost as clear and more effective rhetorically if we had merely said:

(1) Socrates is a man,
Therefore he is mortal;

* "Lessons in Logic", p. 167.

or (*2*) All men are mortal,
Therefore Socrates is mortal;
or (*3*) All men are mortal,
And Socrates is a man.*

Reasoning often takes the form of a chain in which the conclusion of one syllogism is used as one of the premises of another, *e.g.:*

All A's are B's;
All B's are C's;
Therefore all A's are C's.

All A's are C's;
All C's are D's;
Therefore all A's are D's.

The syllogism which supplies such a premise is called a *Prosyllogism;* that which uses it, an *Episyllogism.*

When an Episyllogism depends upon a Prosyllogism which is only partly expressed the argument is called an *Epicheirema, e.g.:*

All A's are C's, for they are B's;
All C's are D's, for they are X's;
Therefore all A's are D's.

This is "a double Epicheirema, containing reasons for both premises".

A *Sorites* is a chain of prosyllogisms and episyllogisms in which all the conclusions but the last are unexpressed, *e.g.:*

All A's are B's;	All Athenians are Greeks;
All B's are C's;	All Greeks are men;
All C's are D's;	All men are mortal;
All D's are E's;	All mortals fear;
Therefore all A's are E's.	∴ All Athenians fear.

* Where the major premise is omitted the enthymeme is said to be of the first order; where the minor, of the second; where the conclusion, of the third.

If we put in the suppressed conclusions, the Sorites is resolved into these syllogisms:

All A's are B's; All B's are C's; ∴ All A's are C's.
All A's are C's; All C's are D's; ∴ All A's are D's.
All A's are D's; All D's are E's; ∴ All A's are E's.

With reference to a Sorites it should be observed:

1) That almost invariably the minor premise of each syllogism involved is written first. A Sorites which begins at the other end seems jagged.

2) That in any valid Sorites every premise but the first (*i.e.*, the minor premise of the first prosyllogism) must be universal and every premise but the last (*i.e.*, the major premise of the last episyllogism) affirmative.

3) That while the last syllogism involved (the episyllogism) may be in any figure, each prosyllogism must be in the first; and that in the case of each prosyllogism it is the minor premise which the previous prosyllogism supports.

CHAPTER XVIII.

BLUNDERS IN WORD AND BLUNDERS IN THOUGHT.

Fallacies or blunders in reasoning are usually divided into two great classes: 'Logical' or 'Formal' (*Fallaciæ in dictione*) and 'Non-logical' or 'Material' (*Fallaciæ extra dictionem* or *in re*). When logic is regarded as a science of the 'forms of thought' or the science which treats of the proper arrangement of words in correct thinking (on the assumption that the 'forms' of thought and the forms or arrangements of words correspond) this distinction presents no difficulties: logical or formal fallacies are those which result from a violation of the rules which logic lays down for correct thinking and the corresponding arrangement of words; and material or non-logical fallacies are those which occur in spite of the observance of these rules—they do not depend upon the general laws of thought or arrangement of words at all, and can only be avoided by a knowledge of the matter thought about.

In the foregoing pages I hope it has been made plain that the same arrangement or form of words cannot be counted upon to always express the same thought. I hope it has been made plain too that the so-called 'laws' and 'forms' of thought with which it is often said that logic deals have no meaning whatever apart from the things thought about and the way in which the relations of these things involve each other. If these views are correct it is quite impossible to make any fundamental distinction between fallacies which

are due to some perversion of the forms of thought and those which are due to some mistake about the relations of things. But we might still distinguish between fallacies which are due to some misconception about the 'matter' under discussion and those which depend in some way upon the 'form' of words used in discussing it. So with the terms 'logical' and 'extra-logical'. They may be taken to mean that some fallacies result from a violation of logic while others do not, or they may be taken to mean that some are, and others are not, concerned with *logoi* or words.

Now it is true that logic has not made such elaborate provision against every fallacy possible as it makes against those already discussed, and yet fallacious thinking is always illogical and there is no reason but one of convenience why books on logic should discuss some and not others. It is not appropriate therefore to divide fallacies into those that violate the rules of logic and those that do not. But there is a reason why we should divide fallacies into those that result in some way from the improper use of words and those that do not. We may therefore accept this division into 'logical' and 'extra-logical' or 'material' on the understanding that it is equivalent to a division into blunders that result mainly from the careless use of words and those that do not.

The 'logical' fallacies are usually subdivided into two classes called 'Purely logical', "where the fallaciousness is apparent from the mere form of expression", and 'Semi-logical', where the fallaciousness is not apparent from the mere form of expression but is due to some ambiguity in the language used or some misunderstanding as to its meaning. Of the 'semi-logical' fallacies we shall have nothing more to say. We have seen already how insidious they are, why they arise, and how best to guard against them.

With the 'purely logical' fallacies we are also familiar. They are such blunders as we make when we ignore the cautions of the syllogism, or convert A simply or O at all, or

reason that because all S is P all non-S is non-P, or infer the falsity of a consequent from the falsity of its antecedent, or the truth of the antecedent from the truth of the consequent, or the falsity of a conclusion from the falsity of the premises, or the truth of the premises from the truth of the conclusion. In all such cases it is 'apparent from the mere form of expression' that the reasoning is inconclusive; the blunder can be detected without inquiring into the truth of the premises or even into the meaning of the terms; so that a purely logical fallacy, unlike any of the others, can be detected when the terms are mere unmeaning words or symbols such as S, M and P, or X, Y and Z. **'Purely logical.'**

The strange thing about these so-called purely logical fallacies is that they are committed so often. How is it possible, we may ask, to think so badly? If the reader will ask the following questions to some unsuspecting person and does not allow very much time to elapse between the answering of one and the asking of the next, the result of the experiment may help to make the matter clear:

Who was the first man?

Who was the first woman?

Who killed Cain?

Abel did not kill Cain, but his name will usually be mentioned, or at least come to mind, merely because it comes naturally at the end of the series 'Adam, Eve, Cain' and fits into the atmosphere of murder. It is largely a mere matter of the verbal jingle, the answer resulting from the same law of habit in the nervous system that accounts for putting one's pen in the paste-pot after using the brush. Most of these so-called purely logical fallacies come in precisely the same way. Inference is in the air and the jingle seems to fit, so we spurt out something when the premises will not justify any inference whatever, or an 'All' or a 'No' that fits the jingle when the premises justify only a 'Some'. If we say:

All X is Y;
All Y is Z;
∴ All X is Z,

the reasoning is valid, but if we say:

No X is Y;
No Y is Z;
∴ No X is Z;

or

No X is Y;
All Y is Z;
∴ No X is Z,

it is not valid though we have merely substituted 'No' for 'All' or 'No X' for 'All X' throughout, without affecting the jingle. Indeed if we had only said 'No-X' instead of 'No X' the reasoning in the last case would have been precisely similar to that in the first and just as valid. Again if we say

Five francs are a dollar;
Four shillings are a dollar;
∴ Five francs are four shillings,

the inference is perfectly valid; but if we say in precisely similar form

Blades of grass are green;
Frogs are green;
∴ Blades of grass are frogs,

the inference is not valid. The reason is, of course, that the copula 'are' is used in different senses in the two syllogisms; but when we do not stop to think of the sense, the familiar jingle, assisted perhaps in this case by some recollection of Euclid's axiom that 'things equal to the same thing are equal to each other', lures us on to danger.

In spite of a real confusion of meaning sometimes associated with some of these 'purely logical fallacies', they can hardly be called the result of bad thinking; because they are not the result of thinking at all, but only of a reflex act. On this account it might have been more appropriate to call them the Reflex Fallacies or the Jingle Fallacies.

As there are two classes of verbal fallacies, so also there

are two classes of the material or non-verbal fallacies, which may be called respectively Fallacies of the Forgotten Issue and Fallacies of the Ill-conceived Universe. Fallacies of the Forgotten Issue are not particularly characteristic of deduction; but some of them are usually discussed in connection with it, and therefore we shall speak of them in the next chapter. In the chapter after that we shall discuss Fallacies of the Ill-conceived Universe. These do not belong to the traditional field of deduction, because there are no rules for verbal manipulation which they break. Yet I have tried to show that all deductive inference depends upon the assumption that things have certain general relations, and that deductive fallacies occur when these relations are overlooked; and if this is correct these fallacies of the Ill-conceived Universe are essentially similar to the fallacies of deduction in their ultimate nature, though they may not be caused like them by a verbal jingle.

Two kinds of material fallacies.

Nothing has been said in the foregoing pages about the fallacy known as 'Non Sequitur'. This name is really applicable, as the words imply, to every argument in which the conclusion does not follow, and in this sense of the words every fallacy is a Non Sequitur. But the phrase is often applied in a more restricted sense to those arguments only in which the conclusion does not even appear to follow, except perhaps to the most hasty and careless of reasoners; as in the following examples: The earth is round; therefore there is no atmosphere on the moon. "Every one desires happiness, and virtuous people are happy, therefore every one desires to be virtuous." "Episcopacy is of Scripture origin, the Church of England is the only established church in England; ergo the church established is the church that should be supported." The subject requires no further consideration.

Non Sequitur.

CHAPTER XIX.

THE FORGOTTEN ISSUE.

A Fallacy of the Forgotten Issue is committed when we forget what it was that an argument was intended to prove, and either take that very thing or something equivalent to it and quite as doubtful for granted, or else prove something which is not equivalent to the point at issue and then assume that we have proved the point itself. In the first case the fallacy is called Petitio Principii or Begging the Question. In the second it is called Ignoratio Elenchi or a fallacy of Irrelevance. Each of these two fallacies of the forgotten issue takes several forms.

Petitio Principii.

The fallacy of Petitio Principii is not committed unless there is a show of proof. Nobody commits it who merely says 'I assume these conclusions to be true, and I do not try to prove them'. But a person does commit it if he thinks he is proving his conclusions when he is really assuming them, or is assuming a premise that is not admitted or would not be admitted if its real significance were understood. Often the premise is actually proved from the conclusion, or 'is such as would naturally and properly be so proved'. But in any case in which the fallacy is present the conclusion seems to be more fully proved by the argument than it really is, because it is not clearly understood how nearly equivalent is that which is taken for granted to that which is to be proved; *e.g.*, "Whoever refuses to believe in the inspiration of the Bible makes the Most High

a deceiver; for has he not told us that 'All scripture is given by inspiration of God'?" Of course we have no reason to believe that it was really God who said that all scripture is given by his inspiration unless we already assume that the Bible or some part of it is inspired.

Whately directs attention to the fact that the English language is peculiarly "suitable for the fallacy of Petitio Principii, from its being formed from two distinct languages, and thus abounding in synonymous expressions which have no resemblance in sound, and no connection in etymology; so that a Sophist may bring forth a proposition expressed in words of Saxon origin, and give as a reason for it the very same proposition stated in words of Norman origin; *e.g.*, 'To allow every man an unbounded freedom of speech must always be, on the whole, advantageous to the State; for it is highly conducive to the interests of the community, that each individual should enjoy a liberty perfectly unlimited of expressing his sentiments'".

A blunder of this same sort is committed when a student says that two chemicals are sure to unite since they *have an affinity* for each other; or that he knows unsupported objects will fall to the earth from the fact that they *are attracted* towards it.

"Connected with this fallacy is the rhetorical device [already discussed] of *Question-begging Epithets.* Thus, though the matter we are discussing is open to dispute, we may speak of a nefarious project, a laudable ambition, an astute act, a far-sighted policy, and so on, attempting, by means of a carefully selected epithet, to assume the point at issue, or at least to create an unfair prejudice in the mind of the hearer or reader whom we address." * **Includes epithets.**

When a conclusion is based upon a premise which in an earlier stage of the argument was itself based upon this very conclusion, the reasoning is said **And circle.**

* Fowler's "Deductive Logic" (Clarendon Press).

to be in a *Circle* (*Circulus in Probando*). Here are some examples:

First Syllogism.	Second Syllogism.
A is B;	A is C;
B is C;	C is B;
∴ A is C.	∴ A is B.

"Some mechanicians attempt to prove (what they ought to lay down as a probable but doubtful hypothesis) that every particle of matter gravitates equally; 'why?' because those bodies which contain more particles ever gravitate more strongly, *i.e.*, are heavier: 'but (it may be urged) those which are heaviest are not always more bulky;' 'no, but still they contain more particles, though more closely condensed;' 'how do you know that?' 'because they are heavier;' 'how does that prove it?' 'because all particles of matter gravitating equally, that mass which is specifically the heavier must needs have the more of them in the same space'" (Whately). 'Any man who would marry such a woman must have something wrong with him.' 'Why, what is the matter with his wife?' 'It is matter enough to be willing to marry such a man as he is.'

If there are a large number of intermediate steps and the argument is a long one it may be very difficult to detect the circle. This fallacy, like a good many others, can be best guarded against by making the shortest and simplest possible summary of any argument that claims our interest.

Ignoratio Elenchi.

Ignoratio Elenchi or Irrelevance, the other fallacy of the forgotten issue, consists merely in arguing beside the point. "I am required by the circumstances of the case (no matter why) to prove a certain conclusion; I prove, not that, but one which is likely to be mistaken for it;—in this lies the fallacy. . . . For instance, instead of proving that 'this prisoner has committed an atrocious fraud', you prove that 'the fraud he is accused of is atrocious'; instead of proving, as in the well-known tale of Cyrus and the two coats, that 'the taller boy had a right

to force the other boy to exchange coats with him', you prove that 'the exchange would have been advantageous to both'; instead of proving that 'a man has not a right to educate his children or dispose of his property in the way *he thinks best*', you show that 'the way in which he educates his children or disposes of his property is not *really the best*'; instead of proving that 'the poor ought to be relieved in this way', you prove that 'they *ought to be relieved*'. . . . A good instance of the employment and exposure of this fallacy occurs in Thucydides, in the speeches of Cleon and Diodotus concerning the Mitylenæans: the former (over and above his appeal to the angry passions of his audience) urges the *justice* of putting the revolters to death; which, as the latter remarked, was nothing to the purpose, since the Athenians were not sitting in *judgment*, but in *deliberation*, of which the proper end is *expediency*." *

It is interesting to find counterparts of this story in the history of our own times. The following sentences from Bismarck's Autobiography (Chapter XX) refer to the Austrian proposals for peace after the Prussian victories in the war of 1866:

"I unfolded to the King [of Prussia] the political and military reasons which opposed the continuation of the war.

"We had to avoid wounding Austria too severely; we had to avoid leaving behind in her any unnecessary bitterness of feeling or desire for revenge; we ought rather to reserve the possibility of becoming friends again with our adversary of the moment, and in any case to regard the Austrian state as a piece on the European chess-board and the renewal of friendly relations with her as a move open to us. If Austria were severely injured, she would become the ally of France and of every other opponent of ours; she would even sacrifice her anti-Russian interests for the sake of revenge on Prussia. . . .

* Whately, "Elements of Logic", third edition, London, 1829.

"To all this the King raised no objection, but declared the actual terms as inadequate, without, however, definitely formulating his own demands. Only so much was clear, that his claims had grown considerably since July 4. He said that the chief culprit [Austria] should not be allowed to escape unpunished, and that justice once satisfied, we could let the misled backsliders [the smaller German states that had sided with Austria] off more easily, and he insisted on the cessions of territory from Austria which I have already mentioned. I replied that we were not there to sit in judgment, but to pursue the German policy. Austria's conflict in rivalry with us was no more culpable than ours with her; *our task was the establishment or initiation of a German national unity under the leadership of the King of Prussia.*

"Passing on to German states, he spoke of various acquisitions by cutting down the territories of our opponents. I repeated that we were not there to administer retributive justice, but to pursue a policy; that I wished to avoid in the German federation of the future the sight of mutilated territories, whose princes and peoples might very easily (such is human weakness) retain a lively wish to recover their former possessions by means of foreign help; such allies would be very unreliable." *

"So Canning, in a speech in the House of Commons in reply to Mr. Percival, says: 'The question is not, as assumed by my opponent, whether we shall continue the war in the Peninsula, but whether it is essential to our success in the war that our present system of currency remain unchanged.' Thus it is not unusual, after a protracted debate, for the cooler thinkers to preface their remarks with reminding the audience of the real nature of the point on which issue is joined; and the longer and more heated the discussion, the greater the need for these monitory exordiums. For, especially when the field of debate is large, the combatants

* "Bismarck the Man and the Statesman". (Harper & Bros., 1899.)

often join issue on the wrong points, or do not join issue at all. One goes to the east, another to the west; one loses the proposition in question, and wanders amidst a crowd of irrelevant details; another mistakes contraries for contradictories, or universals for particulars; and, after some hours of storm, they know not what they have been discussing. One has made out a case which his adversary admits, the more readily as it has not the least bearing on the question; another, having overthrown a similar collateral proposition, makes his pretended triumph resound over the field; yet another, having been rather shattered by reasons, appeals to the prejudices of his auditory, and, overwhelming his more rational antagonist with ridicule and abuse, comes off the apparent and acknowledged victor in the contest." *

There are many subjects that have to be discussed at some length before it is possible to tell precisely what the point or points at issue are; but whenever there is a definite issue we should try very hard in our discussions and in our private thinking to stick to it. In our courts of law the rule that all testimony must be relevant is enforced very strictly. Unfortunately the corresponding rule about parliamentary discussions cannot be enforced so strictly.

Includes 'ad hominem'.

Two special forms of Ignoratio Elenchi are called the Argumentum ad Hominem and the Argumentum ad Populum. These terms are applied in a rather loose sense to any argument that appeals to feeling or that depends, not on the real question at issue, but upon the personality of any of the parties involved, including the hearers. As mere appeals to the desires or passions of one's hearers these arguments have been discussed already. As involving confusion between the real issue and some false issue they have not. As a form of Ignoratio Elenchi the Argumentum ad Hominem often consists in supposing that we have won our case because we have succeeded in em-

* Davis, "Theory of Thought", pp. 277–8. (Harpers, no date.)

barrassing an opponent by some personal reference that has really nothing to do with it. If I am accused of extravagance I do not answer the accusation though I may silence the accuser by proving that he himself is no better; if my opinions are attacked I cannot substantiate them by replying that my opponent once held them himself. Such a retort I may have a perfect right to make, but I have no right to confuse it with a vindication of my own position.

Undoubtedly it was the great danger of confusing issues when questions of personality are once introduced that made our common law exclude as irrelevant all evidence as to the general character of the parties concerned even in cases in which such evidence might seem to the layman very relevant indeed. The courts try to decide each particular case on its individual merits, and not to give any man a favorable or unfavorable verdict merely because his general character is good or bad; they know that in many individual cases good men are in the wrong and bad men in the right; and they know enough about human nature to realize that if questions of general character are dwelt upon the jury will be influenced by them far more than they should be in order to decide the real question at issue on its merits.

The *Argumentum ad Populum* is practically indistinguishable from the *Argumentum ad Hominem.* It is defined as "an appeal to the passions, prejudices, etc., of the multitude". "The fallacy usually occurs in the course of long harangues, where the multitude of words and figures leaves room for confusion of thought and forgetfulness." Here lies the danger of brilliant oratory and startling metaphors. Whatever one may believe to-day about the comparative values of a single and a double monetary standard of value, and the wisdom of suddenly changing from one to the other, it is certainly startling to think how half the country was swept away, in the spring of 1896, and a difficult economic problem for a time apparently settled, by a dazzling though empty metaphor about thrusting a crown of thorns

upon the brow of labor and crucifying mankind upon a cross of gold. The real question at issue was, not whether laboring men should or should not be unjustly treated and mankind in general oppressed, but whether a given policy would tend in the long run to diminish injustice and oppression, or to increase it.

CHAPTER XX.

THE ILL-CONCEIVED UNIVERSE.

Logic deals, as we have seen, with the mutual implications of relations. If the object A possesses the permanent quality B and the object C does not, we know that A and C are not identical. Thus the presence of a certain relation of subject and attribute in one case and its absence in another proves the absence of a certain relation of individual identity. These are the relations concerned, but we can reason as we do about them only because we already know or assume that there are such things as separate individual objects, that these objects can have qualities, that qualities can be more or less permanent, that we are capable of recognizing the difference between qualities that are permanent and those that are not, and so on. These assumptions and probably a great many more form a sort of background for the reasoning in question. They constitute as it were the *universe* in which the relations specified in the syllogism exist, a universe without which they would lose entirely the significance which they now possess. It is a universe which human beings naturally take for granted; but if in the world to come we should discover that there are really no such things as separate individual objects and no such relations as those of subject and attribute, then we should be compelled to revise all our rules of logic and reason in some other way. A simple syllogism which seems to us now to be perfectly valid would then be

The assumed universe.

seen to be absolutely inconclusive. Indeed it would seem so inconsequent as to be utterly incomprehensible unless we could remember our old earthly point of view—the universe in which we reasoned—and judge the argument from that standpoint. We could then say: 'Assuming the fundamental relations of things to be thus and thus, the reasoning is perfectly valid; but then these are *not* the actual relations; the blunder rested upon a wrong conception of the background or universe, and it could not be corrected until that conception was outgrown'.

In this example the relations which I have supposed to be improperly assumed are amongst the most fundamental relations of all reality. That is why the falsity of the assumption would involve the worthlessness of all our rules of formal logic. The universe of discourse included only relations common to the whole of the actual universe. Of course we human beings never in this life can test these most fundamental assumptions of all, and I suppose them to be questioned only for the sake of illustration. But as a matter of fact most of our reasoning is about matters in some special universe, where not only these but a great many other relations are taken for granted. That is why it is so difficult for any one but an expert in that particular field to criticise the logic in any scientific or other technical argument. In even a game of whist, for example, it would not be possible to infer anything about your opponent's hand unless you knew the rules of the game, and even then the very perfection of your reasoning might lead you astray if you supposed he was playing the long game when as a matter of fact he was playing the short. So one might solve no end of chess problems with great ingenuity, yet get them all wrong, if he supposed that pawns always moved straight forward, that a queen could move like a knight, or that all the chessmen moved like checkers.

A more serious example of the sort of thing I have in mind is found in Plato's "Phædo". Many students who

read Socrates' argument for the immortality of the soul, there given, say that where the parties to the dialogue seemed to find the argument more or less conclusive, *they* can find no argument at all; or at least no connection between the premises and the alleged conclusions. The trouble is that they do not realize the conceptions of life which Socrates and his friends accepted as a matter of course. Throughout the dialogue it is assumed, for instance, that every living thing, whether animal or plant, must have a soul to give it life—to *animate* it. If to this is added the further assumption that new souls are not created, does it not follow that life would have ceased to exist ages ago unless the soul which animated one individual were reincarnated after the death of that individual in some other, and so in *in sæcula sæculorum?* Thus by getting back into Plato's universe—into the conceptions which serve as a background of the argument—we find sense where otherwise we find only nonsense; and if his argument seems to us inconclusive it is only because his universe seems unreal.

Another instance of the same kind is found in the controversy between Locke and Leibnitz about innate ideas. Locke said it was absurd to suppose that the mind contained a set of ideas ready-made from birth, though we had no conscious knowledge of some of them until a great many years afterwards. Leibnitz replied that it was much more absurd to suppose that such ideas as those of right and wrong or of cause and effect could be conveyed into the mind through any one or all of the five senses. And so the discussion proceeded, the adherents of each champion seeing the absurdities of the other. The conflict was necessarily interminable until in a later age it was realized that the advocates of both views assumed the same false universe, for everybody assumed that in some way or other the mind contains a kind of things called ideas which must have got there in some way or other. But it is really just as absurd to ask how the mind comes to contain its various ideas as

to ask how a frog comes to contain its various jumps. A frog does not contain things called jumps; it merely acts in a way we call jumping; and the mind does not contain things called ideas; it thinks.

So again with the deistic controversy about the possibility of miracles. Both sides took it for granted that the world had been wound up and started like a clock; but the deists said that God the clockmaker never intervened by a miracle to disturb its running, while the orthodox said he did. The question was one of intervention or non-intervention; and it did not occur to either side that nature was after all nothing but a visible and tangible aspect of God, not something separate with which perhaps he might not be able to interfere.

And so it goes through the whole history of philosophy. Each age is dominated by some particular conception of the general constitution of things; and in that age the mutual relations of any particular facts are necessarily conceived with reference to the assumed nature of the whole of which they are parts—of the background from which they stand out—of the frame into which they must fit. In the next age there is a new conception of the background—a new metaphor, perhaps, to express the deepest relations of things,—and the reasoning that before had seemed absolutely demonstrative now seems almost childish.

In the cases just mentioned the assumption of the universe in question was unconscious and practically inevitable. The same assumption of a universe is made consciously in the old *Fallacy of Many Questions*, *e.g.*: 'Have you cast your horns?' 'Have you left off beating your father?' 'Is the king of Eutopia dead?' 'Why did you take my purse?' 'Have you got over your fit of temper?' etc. Here of course the assumption is that you have had horns, that there is a king of Eutopia, etc., and it is impossible to answer either 'Yes' or 'No' to the question without seeming to admit the assumption.*

* Sometimes the Fallacy of Many Questions is committed to the em-

The confusion of universes.

A Confusion of Universes occurs when we introduce into any universe something which cannot possibly be subject to the relations by which that universe is distinguished, or when we introduce some other relation which is inconsistent with them. All kinds of *absurd questions* rest upon such confusion. If you ask whether this triangle has eaten its dinner, I can hardly say even 'No' without seeming to admit your absurd assumption that a triangle is the kind of thing that eats a dinner. Here of course there is an improperly assumed universe; but the assumption involves a more or less obvious incongruity. In his book on "The Nervous System and the Mind" * Charles Mercier points out with admirable clearness a somewhat less obvious incongruity of the same sort.

"It is not to be denied that there is a large amount of writing about the mind, and about the connection of the mind with the body, which is, strictly speaking, nonsense. . . . Such propositions are neither correct nor erroneous—neither true nor false. They are nonsense. . . . Take an instance. Try to think of a feeling passing along a nerve. We often speak familiarly of a toothache shooting along a nerve; is this an accurate expression? Take the nerve. Dissect it out. Lay it on the table before you. It is a gray thread, four inches long, made up of fibres bound together. Now take a toothache and set it running along the nerve. You cannot. Why? It ran along the nerve, you said, when it was in the body; why cannot it do so now? Because, you will say, the nerve is no longer connected with

barrassment of the questioner instead of the person questioned. Many of the typical Irish jokes belong to this class, such for example as the story of the Irishman who was being tried for assaulting a Chinaman in front of the Palmer House and had arranged with a friend to prove an alibi. He conducted his own defence and when the time came to question the witness he put in some preliminary flourishes and continued: 'Then, Patrick Murphy, on your oath, sir, where was I when I struck the Chinaman in front of the Palmer House?'

* Macmillan, 1888.

the brain. Take another nerve, then, and do not separate it from the body; but pinch it, cut it, burn it, or galvanize it. What torture! what excruciating agony! Surely this pain is in the nerve; you feel it there. Wait a little; let us consider. The nerve is made up of axis-cylinders and padding; in which is the pain? Certainly not in the padding; it must then be in the axis-cylinders. The axis-cylinders are gray threads of protein substance, which is made up, like all other matter, of molecules swinging in space. Now, where is the pain? Is it in the molecules or in the intervening space? And how does it pass along the nerve? Does it jump from molecule to molecule, or does it flow in the interstices? If the former, pain must be a solid; if the latter, it must be a fluid; both of which hypotheses are manifestly nonsense. There is a third alternative. It may be a *movement* communicated from molecule to molecule. . . . Consider again. Imagine the molecules of the nerve swinging in space. Now imagine a wider swing. Does that resemble pain? Turn the circle into a spiral. Is that like pain? But it may be said, Pain, we know, is not really in the nerves, it is in the brain. Again the same problem awaits us. The brain is made of cells and fibres. Is pain in the cells? Is it in the fibres? In either case we must come down to molecules at last, and again the pain eludes our search. No conceivable form of matter and no conceivable movement of matter bears the smallest resemblance to pain, or can by any human imagination be assimilated to pain. We are driven to the conclusion that pain and matter are things with no community of nature, are facts of totally different orders, and cannot be reduced to any common term. Pain is neither in the nerves, nor in the brain, nor in any position in space. It is in the mind." And of course this expression '*in* the mind' simply means that we feel it.

Fallacies of False Analogy may often be regarded as cases of an ill-conceived, and perhaps of a confused, universe.

The bare facts or some of them may be known accurately enough, but the relations between them—the general background in which they are set—is conceived erroneously. "Carlyle's saying that a ship could never be taken round Cape Horn if the crew were consulted every time the captain proposed to alter the course, if taken seriously as an analogical argument against Representative Government, is open to the objection that the differences between a ship and a State are too great for any argument from the one to the other to be of value. It was such fallacious analogies as these that Heine had in view in his humorous prayer, 'Heaven defend us from the Evil One and from metaphors'." *

Often as we turn from one aspect of a situation to another we find some new fact which is not consistent with some general statement that we made about the first aspect; and this may lead to a '*bull*'. 'One man is as good as another', says the Irishman when he resents the claim of superiority made by some one else; but as he thinks of his own excellences and the other's shortcomings he adds, 'and sometimes a long sight better'. Sometimes the bull is due to an unfortunate metaphor; *e.g.*, 'Our cup of sorrow is overflowing, and is not yet full'.

Since every metaphor rests on the assumption, though even for only a moment, of a kind of universe, every case of *mixed metaphors* is a case of confused universes. I take the following from Genung's "Rhetoric": "The very recognition of these or any of them by the jurisprudence of a nation is a *mortal wound* to the very *keystone* upon which the whole arch of morality reposes."—"This world with all its trials is the *furnace* through which the soul must pass and *be developed* before it is *ripe* for the next world."—"I write to you in a state of mind that I really ardly know what I am about, but I cannot indure making no effort to *clear up* the *gaping abiss*

* Minto's "Logic", p. 373.

which the events of the past fatal afternoon *has raised between us.*''

Another kind of ill-conceived universe may be called the Universe with a Neglected Aspect. This phrase is intended to include all arguments in which the existence or influence of some essential relation or object in the universe involved is neglected. In calculating the time it will take a feather dropped from the window to reach the ground we have a right to neglect the attraction exerted upon the feather by some fixed star, for though there is such an attraction it makes no appreciable difference in the result, and we have a right to neglect the death of some Asiatic despot, for it makes no difference at all in the result—is not in the universe under consideration. But we have no right to neglect the resistance of the air, or the influence of the wind with all its gusts and eddies; for they make every possible difference in the result. Similarly we have no right to conclude that free trade is necessarily the best policy for some particular state, merely because it is always or usually the policy most favorable for the accumulation of wealth, unless we have first made sure that there is no question of education, or public morals, or military necessity, or international politics, which demands some other policy.

The neglected aspect.

Every roseate picture of the happiness to be attained when the competition of commercial rivals has ceased, and the State controls all industry and gives every one his due, is painted in happy forgetfulness of the natural discontent, selfishness, laziness, or ambition which would prompt most of the people in such a community to shirk their appointed tasks, to use personal influence in order to get some special privilege, or to gain control of the machinery of government for the particular benefit of themselves and their friends,—forces in human nature which would replace commercial competition with political jobbery.

Under the head of *Composition*, Whately gives several

examples of what I should rather call a *neglected relation of articulation:*

"There is no fallacy more common or more likely to deceive than the one before us: the form in which it is most usually employed is to establish some truth, *separately*, concerning *each single* member of a certain class, and then to infer the same of the *whole collectively;* thus some infidels have labored to prove concerning *some one* of our Lord's miracles, that it might have been the result of the accidental conjuncture of natural circumstances; next they endeavor to prove the same concerning *another*, and so on; and thence infer that *all* of them might have been so. They might argue in like manner, that because it is not very improbable that one may throw sixes in any one out of a hundred throws, therefore it is no more improbable that one may throw sixes a hundred times running."

This is, as we have said, a case of neglected articulation. The miracles in question all took place within a certain short period and were all connected with a single personality. The general fact that one supposed miracle is shown to be the result of accident would be a reason for, and not against, the belief that a great many others could be explained in the same way; but if these others are accidents and not really miracles we should expect to find them scattered, not grouped and articulated as they are in the case in question. So with the throws of sixes. It is not the occurrence of a hundred of them that is remarkable, but of a hundred in succession with the same dice and in the hands of the same player.*

* This argument of Whately's is perfectly valid as a reply to those who try to explain the miracles in question by a series of physical accidents. But if it be taken as an independent proof of their miraculous nature it might be itself regarded as an example of the Neglected Aspect; for it fails to consider the mental influences which tend to produce this very grouping. It is easier to hypnotize one person if others have been hypnotized in his presence, and for much the same reason the 'cures' wrought by our modern faith doctors generally come in groups.

Often the neglected relation is one that is necessarily and obviously involved in some general scheme that is contemplated, and the neglect to consider it must be charged, not to ignorance, but to sheer haste and carelessness. "From the circumstance that some men of humble station, who have been well educated, are apt to think themselves above low drudgery, it is argued that universal education of the lower orders would beget general idleness; this argument rests, of course, on the assumption of parallelism in the two cases, *viz.*, the past and the future; whereas there is a circumstance that is absolutely essential, in which they differ; for when education is *universal* it must cease to be a *distinction;* which is probably the very circumstance that renders men too proud for their work." *

This blunder is like that of the people who clamor for some change in the tariff or in the currency that will give everybody 'more money', forgetting that if dollars were as common as pebbles they would be worth no more than pebbles, and all that one could carry would hardly buy a dinner; or like that committed by the member of a crowded audience who asked that everybody present might be allowed to stand on the back of his seat and thus get an unobstructed view of the performance.

The blunder of a Neglected Aspect is involved in every philosophical theory which resolves *all reality* into mere phenomena, forgetting that there can be no phenomenon or appearance without something to appear and some one to whom it appears. It is involved in the old myth of Atlas supporting the world; for any object that requires to be supported does so because it is heavy, *i.e.*, because it is attracted by every other object and therefore tends to move towards the common centre of gravity. But as there is nothing outside of the world (or at least outside of the universe) towards which it is attracted, the universe as a whole

* Whately, op. cit.

does not tend to fall, or in other words, *it has no weight at all.* Another such absurdity is involved in the question 'Where is the universe?' To tell the position of any particular object is to tell how it lies with reference to some other object or objects; but the universe contains all objects —it would not be the universe if it did not—and therefore there is nothing lying outside of it by which it can be located: *it has no place.* What has been said about the place of the world is applicable, *mutatis mutandis*, to questions about its *cause* and the *time* at which it came into existence. A similar criticism can be applied to the question 'Is life worth living?' To ask what a thing is worth is to ask what people are willing to take in exchange for it—what other object they believe will give them as much satisfaction. But all exchange, all weighing of alternatives, all satisfaction and dissatisfaction presuppose a life in which they take place, and to ask what this life itself is worth is as absurd as to try to weigh your balances in their own pan.

In all such cases the universe as a whole—the total system of related objects—is spoken of as though it could possess relations which exist only between various members within it.

Dilemmas of the epigrammatic sort are almost always based upon a view of only half the universe, and for this reason "can often be retorted by producing as cogent a dilemma to the contrary effect. Thus an Athenian mother, according to Aristotle, addressed her son in the following words: 'Do not enter into public business; for if you say what is just, men will hate you; and if you say what is unjust, the gods will hate you'. To which Aristotle suggests the following retort: 'I ought to enter into public affairs; for if I say what is just, the gods will love me; and if I say what is unjust, men will love me.'" (Jevons.) Again, Epictetus says that if honors which we do not possess are good, we should rejoice that another has them; if bad, that we have them not. But a pessimist might reply that if they are good, we should grieve

that we have them not; if bad, that another has them. What Epictetus says, therefore, amounts merely to this: that we should look on *the bright side* of things and deliberately ignore the other side. This certainly is a healthy practical attitude, but when one begins to theorize he must see both sides.

Often, as Lotze points out,* an aspect of a situation, though mentioned, can be crowded out of view by the mere order of a sentence. To say 'I am a great deal better off than he is' is more cheerful than to say 'He is worse off than I am'; but the facts of the case are the same. This is so also with the saying 'There is a silver lining to every cloud' and its converse, 'There is a cloud to many a silver lining'. "Think of public teachers who say that the farmer is ruined by the cost of transportation, when they mean that he cannot make any profits because his farm is too far from the market." †

The neglected member.

When the whole background is not misconceived or distorted or a part of it overlooked, we may still err by overlooking some member of the universe, coördinate with those under discussion, whose influence (or at least whose presence) ought to have been taken into account. If A, B, C, and D are fixed points on a given plane, I may be able to determine the relations of any one of them to any other without any reference to the rest; but if they are planets, each attracting every other, I cannot do this. If any member of the system is ignored, any prediction I may make with reference to the others is bound to turn out more or less erroneous. It is here that we find the fallacy

* "It may be said that evil appears only in particulars, and that when we take a comprehensive view of the great whole it disappears; but of what use is a consolation the power of which depends upon the arrangement of clauses in a sentence? For what becomes of our consolation if we convert the sentence which contains it thus—The world is indeed harmonious as a whole, but if we look nearer it is full of misery?" Microcosmus, Bk. IX, Ch. V (Scribners).

† Sumner, op. cit., p. 46.

of the old argument in favor of a protective tariff, 'Gold and silver are wealth; a protective tariff, by shutting out imports while not interfering with exports, increases the gold and silver in a country; it therefore increases the country's wealth.' Assuming the truth of the premises, the conclusion does not follow, because while gold and silver are wealth they are not the only forms of wealth, and the imported gold and silver must be paid for by some other kind of wealth exported. The fallacy lies in forgetting this exported wealth—in looking at only a part of the universe or system in question.

So, many an unfortunate maintains that he 'has a right to a living', or to certain comforts; forgetting that such a right on his part implies a duty to provide him with these things on the part of somebody else.

The best account that I know of this fallacy of the forgotten member of the universe is to be found in the little book by Professor Sumner already quoted.

"THE FORGOTTEN MAN."

"In all jobbery the case is the same. There is a victim somewhere who is paying for it all. The doors of waste and extravagance stand open, and there seems to be a general agreement to squander and spend. It all belongs to somebody. There is somebody who had to contribute it, and who will have to find more. Nothing is ever said about him. Attention is all absorbed by the clamorous interests, the importunate petitioners, the plausible schemers, the pitiless bores. Now, who is the victim? He is the Forgotten Man. If we go to find him, we shall find him hard at work tilling the soil to get out of it the fund for all the jobbery, the object of all the plunder, the cost of all the economic quackery, and the pay of all the politicians and statesmen who have sacrificed his interests to his enemies. We shall find him an honest, sober, industrious citizen, unknown outside his little circle, paying his debts and his

taxes, supporting the church and the school, reading his party newspaper, and cheering for his pet politician.

"We must not overlook the fact that the Forgotten Man is not infrequently a woman. I have before me a newspaper which contains five letters from corset-stitchers who complain that they cannot earn more than seventy-five cents a day with a machine, and that they have to provide the thread. The tax on the grade of thread used by them is prohibitory as to all importation, and it is the corset-stitchers who have to pay day by day out of their time and labor the total enhancement of price due to the tax. Women who earn their own living probably earn on an average seventy-five cents per day of ten hours. Twenty-four minutes' work ought to buy a spool of thread at the retail price, if the American workwoman were allowed to exchange her labor for thread on the best terms that the art and commerce of to-day would allow; but after she has done twenty-four minutes' work for the thread she is forced by the laws of her country to go back and work sixteen minutes longer to pay the tax—that is to support the thread-mill. The thread-mill, therefore, is not an institution for getting thread for the American people, but for making thread harder to get than it would be if there were no such institution. . . . It makes a great impression on the imagination, however, to go to a manufacturing town and see great mills and a crowd of operatives; and such a sight is put forward, *under the special allegation that it would not exist but for the protective tax*, as a proof that protective taxes are wise. But if it be true that the thread-mill would not exist but for the tax, then how can we form a judgment as to whether the protective system is wise or not unless we call to mind all the seamstresses, washerwomen, servants, factory-hands, saleswomen, teachers, and laborers' wives and daughters, scattered in the garrets and tenements of great cities and in cottages all over the country, who are paying the tax which keeps the mill going and pays the extra wages? If the sewingwomen,

teachers, and washerwomen could once be collected over against the thread-mill, then some inferences could be drawn which would be worth something. Then some light might be thrown upon the obstinate fallacy of 'creating an industry', and we might begin to understand the difference between wanting thread and wanting a thread-mill. Some nations spend capital on great palaces, others on standing armies, others on iron-clad ships of war. Those things are all glorious, and strike the imagination with great force when they are seen; but no one doubts that they make life harder for the scattered insignificant peasants and laborers who have to pay for them all. They 'support a great many people', they 'make work', they 'give employment to other industries'. We Americans have no palaces, armies, or iron-clads, but we spend our earnings on protected industries. A big protected factory, if it really needs the protection for its support, is a heavier load for the Forgotten Men and Women than an iron-clad ship of war in time of peace.

"It is plain that the Forgotten Man and the Forgotten Woman are the real productive strength of the country. The Forgotten Man works and votes—generally he prays—but his chief business in life is to pay. His name never gets into the newspapers except when he marries or dies. He is an obscure man. He may grumble sometimes to his wife, but he does not frequent the grocery, and he does not talk politics at the tavern. So he is forgotten. Yet who is there whom the statesman, economist, and social philosopher ought to think of before this man? If any student of social science comes to appreciate the case of the Forgotten Man, he will become an unflinching advocate of strict scientific thinking in sociology, and a hard-hearted sceptic as regards any scheme of social amelioration. He will always want to know, Who and where is the Forgotten Man in this case, and who will have to pay for it all?" *

* Sumner, op. cit., pp. 145–149.

Since the forgotten member is always a member related to the rest of

The examples so far given have all shown a universe unduly curtailed or simplified through the neglect to consider some one or more of its essential relations or members. But there are cases in which the very existence of a universe of interrelated members seems to be ignored. To quote Whately once more (though here again he gave the example under another head):

The neglected whole.

"When a multitude of particulars are presented to the mind, many are too weak or too indolent to take a comprehensive view of them; but confine their attention to each single point, by turns: and thus decide, infer, and act accordingly: *e.g.*, the imprudent spendthrift, finding that he is able to afford this, *or* that, *or* the other expense, forgets that *all of them together* will ruin him.

"To the same head may be reduced that fallacious reasoning by which men vindicate themselves to their own conscience and to others, for the neglect of those *undefined* duties, which though indispensable, and therefore not left to our choice *whether* we will practise them or not, are left to our discretion as to the *mode*, and the particular occasions, of practising them; *e.g.*, 'I am not bound to contribute to this charity in particular; nor to that; nor to the other': the *practical* conclusion which they draw is, that *all* charity may be dispensed with."

In each of these cases the trouble lies, as Whately so clearly says, in the failure to "take a comprehensive view" of the universe as a whole. In the former case we forget that when a given variable (namely, the money at one's disposal) is placed there it cannot also be placed here; in the latter we forget that if the variable (namely, charity) is not placed there, it ought to be placed here, since it ought to have a place somewhere or other in the universe.

The same failure to keep the whole universe and its rela-

the system, no hard and fast line can be drawn between a case of the forgotten member and a case of the forgotten relation.

tions in view accounts for what Whately calls the *Fallacy of Objections, i.e.*, "showing that *there are* objections against some plan, theory, or system, and thence inferring that it should be rejected; when that which ought to have been proved is, that there are *more*, or *stronger*, objections against the receiving than the rejecting of it. . . . For there never was, nor will be, any plan executed or proposed, against which strong and even unanswerable objections may not be urged; so that unless the opposite objections be set in the balance on the other side, we can never advance a step."

This Fallacy of Objections is peculiarly characteristic of people whose energy is small but whose moral or æsthetic sensibilities are morbidly developed. They should help to pay a detective to catch a thief, and refuse because a detective's work is not straightforward and frank; they should help to hang a murderer, or should shoot a murdering burglar to secure the safety of honest people, and hold back because it is cruel; they want the liquor traffic cut down and they are convinced that a license system is the only thing that will do it, but they object to this because it makes the government 'a partner in sin'; they should wash their clothes, but they are afraid of soiling their fingers. Perhaps there is some relation in life in which each of us strains for ever at gnats and swallows camels. We see the gap in the universe, but there is no ideal material at hand, so we let it go unfilled:

> "Our common problem, yours, mine, every one's,
> Is, not to fancy what were fair in life
> Provided it could be, but finding first
> What can be, then find how to make it fair
> Up to our means; a very different thing."

CHAPTER XXI.

THE DIFFERENCE BETWEEN INDUCTION AND DEDUCTION.

THE student who has followed the preceding chapters at all thoughtfully has doubtless been impressed by a keen and disappointing sense of limitation: there are so many things that we must not do, and so few that we may, so many cautions to hold us back, and so few positive principles to help us on. This impression is well founded. The principles of deductive logic are important indeed and should never be forgotten: and yet if any one dwells on them exclusively, and forgets that cautions would be worthless in the absence of some positive forward impulse, he is likely to become a mere carping critic; a fault-finder, who is forever detecting flaws in the reasoning of others, but utterly incapable of doing any constructive work of his own.

Difference in limitations.

Induction is that part of logic which is concerned with the onward movement that deduction cannot undertake. Its essential task can be best understood by looking once more at the limitations of the syllogism and then seeing how it tries to supplement them. The first figure teaches that if every A is a B and every B is a C, it is safe to conclude that every A is a C. But it does not tell how to find out in the first place that every A is a B or that every B is a C. It proceeds in this case, as they say, from generals to particulars, but not from particulars to generals. Induction tells us how to prove in the first place that every A is a B or that

every B is a C; how to get from particulars to generals. The second figure teaches that dissimilarity proves objects not to be identical; but it does not help us to prove that objects are identical. Induction does. The third figure teaches that the coexistence of relations proves that they are compatible, that they *may* come together; but it does not try to prove that they are necessarily connected, so that they *must* come together. Induction does. Thus in the case of each figure induction attempts a task in the presence of which deduction is helpless. This work of getting more general or more positive or more emphatic results than those reached by the syllogism is not the whole of inductive logic any more than the syllogism itself is the whole of deductive; but, like the syllogism in deduction, it is the heart of the subject.*

Why Induction is able to go ahead and do more than Deduction we shall understand better when we have reached the close of the next chapter, and better still when we have gone farther. But there are three things about Induction by which we can explain at least a part of the difference now.

In the first place, every system of Induction rests on the assumption that facts of true logical significance can be attained by direct observation. If a person whose mental life was limited to deductive reasoning were asked whether the man in front of him had light hair or dark, he might be in possession of some premises that would enable him to answer the question or he might not; and if he had no such premises, he

* It is often said that the difference between Deduction and Induction is that the one proceeds from generals to particulars, while the other proceeds from particulars to generals; that is to say, that deduction proceeds from statements about classes of things to statements about smaller classes or about individuals, while induction proceeds from statements about individuals to statements about classes. But in deduction it is only the first figure of the syllogism that goes from statements about classes to statements about the individuals in them; and in induction it is only the process corresponding to the first figure that is concerned with mere generalization. This statement, therefore, is based upon too narrow a view of the scope of both branches of Logic.

could not answer it. But the one thing which he could not do would be to look at the man and see. Reasoning absolutely limited to deduction is eminently suited to people like the monastic scholars of the middle ages who did not profess to possess any other source of true knowledge than the written word of the Church as found in some recognized authority, such as the Bible or the works of Aristotle or St. Thomas. To the orthodox Scholastic every opinion at variance with such authorities was not only false but wicked. "There is a characteristic anecdote of Scheiner, who contests with Galileo the honor of being the first to observe the spots in the sun. Scheiner was a monk; and, on communicating to the superior of his order the account of the spots, he received in reply from that learned father a solemn admonition against such heretical notions: 'I have searched through Aristotle,' he said, 'and can find nothing of the kind mentioned; be assured, therefore, that it is a deception of your senses or of your glasses.'" * Thus, to the Scholastic, the data for reasoning were all given by some authority, and usually in the form of general propositions. In contrast with this, Induction supposes us to be set loose in the world with all our senses about us to collect our own data; to find the straw as well as to make the bricks.

Now all observation is of individual facts. General truths may be inferred; but the facts on which they are (or should be) based must be observed one at a time. Consequently, while the Deductive reasoner is accustomed to appeal for the most part to general principles of some sort by way of premises, and thus learns to regard such principles with a certain veneration, the Inductive reasoner is appealing all the time to individual facts as revealed by the senses, and he gives a large share of his veneration to them. Hence the saying that in science a single fact is worth a bushel of theory.

In the second place, Induction distinctly recognizes the

* Baden Powell's "History of Natural Philosophy", p. 171. Quoted in Fowler's "Inductive Logic", p. 293.

presence of change in the world. It may be remembered that the relations of things (as distinguished from the laws or thought pure and simple) upon which deductive reasoning turns are individual identity, similarity, and the coexistence of qualities or other attributes. Now all three of these relations could exist—identity after a fashion and the other two very well—in a world in which there was no change, in which nothing ever happened or came to pass; and it may be remembered that this notion of change was so remote from the conception of the universe involved in deduction that the rules of the syllogism have been declaring for centuries that contrary relations prove objects to be non-identical, without ever stopping to say that this rule only holds when the objects in question are observed at the same time, or at least nearly enough at the same time to make it certain that these particular relations could not have changed. Induction on the other hand has to do almost altogether with a world of change; and for it the most important relation of all is the relation of one change or event to another.

A third characteristic of induction is that it rests at bottom upon a tendency, unconscious or conscious, vague or definite, to act as though we took for granted that in the changes and other relations of things we might expect a certain uniformity. This tendency is so important that we must presently give a longer account of it.

The fourth characteristic of Induction is that when it reaches the critical stage it attempts more or less seriously to find out and examine every case of a given sort in the world or the part of the world under consideration. Let us explain this with reference to each of the figures separately.

If we wish to find out whether it is true that every member of a certain family has light hair, the simplest inductive method is to look at each one of them, and if we find none that have not and if we are sure that we have examined them all, we can be sure that the statement is true. In this way we are able to gain the singular or the universal proposition

which is required as one of the premises of a deductive argument in the first figure.

According to the second figure dissimilarity proves that objects are not identical, but similarity does not prove that they are. What then will prove it? If I leave a book of mine in a library and the next day find one there which looks precisely like it, how can I make sure that it is mine, and not some other copy of the same work? I can make sure of it if I make sure that no book has gone out of the library since mine was left there, and then examine every book in it, but find no other that looks precisely like the one that I left. Similarity will not ordinarily prove identity, for there is no limit to the number of things that may resemble each other; but if we are certain that some former object still exists in a certain part of the world and still retains its former appearance, and if we are equally certain that in that part of the world there is now only one object with that appearance, we may be certain that the two are identical. Thus by a sufficiently exhaustive examination of other things we can prove the identity of those in question, where the second figure of the syllogism which does not look beyond the objects given can prove only non-identity.

The third figure proves from the coincidence of two given relations that they are compatible; but it can never go further and prove that they are necessarily connected. Induction can; and again it proceeds by going beyond the two given relations and attempting to consider a worldful. Suppose for example that an event A is immediately followed by the event K. For deduction this proves that the succession of these two events, A and K, is not impossible; and for induction this one case taken by itself proves nothing more. But if we assume, as induction does, that every event has a cause with which it is necessarily connected, and if there is any way—whether direct or indirect—of going over all the events which preceded K and of showing that there was no event but A which could have caused it, then we can be

quite sure that A did cause it. And something of this sort is what induction attempts.

Thus there is an inductive process corresponding to each figure of the syllogism, and in each case the inductive process attempts to reach conclusions which the deductive does not, by looking beyond a mere pair of given relations and attempting to exhaust the universe.

Difference in certainty.

One of the most obvious differences between deduction and induction is that in deduction any conclusion that follows from the given premises at all seems to follow with absolute certainty, while in inductive reasoning it only follows with a greater or less degree of probability. This difference is striking and important, and it is sometimes thought to depend upon some absolutely fundamental difference between the two methods. But it does not. So far as the strictly demonstrative side of the two is concerned their problems are precisely the same, namely: to find whether the conditions named in the premises could exist in the assumed universe in the absence of the conditions named in the conclusion. The difference in certainty is due merely to the greater complexity of the material which induction attempts to handle. In the first place, the relations of things—the laws of the universe—which induction has to assume are more complex; in induction we must reckon with the uniformity of nature and all that that implies as well as with the simpler relations of things assumed in deduction. Then, in the second place, the particular facts that have to be built together according to the general relations of our universe are—as we have seen—ever so much more numerous in induction—often indeed innumerable, and even when we know that all the facts are in our possession it is a much more difficult and uncertain thing to realize how they must all fit together when there are a great number of them than when there are only two. Moreover, in induction, where the ideal is to search through the whole world and find every fact of a given sort, we usually know perfectly

well that the search has been incomplete, and even when we have tried to make it complete there is always the chance that something has been overlooked and that we really have not exhausted the universe after all. This is why the inductive sciences are being continually corrected, while such a science as geometry (which does not demand this exhaustive search through the world) stands from the first with little or no correction. Assume the uniformity of nature, and a 'perfect induction', or one which really exhausted the universe in question, would give quite as much certainty as deduction; but in this complex world of which we are so ignorant, perfect induction is little more than an ideal. We come as near to it as we conveniently can and then begin to guess, trusting to future experience to correct us if the guess is wrong. But the fact that in induction we often have to guess for lack of premises or for lack of skill to put them together does not prove that if anybody had the premises and had the skill his inductive construction of facts would be any less infallible than his deductive.

CHAPTER XXII.

THE UNIFORMITY OF NATURE.

ACCORDING to the old definition, "Man is a rational animal"; and to say this means much more than merely to say he is rational. Animals are distinguished from sticks and stones by the fact that they can feel and move. The movement comes in response to the feeling; but animals differ from each other with respect to the connection between the two. With some the movement follows upon the feeling directly, uniformly, and, so far as we can judge, inevitably. Others can postpone the movement until they have stopped and considered. Man can stop longer and consider more than any other animal, and for that reason he is called pre-eminently rational. But even man cannot always stop and consider how to act in response to his impressions. He cannot help swallowing anything which has begun to go down his throat; he cannot endure more than a certain amount of pain without crying out; and without considerable training he cannot avoid raising his arms, shutting his eyes, or drawing his head back when a blow is aimed at his face. Reflex acts like these shade so imperceptibly into the acts which we call purely voluntary that it is often impossible to say whether some act, even if it is our own, belongs to the one class or to the other. Moreover, there is good reason to believe that all voluntary acts are built up on a basis

How we come to believe in it.

of those which are purely reflex and instinctive; so that if we did not resemble the lower animals in performing the latter we should never be capable of the former; and if this were the case—if we could not perform a voluntary act—there would be no use in considering what act would be best. This means there would be no use in reasoning; for sooner or later all reasoning in this world has reference to some possible act. Therefore, as man is actually constituted, his being rational is connected in the closest possible way with being an animal, subject to all sorts of impressions from the world outside, and bound to respond sooner or later to these impressions by some kind of action.

The advantage which man possesses of being able to stop and consider how to act longer than any of the other animals is not without its dangers. To let an opportunity for action go by is often quite as fatal as to act rather stupidly. A fox that considered too long because he feared a trap would soon starve to death, and a man who never did anything until it was too late would get along as badly as one who acted on every impulse as soon as it arose. What we need is some way of combining the advantages of both kinds of action, the wisdom of the deliberate with the rapidity of the purely reflex or impulsive; and this we get in a large measure through our capacity for forming habits. When an act has been advantageous we tend to do it again under similar circumstances, and every time we repeat it it becomes more spontaneous, easy and rapid, until at last it becomes to all intents and purposes purely instinctive or reflex. These results of habit are very useful in the main. Of course habit has its drawbacks as well as its advantages; for occasionally we form habits that are bad from the beginning, and sometimes an exceptional condition of things will make a mode of action which is generally good extremely inappropriate. But the race would not have acquired and kept the habit-forming tendency at all except in adjustment to a world in which there is uniformity enough to make it profitable.

Mental anticipations are one form of habit. These do not seem to come at first in the form of anything so definite as a judgment. The new-born child does not say 'Now I shall see this or feel that'. He does not ask 'Is it to be so or is it not?' and then answer his own question in the affirmative. There is no antithesis between 'Yes' and 'No'. There is only a purely reflex expectation. The image of the anticipated experience enters the mind and fills it. But the anticipation comes nevertheless, and thus in the imagination, as in the muscles, the nervous system produces a very naïve and elementary form of expectation. This expectation when it arises is not, as some philosophers used to think, derived from any 'innate principle' or 'implicit' or 'unconscious' thought that nature is always uniform; but in each particular case the particular expectation arises spontaneously and mechanically as a result of habit.

All this is only to say that man is a creature of habit because he has to live in a world in which Nature is uniform. It does not tell how we came to believe in that uniformity. The blind tendency to form habits and thus act as though order were to be expected in the world is something very different from the explicit thought that it exists; and yet without the former it is difficult to see how we could have gained the latter. The process by which the one leads to the other is something like the following.

As the child gets along in life and his expectations are sometimes disappointed he gains a sense of the difference between the images and expectations that arise from within and the experiences that come from without, *i.e.*, between thoughts and realities; and at the same time he gains a sense of the difference between the thoughts that correspond to the realities and those that do not; *i.e.*, between the true which he can welcome with the judgment 'Yes, it is so' and the false that he must reject with the judgment 'No, it is not so'. Some of his expectations, however, are neither grati-

fied immediately nor finally disappointed, but only delayed. If he smells something good to eat he may have to wait before he can get it or even see it. But everything that he does in the interval is done with the anticipated experience in mind, and thus his activity takes the form of a search.

Thus a period of delay between two connected experiences produces a modification in the original association. It is now no longer a matter of smell and sight or taste, but of smell, expectation and search, and then sight or taste. Moreover, the period of expectation and search is sometimes longer and sometimes shorter, and consequently if the child had to go away some day without finding the object of his search he would still think of it as there and picture himself finding it at the end of a longer search; so that at last he might say to himself, 'When you get the smell (or other sign, whatever it is) you can always find the object if you look long enough'. In some such way as this we come to believe that a uniform connection between two given circumstances exists whether we happen to observe it or not. The same sort of experience taking place in a thousand other relations, we learn with each one of them to look for the circumstance that is necessary to fulfil our spontaneous expectations, or, in other words, to look for the relations of things necessary to make our experience of the world uniform.

In cases where our expectations are not only delayed but positively disappointed, the disappointment or surprise is apt to prick our attention and make us notice the presence of some variation in the conditions that we had not observed at first; and if this circumstance happens to be connected in any real way with our disappointment and the experience is repeated, we soon learn to modify our expectations and to say 'When A is present expect B, *except* when C is present also'. In this way our expectations of uniformity become more and more refined. Moreover, since the feeling of surprise is present in every case of disappointed expectations, if

the feeling once makes us open our eyes and look around for the relation necessary to make our experience uniform and if the search proves useful, we shall be more likely to look around the next time we are surprised; and thus form the general habit of seeking for uniformities when they are not apparent.

Even this habit of seeking for uniformities in all the situations in which we are placed is something different from a formal conviction that Nature is always uniform, just as the habit of meeting one's obligations is different from any theory that he should be honest; but for practical purposes the habit without the theory is better than the theory without the habit. The habit alone is sufficient to give the practical confidence in uniformity with which induction starts. When the logician asks the man of sound sense and practical usefulness what reason he has for believing that the uniformity he looks for is always present the answer will probably not satisfy the logician. The man has never tried to give his habit of thought a logical basis; perhaps he has hardly recognized that he had it. It was merely one of the unconscious products of his nature that happened to help keep him alive. Yet, after all, when the general truth that Nature is always uniform does dawn upon us and does become explicitly recognized, it is through the habit. For if the question of the uniformity of Nature is ever put to us in this general way, we try to think of some situation in which we should not or could not feel the impulse to seek uniformity whether we actually found it or not, and when we cannot think of any we agree that uniformity is always to be sought; and this implies that it always exists.

Its two main aspects.

There is nothing in the world but a vast number of things of various kinds acting upon each other in various ways under various circumstances, and we cannot possibly conceive of an event which could not be explained, if we knew enough about it, as due to the action of a thing or things of some particular kind

under some particular kind of circumstance. This is only another way of saying that the uniformities we expect to find in the world take two main aspects, one of which is indicated by the term 'thing' and the other by the term 'circumstance'.

This distinction we make between things and circumstances enables us to find uniformity in phenomena even when they are not precisely alike. If the same series of phenomena should persist or recur with absolutely no variation, like the notes in a piece of music reeled off by a machine, the distinction between things and circumstances would not be necessary; each part would be determined in every respect by its relation to the inflexible whole, and there would be no other particular part to which it bore any special relation. But, as our experience actually occurs, this is not the case. Experience is capable of infinite variation, and still it can be reduced to uniformity of the sort that we believe in by distinguishing between these two aspects, the thing-aspect and the circumstance-aspect, and by supposing that either the thing or the circumstance can be changed while the other remains the same, or that both can be changed or remain the same together. In this way our belief in uniformity does not require us to expect an absolute repetition of events except when things and circumstances are both the same.

To explain first of all what we mean by the thing-aspect of uniformity. We distinguish between different things, and therefore this thing-aspect of uniformity does not mean that all things are alike. For if everything in the world were to appear and act precisely like everything else no one would be able to distinguish his hat from the cat or a bar of soap, and there would be no reason why he should. On the other hand, if everything in the world were not only to appear different and act differently from everything else, but were also to be continually changing its appearance and mode of acting in a perfectly arbitrary way, it would still be impossible to distinguish between one thing and another, for the

points of difference found at one moment would cease to exist in the next. In the first case any expectation would be fulfilled by one thing as well as by any other; in this latter case no expectation would be fulfilled by anything. Thus while the possibility of distinguishing between things implies that they differ from each other, it also implies that each of the things distinguished has some characteristic uniformity of appearance or way of acting which marks or constitutes its own individuality or nature.

An oxygen atom acts in one way in the presence of hydrogen and in another in the presence of nitrogen, but it never ceases to act like oxygen and begins to act like iron or chlorine. A formula which told the various ways in which it does act under all possible circumstances would be a definition of its nature.

We not only assume that every thing has a nature of its own, but we assume also that there are absolute similarities between them, so that they can be divided into various 'kinds', all the things of a given kind having so much the same 'nature' that one might be substituted for another without producing any perceptible change in the result.

Along with the uniformities characteristic of certain individuals and classes of things we come to have also some idea of the uniformities characteristic of things in general, particularly of material things. A material thing, as contrasted with a mere phantasm, is expected to be tangible, to resist pressure more or less, to have a continuous existence in time and space in the sense of not passing from one point to another without passing through all the intermediate points; and perhaps also to obey the laws of inertia and gravitation. Finally, since things are only given as aspects of the sum total of experience called the world as a whole, by reference to which aspects the essential uniformity of the whole can be conceived, it is a part of the very nature of a thing to be bound up with others in a whole world, to act upon them and to be acted upon by them, and to observe the same

general laws. To speak of a thing out of all relations to everything else is to speak of an aspect apart from that whose aspect it is, and is therefore absurd.

By the Circumstances of a thing as contrasted with its nature we mean, not the general rule of its action, but the particular conditions to which the rule must be adapted at any given instant. These conditions include the present or vanishing state or activity of the thing itself. An iron, for example, that has just been heated will not act precisely like one that has not. They include also the position and nature, and present or vanishing states or acts, of other things. Oxygen does not act in the presence of hydrogen precisely as it acts in a vacuum, and it does not act in the presence of hot hydrogen precisely as it does in the presence of cold.

When I say that the conditions which help to determine an act include the position and state of other things besides the agent, I mean, not merely that sometimes an act is determined in part by something outside of the agent, but that it is always. The moth in its chrysalis seems to develop wholly from within, shut off from all the rest of the world; but take away the warmth of the sun outside and how long would the development last? Even within the chrysalis we have not one simple thing moving by a wholly inward law from one state to another, as we may be inclined to assume at first, but rather a whole system of cells acting and re-acting upon each other; and each one of these cells again is composed of atoms, all acting with reference to what lies beyond them. In short, all explanations in natural science come down finally to atoms; and no explanation assumes that an atom ever acts wholly from within, regardless of the rest of the world. Therefore every ultimate explanation is made on the assumption that every act or state of a thing is determined partly from without. In other words, causation always involves interaction.*

* This is quite as true in psychology as it is in physical science. When we 'explain' a person's thought we either regard it as due to the

Circumstances, like things, can be divided into various 'kinds'; but in their case the division is always rather loose. Two things can be absolutely alike in their nature, for there is no reason why they cannot both act according to precisely the same general law. But two circumstances can never be precisely alike. If one exists here at this time the other must exist somewhere else or at some other time, and consequently they must always differ in time or place if in nothing else.* We often speak of two circumstances being exactly alike, but when we do so we only mean that they are alike in every respect that is worth considering for the purpose in hand. Circumstances of essentially the same kind, like things of the same kind, can be substituted for each other without any essential change in the resulting phenomena. If they could not, the division of experience into the two aspects of things and circumstances would not help us in our conception of its uniformity. Wind blowing upon the green leaves of an aspen-tree here to-day makes them move here and to-day, if every other circumstance is the same, just as wind blowing upon them there and yesterday made them move there and yesterday.

play of different feelings and ideas or else we turn to physiology and regard it as due to various 'currents' in the cells and fibres of the brain. In either case the explanation involves something beyond what is explained. Of course this causal analysis does not prevent us from believing in the real unity of the mind, and a similar analysis in natural science does not prevent us from believing in the unity of the world. The fact that explanations take account of aspects of reality does not turn these aspects into something independent and self-existent.

* I do not refer to their absolute position in time and space,—to mere position apart altogether from the relation of events to other events that precede or coexist with them and of things to other things that surround them. Such absolute position does not enter into our explanations at all, and is therefore not a 'circumstance' in the sense in which the word is used here. When we say that the result of an act was different in two different cases because the act occurred at two different times it is not the bare time as such that we have in mind, but the preceding or concurrent events; and the same is true, *mutatis mutandis*, of differences in place.

Having divided our world into these two aspects of thing and circumstance we are not satisfied to say merely that under precisely similar circumstances precisely similar things will produce precisely similar results. We go further, and seek for laws, which take account of the differences between things and circumstances as well as of their resemblances, and which tell not merely what takes place when the same things or precisely similar things are placed in precisely similar circumstances, but tell also how much variation in the circumstances or the things produces a given amount of variation in the result. We do not merely say that bodies attract each other; but we say that they attract each other in direct proportion to their mass and in inverse proportion to the square of their distance. This would not be possible unless we took account of the amount of difference between two situations and measured off one against another. It is this recognition of differences in amounts of difference, and their precise measurement, that enables us to introduce the conception of proportion into our formulæ and to deal with them by mathematics. **Law.**

The notion of law would be impossible and our search for uniformity in the world would be doomed to failure if our ability to see different aspects of reality were limited to the general distinction between things and circumstances. But the very ability to enumerate a thing's different attributes involves the power on our part of attending to one of them at a time, and in the same way the very possibility of stating some general law such as that of gravitation involves the power of attending to some one of a thing's relations and accounting for it without saying anything about the rest. In accounting for the fall of an apple by this law, we consider its mass and the mass of the earth, its motion towards the earth, and the original distance between them. We do not say anything about its taste or color or even perhaps about its size and shape and its motion around the sun. Of course these neglected relations must be accounted for too,

and the state of affairs that produces them must be compatible with the state of affairs that produces the apple's fall. The total state of the world at every instant is the cause of its total state at the next; but we feel that we have a right in our explanations to break up this complex whole of each instant into as many aspects or different relations as we please and account for some of them at a time.

Thus the uniformity discovered by each one of our explanations is a uniformity in some one definite respect, and when we speak of the 'Cause' of an event we are almost always trying to pick out the essential elements of its nature and the relations to its own past and to other things that account for some few of its salient features. Nothing short of the whole universe would account for the event as an absolutely complete whole.

The ability of which we have just spoken to attend to some one aspect of a situation involves certain dangers.

Precision in uniformity.

For we may forget that the situation has other aspects which might also be accounted for, and therefore rest with the feeling that our explanation of the situation as a whole is complete when it is really very incomplete. Worse still, the explanation which we give of the aspect of the situation that we happened to notice may be quite inconsistent with any reasonable explanation of the other aspects that we did not happen to notice, and therefore wrong, without our detecting the fact, as we should have done if we had realized how much there was to explain. We may say that the light on the wall comes through a certain window, and be perfectly satisfied with the explanation so long as we fail to notice that the glass in the window is blue and the light on the wall is not.

Nay, even if we notice such a discrepancy, we may deliberately disregard it, with the feeling that in some way or other it can be detached from the rest of the phenomenon, and therefore makes no difference, and that in any case it must not be allowed to interfere with the conclusion already

reached. This is more likely to be true when our emotions are involved. If a storekeeper has been robbed and there are five or six impressive circumstances which all suggest a certain clerk as the culprit, he may jump to the conclusion that the clerk is guilty, although he is perfectly aware of some other circumstance, which is less striking, but which is nevertheless absolutely incompatible with the clerk's guilt. If this circumstance is mentioned and the storekeeper is forced by it to admit that the clerk is not guilty there is considerable chance that he will admit it very reluctantly, and will still feel that he 'nearly did it', and bear him a grudge accordingly. The good reasoner, whether he be a scout finding a trail, a detective tracing a crime, a physician diagnosing a case, or a scientist pure and simple, is the one who has not only skill enough to observe the less striking circumstance but strength enough to hold it in mind until it is accounted for, instead of allowing it to be swept away like the still small voice of conscience by the larger mass of more vigorous impressions and associations that hurry us on to a more apparent goal.

Even in the aspect of a situation that we really do attend to we may overlook the necessity for explaining the finer details. Every one knows in a general way that rough water is caused by wind; but when we have accounted for this general appearance of roughness most of us are satisfied. As Professor Huxley says: "Even thoughtful men receive with surprise the suggestion" that the form of every wave and the direction taken by every particle of foam "are the exact effects of definite causes"; and so long as we fail to recognize this precision of the causal relation it is perfectly evident that we do not realize the complete uniformity of nature.

Thus while it is absolutely necessary that we should ignore certain aspects of a situation when we try to explain others, it is quite as necessary that we should not ignore the wrong ones. The only thing to do under these circumstances is

first to make an explanation that will account for the aspect of the situation in which we are interested and then to inquire in a cold-blooded, critical way whether there are not other aspects of the situation with which the explanation is not consistent. If there are we must reject it even though it has cost us years of labor. It may be that every one has a lurking tendency to feel that a plausible explanation has some value, whether it be true or not, until it is proved to be false. But—except as it helps us to remember the facts themselves—it has not, and the sooner we recognize its falseness and try to find one that will stand in spite of criticism, the better it is for ourselves and everybody else. It is useless to try and live in a fool's paradise or to bury our heads in the sand and refuse to recognize the disagreeable facts that upset our theories.

Besides the necessity for looking at one aspect of a situation at a time there is another reason why it is hard to realize that every event "is the exact effect of definite causes"; namely, because it is hard to realize that the events themselves are exact and definite. Most of our ideas are very hazy, and it takes hard training to make us realize that the realities which these ideas profess to represent are not as hazy as the ideas themselves; that though we can form no clear idea of the beginning of things, there was no chaos, no mere 'stuff' without definite attributes and relations; and that though we may be in doubt about some state of affairs, there is no uncertainty or hesitation in the state of affairs itself. Hazy thoughts claim to represent reality as much as clear ones, and so long as all our thoughts are hazy we cannot realize that the claim is false. We must know some things definitely before we can begin to realize that all things are definite whether we know them definitely or not. Whenever we have used exact measurements so often that we feel the tendency to apply them to everything and no description seems complete without accurate statements about size, shape, direction, number, duration, degree, and

so on, we are then in a position to realize that these quantitative relations of things fit into the general law of uniformity as well as the qualitative. Then, but hardly any sooner, do we realize that the form of every wave and the path of every falling leaf are 'the exact effects of definite causes'; that the law of uniformity is not only universal but precise.

The thing-aspect of this precision.

To realize the uniformity of nature it is quite as necessary to keep a clear view of the individuality or separateness of different things as to make clear distinctions between different circumstances. Yet this absolute distinction between different things we often tend in our instinctive reactions to ignore. If we are stung by one hornet it seems appropriate in revenge to wipe out the entire nestful, whether it includes the one that hurt us or not. If certain Americans are massacred by a set of Chinamen in Asia it is perfectly natural for a mob of other Americans to revenge itself by attacking some innocent laundryman in Kansas City. If two or three members of a household do us an injury it is difficult, especially if we are not brought into close contact with them, not to harbor resentment against the whole household. To the individuals themselves the difference between them may mean everything; to us it means nothing at all.

In much the same way as we ignore the numerical difference between several individuals and treat one as though in some way it were actually identical with another because it belongs to the same group, so we ignore also the distinction between different kinds of individuals. To make an accurate definition is one of the hardest things in the world; but the curious thing is that it often comes to people as a sort of revelation that such definitions can be made at all. 'Definition is possible!' This thought it was, perhaps more than any other, which gave Socrates a life-long inspiration. But to assert the possibility of definition—to say that some moral attribute or some material thing can be defined in such a way as to include by the very definition all that we think

the word should stand for, and to exclude all that we think it should not—is only another way of saying that whether we can find it or not the difference between things or kinds of things is always perfectly definite.

In still another way uniformity of nature involves more about things than is evident at first. It involves the absolute permanence of whatever can truly be called a Thing. People often suppose that when things are 'burned up' the total amount of matter or of ultimate things in the world is diminished and that when plants grow it is increased. But such a supposition is inconsistent with the very idea of a thing and with all our explanations of events that involve it. When we say that some given event is due to the circumstances in which certain things were placed, we take it for granted that the things existed before the circumstances, and were thus at least relatively permanent; and when we learn that the permanence of sticks and stones and other such things by which we have explained events is not absolute, we account for it by saying that after all they are not really things at all in the ultimate sense of the word, but only temporary combinations of atoms, which latter are truly permanent; and that these atoms are the true things. Thus we come to realize what the scientists call the conservation of matter. The permanence of things is one of the two great aspects under which we think of the general law of uniformity in the world; and to assume that any real thing is not permanent is therefore to deny the existence of absolute uniformity. Yet it is difficult to realize in a positive way that the law of uniformity implies the absolute permanence of every ultimate 'thing'. This scientific principle of the conservation of matter (and of any other ultimate reality) states for the things involved the same absolute uniformity that the law of absolutely precise causation states for the circumstances in which they are placed.

Thus far we have explained how we grow into the belief that Nature is uniform, and we have shown the forms which

we expect that uniformity to take; and yet after all we have not proved that such uniformity exists, and it is the proof of facts and not the history of beliefs with which we are concerned in logic. But where would such proof begin? If we do not take uniformity of any sort for granted we should have to begin like Descartes by pretending to doubt our own existence and the existence of other people to whom the proof might be addressed, for even personal identity is a principle of uniformity; and we should assuredly share in Descartes' failure. If, on the other hand, we are willing to be less thoroughgoing than Descartes and take something for granted to begin with, and thus begin our attempted demonstration with the assumption that there are things and that various uniformities have existed in the past, why is this any evidence that other kinds of uniformity exist now or that any whatever will exist in the future,—unless we already take for granted some wider principle of uniformity which decrees that if uniformity exists anywhere or at any time it must exist everywhere and always? The fact that the sun has risen (as we believe) every twenty-four hours throughout long ages makes us expect that it will rise to-morrow; but unless we already assume that the future will resemble the past it does not prove it. **Proof of uniformity.**

Possibly the best thing that any one can say in justification of his conviction that experience depends upon a world of uniform relations is merely this: that it is a faith growing out of his very nature as an active being (if he exists and has a nature), that he has lived by it in the past (if he has had a past), that the longer he has taken it for granted the more it has seemed to justify itself, and that he means to take it for granted in the future (if there is a future). Our belief in the uniformity of things is thus something which we can account for psychologically, and we can show that to deny this uniformity *in toto* involves conclusions which no sane man is willing to act upon; but there is no direct way of proving its existence.

CHAPTER XXIII.

SCIENCE AND THE PECULIARITIES OF THE RELATIONS THAT IT TRACES.

Twofold work of science.

THE conviction that there is a distinction between circumstances and things is only a starting-point in our search for the uniformity that we believe to pervade the world. The vast differences between one complex group of phenomena and another are not explained to our satisfaction by the mere general statement that they are all due to differences between things and the circumstances in which they are placed. The difference between one complex situation and another is always perfectly definite, and what we want to find out is the precise difference in the things and in the circumstances that accounts for this definite difference in the situations as a whole. This is the business of Science, and its work has two sides: (1) Observing as much as we can, and finding out from what we observe the general laws according to which things always act; and (2) inferring further from the concrete combination of circumstances or events that we observe and from the general laws that we have discovered what must be the concrete state of affairs where we cannot, or cannot yet, observe. As a result of various concrete observations made by himself and others Newton discovered and proved the law of gravitation; and an astronomer who knows this law and also observes the position of some heavenly body at various intervals is able to tell where it was before he saw it, when it will come within a certain distance

of the earth or the sun, and what will happen to it and to them when it does. In this way any one who was master of an absolutely perfect science would be able (1) to find every law in the universe, and then (2) starting with the present, to go back indefinitely and tell the history of the past and to look forward just as far and tell the story of the future. In the following chapters we shall speak first of the logical method pursued by scientists in the discovery of general laws, and afterwards of the application of these laws for the discovery of particular concrete facts. But before we begin it is desirable to say a few words more about the nature of the identity and causal interaction assumed in all such investigations.

Peculiarities of individual identity and causal interaction.

To tell whether one object resembles another we need only look at the two and compare them, and in the same way we can often tell by direct observation whether one event succeeds another; but to tell whether one object is identical with another, or whether some given event is the cause of another, simple inspection is not sufficient. If we lose sight of a thing for a single instant how can we tell that it has not been removed and another put in its place? Indeed, until we know something more about it than its outward appearance, how can we even be sure that such a substitution has not taken place by some jugglery before our very eyes? So also with the causal relation between one thing and another; however certain we may be that a change we observe in one thing followed immediately after a change we observed in something else, how can we be sure that it was caused by this rather than by a change in some third thing, miles away perhaps, that we did not happen to observe at all? No one can directly observe either the identity of an object with itself or its causal action upon something else, and therefore our identifications and causal explanations have to be reached by guesswork in the first instance, and if they are afterwards proved to be correct the proof has to be indirect.

Another thing to notice about these relations of identity and causal interaction is the conviction we feel that they are always present. Since they are aspects of the uniformity of nature that we believe in, we have a right to believe that at every moment in the past there were things identical with the things of the present, and that there are and always have been causal relations to other things that help to explain their present conditions; and the same is true, *mutatis mutandis*, of the future.

Still another peculiarity of these two relations is their exclusiveness. In this respect identity and causal interaction are very different from resemblance and succession. The house I live in can be similar to each one of a hundred others, wholly regardless of the place and time of their existence, and its similarity to one does not interfere in the slightest with its similarity to some other. It is true too that the house that keeps out the rain to-day can be identical with the house that was bathed in sunshine yesterday and none the less identical on that account with the house that was covered with snow two months ago, for these relations of identity and causation go back from instant to instant without a break to all eternity; but the house cannot be identical with one existing in some other place or state at the same time. There is only one thing at a time with which it can be identical, and to be identical with this means not to be identical with some other. The same exclusiveness is found also in the case of causation. The house we have been talking about can exist at the same time as an unlimited number of others, and an unlimited number of things might have been acting before or during its erection; but if the house was built by John Smith it could not have been built by anybody else; and if a spark from a certain locomotive destroyed it, it is quite certain that an earthquake or a stroke of lightning did not. It is perfectly true, to be sure, that if we try to explain absolutely everything about the house as it exists at a given instant the explanation will have to include

a statement about absolutely everything in the universe the instant before. Thus, inasmuch as the position in space of every object depends upon that of every other, the position of the house would certainly be affected by a boy throwing a stone in China. And if we use the word 'cause' with reference to some single aspect or relation of things that we are trying to explain it is true also that even this single relation may be due to the co-operation of several causes. John Smith may not have built the house alone. But when causes co-operate in this way no one of them is a complete cause. If anything is the complete cause of a condition, so far at least as one instant of time is concerned, it is its sole cause. A resemblance or coexistence or succession can belong at once to many things without diminishing the share of any. A relation of identity or causal interaction can not.

Our knowledge, or rather our assumption, that these relations of individual identity and causal interaction always exist and are always exclusive is very helpful in our search for uniformity in nature. If we have any way of showing that this, that, and the other thing of one instant is not identical with the thing of another, the first of these assumptions gives us a right to infer that some other—perhaps the only one left—is. In the same way, if we have any means of showing that this, that, and the other thing had nothing to do with some present condition of a thing that we are inquiring about, our assumption gives us the right to infer that something else had. In this way we can apply the 'method of exhaustion' in our search for particular uniformities. So with the second assumption, if we know that this thing of one instant is identical with a given thing of another, we know perfectly well that we need not look for any other thing of the same instant to be identical with it. Here again the same is true, *mutatis mutandis*, of causation. In this way our search for uniformity is simplified and shortened.

The fact that causes may be endlessly complex gives one

of our two relations—that of causal interaction—still another peculiarity, which is very important. The relatively simple relations between two or more things that we are accustomed to pick out and call relations of cause and effect are not independent; for the sequence of what we call the 'effect' upon what we call the 'cause' is at the mercy of other causal relations. If two things are similar their similarity is neither increased nor diminished by their likeness to other things or by the likeness of other things to each other. John does not look any the less like James because he looks like Henry also, or because Henry looks like Thomas. In the same way, if the line AB meets the line AC at an angle of 45 degrees you can draw as many more lines as you like to the point A from as many different directions as you like, without affecting the size of the angle in the slightest. As a result of this we are justified in absolutely ignoring the existence of Henry when we are discussing the resemblance of John and James, and in ignoring all the other lines which meet at A when discussing the relations of AB and AC.

This possibility of considering two relations regardless of everything else in the world is what makes the problems of deduction and of geometry so relatively simple. But with causation all this is changed. John can beat James in a fight if they are left alone, but if Thomas warns James to run or takes part too he cannot; the sun will keep a comet in a certain path if they are left alone, but if Jupiter happens to come too near, the comet may swing out of its path; and so on. Whether A is similar to B is a mere question between A and B; but whether A will cause B to act in a certain way is a question that also involves C and D and E; and when we are trying to find out what A will make B do we must know whether C and D and E are present and how they are acting. We cannot ignore them. If they are not affecting the relations of A and B we must make sure that they are not; and this, of course, makes our problem much more complex. Deduction and geometry can neglect irrelevant

circumstances; inductive reasoning about relations of cause and effect must eliminate them. This is often very difficult; and when the elimination has been made and we are thus able to conclude from the cases examined that A's action was the cause of B's, we must not conclude from this that this same act on A's part will always be followed by the same act on B's; but only that such will always be the case as long as nothing else interferes.*

This fact that one cause can interfere with another is what makes a knowledge of causal relations so very important practically as well as theoretically. No human effort can change a thing's identity, but if we know enough we can use our bodies in such a way as to pit one cause in the world against another and change its effects in accordance with our purposes. This is why knowledge is power. The thing that makes the knowledge of causal relations most difficult is the very thing that makes it useful.

* In this connection the reader may recall the statement made on p. 80 that causal relations seem to penetrate into the very being of things, while non-dynamic relations exist only externally for some observer. In this respect identity is like causation.

CHAPTER XXIV.

THE METHOD OF EXHAUSTION AND THE SEARCH FOR PARTICULAR UNIFORMITIES.

We use the method of exhaustion when we base a conclusion upon the results of a more or less direct and serious attempt to examine every case of a given sort in the universe.

The simplest application of this principle is found in the so-called Aristotelian, or Perfect, Induction. The first example of induction given on page 224 was of this sort, namely: This, that, and the other member of a certain family each has light hair; these members constitute the whole family; therefore every member of the family has light hair. In the same way we can say: 'The apostle James was a Jew; so was John; so was Peter'; and so on through the twelve; 'therefore the twelve apostles were all Jews'. January contains less than thirty-two days; so do February, March, etc.; therefore each month of the year contains less than thirty-two days. In this 'perfect induction' the conclusion is something more than a mere summary of the particular facts stated in the premises; for we might know that James was a Jew and that John was a Jew, and so on through the twelve, without knowing or without thinking that these twelve were all the apostles.

'Perfect induction.'

This Perfect Induction is relatively rare; for it is only in comparatively few cases that we can be sure that the individuals named constitute the whole class in question.

"The assertion that all the months of the year are of less length than thirty-two days . . . is a certain conclusion because the calendar is a human institution, so that we can know beyond doubt how many months there are. . . . But the assertion that all the planets move in one direction round the sun, from West to East, is derived from Imperfect Induction; for it is possible that there exist planets more distant than the most distant known planet Neptune, and" of "such a planet of course the assertion would" not hold true.*

If a being who was purely rational found that a Perfect Induction was impossible, he might go no further. But we men are animals as well as rational, and have an animal's tendency to react to every impression, and to react in the same way when the impressions are similar. Consequently we will often risk a conclusion that the premises will not altogether warrant, and when all the individuals with a given general appearance that we have examined have a certain particular characteristic we almost always take it for granted that those we have not examined have it also. "Practically in inductive argument an opponent" who maintains that some general statement is not true "is worsted when he cannot produce an instance to the contrary. Suppose he admits the predicate in question to be true of this, that, and the other, but denies that this, that, and the other constitute the whole class in question, he is defeated in common judgment if he cannot instance a member of the class about which the predicate does not hold. Hence this mode of induction becomes technically known as *Inductio per enumerationem simplicem ubi non reperitur instantia contradictoria.* When this phrase is applied to a generalization of fact, Nature or Experience is put figuratively in the position of a Respondent unable to contradict the inquirer." †

Why we accept less.

Thus we see how the inductive process by which we make

* Jevons' "Lessons", p. 213. † Minto, pp. 236–7.

and try to justify a universal proposition falls short of its ideal. We set out to exhaust the universe, and stop when we have exhausted our own knowledge or when we get tired of going any further, but we draw our conclusion nevertheless. The only justification we can give for such a proceeding is practical; that on the whole we get along better if we jump to such conclusions after a reasonable amount of investigation than if we always suspend our judgment and refuse to act until our data are absolutely complete.

Justification practical.

How many cases constitute a 'reasonable' number upon which to base a general conclusion depends altogether upon circumstances. If the general conclusion in question fits in with what we know about other things it will not usually require so much evidence in its favor as it would if it did not. In view of what everybody knows about other animals it requires very little evidence to prove that all sheep are mortal. How many cases make a reasonable number depends too upon how likely it is that we should know of an exception to the general rule if one existed. That every man in the civilized world is less than twenty feet tall we have a right to say at once, because we know that if taller men than that existed anywhere within the bounds of civilization we should be sure to have heard of them. A third consideration which helps to determine how many cases we should investigate before venturing upon a general statement is the practical importance of the question at issue. If the eternal salvation of every human being depended upon the truth of our statement the number of cases investigated would have to be very great indeed before any one of us would think it reasonable to draw the general conclusion. Anything short of an absolute exhaustion of the group in question fails to give absolute certainty, and the completeness of the exhaustion which we feel compelled to make will always depend upon the amount of certainty that we require.

When may we guess?

Though the circumstances which help to determine how much evidence for a general conclusion is reasonable are related to the case in question, they all lie outside of it. That is to say, the question of how much evidence is reasonable does not depend so much upon the nature of the problem in itself as upon its relation to what we know about other things—*e.g.*, the constitution of society that makes it likely that we should hear of a man twenty feet tall if he existed, the supposed divine law that would lead to our damnation for a false guess, etc.

The one thing in this connection which the student of logic should not overlook is this: As civilization advances, the need for accuracy and certainty of thought and action constantly increases. Our environment in this respect is changing very rapidly, while our natures change very slowly. The consequence is that the average man is apt to be too impatient of suspense and to jump to his conclusions too rapidly for his own good. And if the individual happens to be concerned with science, in the very front of the forward movement, where the need for accuracy and the means of attaining it are growing most rapidly, then intellectual patience becomes a virtue of which he can hardly have too much. An incautious or inaccurate farmer may get along after a fashion even in this day; but an inaccurate scientist is almost certainly bound to be an utter failure.

Perfect Induction and the Inductio per Enumerationem Simplicem may be regarded as the inductive processes corresponding to the first figure of the syllogism. They are not concerned pre-eminently with any particular kind of relations; they involve no refined analysis; and their only positive characteristic is the generality of their conclusions.

How problems and relations are interwoven.

On page 225 we saw how the principle of exhaustion is used to ascertain relations of identity and causation as well as to prove propositions that are merely general; and how the principle therefore furnishes an inductive process correspond-

ing to each figure of the syllogism. In the case of the general proposition the method of exhaustion is one of addition,—such and such a thing is true of this *and* that *and* the other, therefore of all. In the case of identity and causation the method of exhaustion is one of subtraction or elimination,—the thing or the cause in question cannot be this or that, therefore it must be the other. This method of elimination would not be possible unless we were sure enough of the uniformity of nature to assume that a thing identical with the one in question or that a cause for the given event must surely exist somewhere.

It is noteworthy that in practice questions concerning general truths, concerning identity and concerning causation are interwoven in many ways. When we have proved either by a 'perfect induction' or by the Inductio per Enumerationem Simplicem that everything with the characteristic A has also the characteristic B we can hardly avoid suspecting that between A and B there is some rather direct relation of causation. Conversely we can prove that everything with the characteristic A has also the characteristic B without the Perfect Induction or Inductio per Enumerationem Simplicem if we have any means of proving that A inevitably causes B. So with Identity. A person can be sure that the book he finds in a certain library to-day is the one he left there yesterday if he can prove that it is the only book like his that is now in the library, and that no books have gone out and come in since his was left. But how can one be sure that a book has not gone out of the library and another come in to take its place? Is it not only because he knows that books cannot pass through solid walls or throw themselves spontaneously through windows and doors, and because he knows also that if anybody had carried one book in and another out the librarian would have seen him or the lock on the door would have been broken, or some other perceptible change would have been caused, when it was not?

In this way knowledge of causal relations helps us to prove identity just as it may help us to prove the truth of a universal proposition. Such a case as this is not exceptional. We prove that the comet now visible is the one that appeared ten years ago because calculations made at that time showed that at precisely this time it must appear at the very spot where a comet is now seen; and since there cannot be two comets in the same place the one we see now must be the one we saw then. So too when a 'medium' tries to prove to you that the spirit with which she now claims to be in communication really is the spirit of your departed friend she tries to show that the spirit does and knows what nobody but that friend would or could do or know; and thus she leaves this problem on your hands: If my friend did not speak through the medium, how in the world am I to account for all the things the medium or the spirit that spoke through her did? In the same way once more, when a prosecuting attorney tries to prove that the prisoner before the court was the person who broke into the bank he attempts to show that there is some effect which could not possibly have been produced if the burglary had been committed by anybody else. Thus a question of identity, like the question of the truth of some general statement, can be resolved into a question of causation.

So, *vice versa*, when we are investigating a question of causation we always take for granted various relations of identity and various relations of kind such as are expressed in general propositions. When we conclude that a certain man must have been poisoned by arsenic because nothing else would have caused the same symptoms we assume the existence of the man and of the arsenic as real things retaining their identity from the time of the supposed poisoning to the time that the symptoms appeared. We also assume that every bit of arsenic has the same nature as every other bit and acts in the same way under the same conditions, and that men also are similar to each other, so that at least some

of the symptoms produced by the arsenic are substantially alike in them all.

In courts of law individuals as such are everything, and general laws or rules of action are only means for adjusting their conflicting claims. Consequently problems of identity in the courts are all-important, and all sorts of devices are resorted to for answering such questions as whether or not this particular man is the one that owned this particular horse or signed this particular bit of paper. But with science it is different; for scientists have very much more to say about the conditions under which certain effects are produced than about the identity of the agents. Since the chemist assumes all atoms of hydrogen to be alike he neither knows nor cares whether the one involved in a given reaction is Atom No. 49 or Atom No. 63. His identification stops when he finds out that the atom in question is some one or other of the many billions of similar atoms that we call hydrogen, and that it has recently been put through such and such a process. For that particular atom as such he cares not a snap of his finger. Its only value for him is to show what any atom of the sort will do under a given set of conditions. This distinction, however, is not absolute. The jury in a trial for murder has to determine whether the injury inflicted by the prisoner upon the deceased really was the cause of his death, and such sciences as geology and history are concerned very largely with the question of what individual person or thing it was that produced this or that given effect. After all, the question of identity and the question of circumstance are inextricably interwoven; for thing and circumstance are only different aspects of the same concrete fact and neither could act or even exist without the other.

CHAPTER XXV.

INDUCTION BY SIMPLE ENUMERATION AND THE SEARCH FOR CAUSES.

THE Inductio per Enumerationem Simplicem is regarded by scientists nowadays as a primitive and unsatisfactory kind of inference that should be replaced or supplemented wherever possible by an inference based upon the knowledge of causes. The essential difference between the two methods is that the one based upon a knowledge of causes involves analysis and the other does not. Inductio per Enumerationem Simplicem takes each of the observed relations in the mass just as it stands, notices how often they coincide, and then makes a guess about the future. Causal analysis is not content to take the observed uniformities so roughly; but splits them up, in order to find the simpler and more general uniformities which are involved. And then when this is done it may try to determine the conditions under which they will work together in the same way again. An inference based upon the knowledge of causes is therefore based upon a definite knowledge of all the details rather than upon a confused impression of a whole.

Analysis, inference, and explanation.

A farmer may happen to notice that red clover grows better near the homes of old maids than elsewhere in the neighborhood. Reasoning *per enumerationem simplicem* he might conclude that there is always something about an old maid that helps the growth of clover. But he would have no

means of knowing how much faith should be attached to this conclusion. He could not say how likely it was that the next case he noticed would conform to the rule. But suppose he should happen to think that old maids often keep cats, that cats kill mice, that mice destroy bumblebees, and that without bumblebees to carry the pollen from one plant to another red clover cannot develop its seed. Then he would have broken up the first uniformity—between the old maids and the clover—into a series of others, most of which, at least, are much more familiar and therefore much more thoroughly tested; he would know almost exactly how much reliance could be placed upon each of them; and putting them all together he could say without any hesitation whatever whether it is always true that old maids help the clover-meadows. In this way, a knowledge of causes gives much greater certainty than the bare, unanalytic Inductio per Enumerationem Simplicem.

In this example, as in almost any other that might be given, the analysis into causes or simpler uniformities is very incomplete. Each of the causal relations given might be itself reduced to others still simpler. That old maids keep cats; that cats kill and eat mice; that mice destroy the nests and young of bumblebees; and that red clover needs these bees to fertilize it: all these may be themselves mere Inductiones per Enumerationem Simplicem, about which we may feel sceptical; and we may wish to test any or all of them by finding the still simpler uniformities that would account for them if they really existed. If cats do always eat mice, the mice are probably good for them; if mice are good for cats to eat, it must be because they can be digested and pass into the system; if digested food passes into the system of a cat or any other animal, it must be because it can get through the membranes lining the alimentary canal; if food can get through these membranes, it must be because certain fluids will pass through moist membranes anywhere. In this way one uniformity after another can be reduced to others still more general, until we can carry the process no further and we

have to content ourselves at last with certain simple laws of chemistry and physics.

When any event is shown to have taken place in accordance with uniform laws, or when some uniformity is reduced to others more simple and more general it is said to be 'explained'. The simplest and most general laws of all must be accepted without explanation on the strength of an Inductio per Enumerationem Simplicem. The most that we can say for the existence of any of them is that they seem to be involved by a vast number of experiences and to be contradicted by none.*

When we find the causes back of any observed uniformity, the things with which we started become more fully known as well as the relations between them, and the knowledge of both things and relations in becoming fuller becomes also clearer. In the example given 'red clover' took a definite character as a plant dependent for its propagation upon cross-fertilization by an insect able to reach its nectar, and if we had asked why the pollen must be carried, why the bumblebee is better able to get at the nectar than other insects, and so on, we should have gained still clearer and fuller ideas about the clover. As the relations with which we begin are not fully analyzed and explained until they are reduced to ultimate laws, so the things with which we start are not completely understood until each of them is analyzed at last into a definite group of various kinds of atoms.

Analysis and clear thinking.

The clearness and definiteness of thought which causal analysis gives is as valuable in itself as the greater certainty of inference that goes along with it. General appearances ob-

* The sociologist, for example, cannot get along without assuming certain laws of mind; the psychologist tries to account for mental laws by reference to nerve-physiology; the physiologist tries to reduce his data to laws of chemistry and physics; the chemist tries to explain his data by molecular physics; and the physicist tries to state all his facts in the formulæ of mathematics.

scurely apprehended are often sufficient to call forth a fairly definite and appropriate reaction on our part. The lines on a companion's face may be quite indescribable by us and yet suggest the words 'He is angry'. In the same way a number of very indescribable impressions may suggest the word 'Iron' or 'Bewitched' or 'Tyrannizing'. These vague impressions are valuable because the words and other reactions to which they lead are generally fairly appropriate and useful. And yet real things and relations are never vague, and vague impressions can never represent them. They do not precisely misrepresent them, for vague ideas neither represent nor misrepresent, since they cannot be measured against the facts at all. How can we ever prove that Mother Hubbard does not 'hoodoo' her dog or 'project' her thoughts in such a way as to 'impress' the brain of the Czar, until we know precisely what it means to be 'hoodooed' or what a 'projected' thought is supposed to do to the brain that it 'impresses'? Hence if one's expectations are only vague enough there is no such thing as definite fulfilment or definite disappointment. Definite conceptions, on the other hand, can represent realities; and therefore there is some chance of having one that does. If it does not, its very definiteness makes it possible to prove that it does not. If it does, we can count upon it always. Thus a second reason for seeking to reduce observed uniformities to their causes is the clearness of conception which it gives.

CHAPTER XXVI.

THE METHODS OF DIFFERENCE AND AGREEMENT.

In the preceding chapters we have discussed the general principles involved in inductive reasoning. We must now see how the principles are applied to various kinds of concrete problems. We have seen already that these problems may be divided roughly into two classes: the discovery or verifying of general laws, and the ascertaining of concrete individual facts. Questions of concrete fact will not be discussed until after we come to Chapter XXXIII. At present we shall consider only questions of general law.

Different ways of exhausting the universe.

The principles involved in these questions are always the same, yet there are differences in the data to which they are applied which involve corresponding differences in the applications; and if our knowledge of the principles is to have any richness, we ought to know something about these different ways in which they are applied. The most striking applications are to be found in scientific investigations. Many of these are described in Herschel's "Discourse on the Study of Natural Philosophy" (1832) and in Whewell's two large volumes on the "History of the Inductive Sciences" (1837) and his "Philosophy of the Inductive Sciences" (1840), from which subsequent writers have drawn much of their material. With the facts and theories of these writers before him John Stuart Mill set out in his "Logic" (published in 1843) to 'generalize the modes of investigating truth and

estimating evidence, by which so many important and recondite laws of nature have, in the various sciences, been aggregated to the stock of human knowledge', and Mill's chapter on "The Four Methods of Experimental Inquiry" contains the classical account of the various ways in which the principle is applied for the discovery of causal relations.* These different ways of applying the principle are called by Mill the Method of Agreement, the Method of Difference (including the Joint Method of Agreement and Difference, or Indirect Method of Difference), the Method of Residues, and the Method of Concomitant Variations. We shall give an account of each of them, beginning with the Method of Difference.

The Method of Difference.

If a baby strikes or pushes a hanging ball and the ball moves, and if the experience happens to be repeated several times, the baby gets in the way after a while of expecting that each new stroke or push will be followed by a new movement of the ball, and years afterwards it learns to say that the stroke of the hand caused the movement of the ball. If a man were to try the same experiment as the child he would reach the same conclusion, but, unlike the child, he might try to explain why the conclusion was reasonable. If he did, his reasoning would be something like this: 'Before I touched the ball it was motionless. I struck it a great many times. Every time I struck it it moved. When I left it alone it gradually stopped moving. If my blows did not move it, what did? I was

* It is hardly necessary to say that inductive inquiry existed long before the theories of these writers, and even a very clear theoretical conception of the principles on which it is founded. Minto's "Logic" (pp. 243–272) gives a good account of the whole matter, including the views and influence of Francis Bacon and of Roger Bacon (1214–1292), whom Minto calls his greater namesake. Minto, however, does not mention Hume and his remarkably clear statement of the canons for the methods of Agreement, Difference, and Concomitant Variations (See the 'Rules by which to judge of causes and effects' in the "Treatise of Human Nature").

alone in the room; the house was quiet; there were no sudden draughts of air. It may be that a ball might be moved before my eyes by some cause that I could not see and do not know about; but if there was any such cause, why did it always wait to move the ball until I struck it?'

The force of this argument lies in the question 'If I did not, what did?' The man assumes that the event must have been caused, and the harder he looks without finding anything that might have caused it except his own activity the surer he feels that that was the real cause; and if he knew for certain that his blow was the only change introduced into the situation before the ball began to move—the *only point of difference* except the movement itself between the situation in which the ball did not move and that in which it did—then he could be absolutely certain that it was his blow that caused the movement of the ball. It is clear enough that this reasoning is based on our principle of exclusion (one of the forms taken by the wider principle of exhaustion, see p. 225); and the italicized words explain why Mill calls this special application of the principle the Method of Difference.

It makes no difference in the method whether it is used to explain an actual change in one situation or the difference between two. If there are two precisely similar balls suspended in precisely similar manners, except that one is exposed to a steady wind and moving while the other is sheltered and stationary, we can conclude that the wind is the only possible cause of the movement, since it is the only circumstance (except the movement itself) present in one case and not in the other and we cannot suppose that a circumstance present in both cases would move one ball which happens to be here and not move another precisely like it which happens to be there.

The principle of exclusion as used in this method of difference can be stated in the following abstract canon of Mill's: "*If an instance in which the phenomenon under in-*

vestigation occurs, and an instance in which it does not occur, have every [*other*] *circumstance in common save one, that one occurring only in the former; the circumstance in which alone the two instances differ, is the effect, or the cause, or an indispensable part of the cause, of the phenomenon.*" To put the same thing more symbolically: If there is a case in which all the antecedents can be represented by the letters A, B, C, and D and all the consequents by the letters W, X, Y, and Z, and another case in which all the antecedents can be represented by the letters A, B, and C, and all the consequents by W, X, and Y, then the antecedent D is the cause or part of the cause of the consequent Z.

The Method of Agreement.

A girl dressed in a blue gown and carrying some books walks quietly across a room and as she passes over a certain place a squeaky noise is heard. Soon after, a boy dressed wholly differently, talking, and carrying nothing at all, walks over the same place, and as he does so the noise is heard again. Now if we assume that the noise has the same cause in both cases and if we can be sure that walking over the same spot was the only circumstance except the noise that was the same in both, *i.e.*, the only one *in which the two instances agreed*, then we cannot help concluding that walking over that spot caused the noise. The italicized words explain why this inference is drawn by the Method of Agreement.

This method, like the last, depends on the principle of exclusion; for we cannot be sure that the circumstance we have picked out is the only point of agreement until we have examined every other circumstance and found that no one of them is the same in both cases.

In the example just given the effect was produced at the time of the observation, so that the things involved underwent a change. But the method of agreement is also applicable to cases in which we cannot observe the origin of the effect. Suppose, for example, that on the top of every high mountain

which we climb we find the air to be cooler than in the surrounding country, no matter where the mountains are, what their shape, or what they are made of. As soon as we can be sure that the elevation is the only point of agreement between them all (except the lower temperature) we can infer that the greater elevation has something to do with the lower temperature.

Mill's canon for the method of agreement is this: "*If two or more instances of the phenomenon under investigation have only one circumstance in common* [and if that phenomenon is always produced by the same circumstance; then] *the circumstance in which alone all the instances agree is the cause* (*or effect*) *of the given phenomenon.*" To put it more symbolically: If all the antecedents in one case can be represented by the letters ABCD and all the consequents by WXYZ, and in another case all the antecedents can be represented by EFGD and all the consequents by KLMZ, and if Z always has the same cause, then D is the cause of Z.

Does either method really exhaust?

The methods of agreement and difference both depend upon the principle of exclusion, and in both methods this principle has been properly applied only if we have been correct in assuming that no point of agreement or difference (as the case may be) escaped our observation. But this is no small assumption. How do we know that the mountains we visit do not happen by the merest chance to lie over relatively cool places in the centre of the earth, or to be packed with ice, or to lie in places where cool currents of air turn downwards towards the earth? How do we know that when the girl and the boy both passed the same spot a cat in the cellar did not happen to catch a mouse which made the noise as it was caught? How do we know that the ball the child struck was not possessed by a demon which happened to move it at that instant, and that the one swinging in the wind was not similarly possessed? In short, it is only an infinitesimal part of nature that we can pretend to have observed, and how can we possibly prove that the

only point of agreement or the only point of difference we happen to have noticed is the only one there is?

We can not; and for that reason our inductive argument will always be at least theoretically inconclusive. But all reasoning, even deductive, is for the sake of practice; and from the practical standpoint the force of this objection can be considerably weakened. As mere theorists the thought of the almost infinite search that must be made before we can say we have found the only point of agreement or the only point of difference between two situations appals us. As practical beings it does not; for when we begin to reason about the connections in Nature we are not in the position of strangers dropped from another universe. We are in the midst of affairs. We have learned in a practical way that a vast number of agreements and differences between situations can be thrown out immediately as immaterial; and we dismiss them so unhesitatingly that we are inclined to laugh at the theorist instead of answering him when he asks how we know that the ball did not swing because a Chinaman laughed on the other side of the globe, or that the mountains we saw were not cool because people lived in the valleys. Thus the practical man approaches his inductive problem with vastly more data than he states; he does not hesitate to distinguish between kinds of agreement and difference that may be material and those that are certainly immaterial; and thus most of the elimination is already performed.

Not only have we this practical belief that the possible causes are after all not so very numerous, but we have it in our power to diminish the chance that any of them have been overlooked by observing as many and as different cases as possible.

The more mountains we examine and the more differenc there seems to be between them in every respect except height and coolness, the greater is the chance that there is no other point of resemblance common to them all. The more often we notice that light things begin to move when the wind begins to blow, the less chance there is that, in every

case we noticed, some entirely different event (which we did not notice but which might have caused the movement) took place at the very instant the wind rose. Mere coincidences—whether in space or in time—may be expected occasionally, but occasionally only.

Whether we shall use the Method of Difference or the Method of Agreement in any particular case often depends upon what data we have to work with; and therefore we cannot always choose between them. It is worth while nevertheless to discuss some of their relative advantages and disadvantages.

Advantages of each.

The methods of Difference and Agreement are both subject to the defect already pointed out that we may fail to observe or consider some essential circumstance, and may thus mistake some merely accidental concomitant for the true cause. With the method of Difference the circumstance overlooked would be a point of difference between the objects compared; with the method of Agreement it would be a point of agreement.

A much more certain and important difference in the practical application of the two methods grows out of the fact that much the same result can often be produced by any one of several causes. You can kill a man in a great many different ways; you can poison him with any one of a great many different drugs. Rain is not the only thing that can wet a lawn; and sunshine is not the only thing that can dry it. This is called the principle of the *Plurality of Possible Causes* or the *Vicariousness of Causes*. Let us see how it affects each of our methods.

Plurality of possible causes.

If we find the antecedents ABC accompanied by the consequents XYZ, and the antecedents AB accompanied by the consequents XY, and if we know that these are all the essential facts, we can conclude that, under the circumstances AB, C is the cause or a necessary part of the cause of Z. This is the method of difference, and it is not affected by the plurality of possible causes.

But suppose that in one case we have the antecedents ABC and the consequents XYZ, and in another the antecedents EFC and the consequents VWZ, and that these are all the essential circumstances. If Z has the same cause in both cases, we know that it cannot be anything but C; but it need not have the same cause in both cases. How do we know, then, that it is not caused by A in one case and by E in the other? If I have coffee, toast, and eggs for breakfast one day, and water, hot biscuit, and eggs the next, and if I have indigestion on both days and know or assume that in each case it was caused by something which I had for breakfast, it may have been caused by the eggs on both occasions; and in the absence of any further information that is the most natural inference. And yet it is possible that the trouble was due to the coffee on the first day and to the hot biscuit on the second, while the eggs were all the time perfectly harmless.

This plurality of possible causes constitutes what Mill calls "*the characteristic imperfection*" of the Method of Agreement,—an imperfection which makes it distinctly inferior to the Method of Difference.

Where answers suggested by the method of agreement cannot be tested by some form of the method of difference, the uncertainty arising from the plurality of possible causes can be indefinitely diminished by the multiplication of variations. If we cannot try the eggs alone, it may be that we can try them with different accompaniments on a great many different days, until we are driven to the conclusion that if they do not cause the trouble, they are about the only things one can eat that do not. But farther than this we cannot go by the Method of Agreement; and it is not often we can go so far.

This difficulty growing out of the plurality of possible causes I have spoken of as practical. It is due partly to the practical impossibility of ascertaining all the antecedents and all the consequents in any set of cases, partly to the practical

impossibility of distinguishing with absolute accuracy or certainty between relevant and irrelevant circumstances, and partly to our careless way of ignoring the distinction between different conditions so long as they are called by the same name. So long as we know merely that 'we have indigestion after breakfast' in all the cases mentioned, we cannot tell whether it was caused by the same thing every time or by different things; but if we took the trouble to find out each day how soon after the meal the attack arose, precisely how severe it was, how long it lasted, and every other observable detail, we should soon be able to say whether the trouble really was the same in all the cases, as our first, rough statement implied, or in fact quite different. If it was precisely the same in every relevant respect, we could be quite sure that in each case the attack was caused by the same thing—namely, the eggs; but if we found constant variations in the symptoms, and if we could be sure (as we cannot) that variations in the things eaten for breakfast, as distinguished from previous conditions or the manner of eating, were the only possible cause of these variations, and that the eggs eaten were all precisely alike in every essential respect, then we could be quite certain that the eggs were not the cause, or at least not the only cause, of the symptoms.

Theoretically I think we must admit that no two causes in the world could be substituted for each other and leave precisely the same results everywhere; and therefore to a perfect intelligence dealing with perfect data there would be no such thing as a plurality of possible causes. I think we must admit, too, that in many cases where we seem to be confronted by such a plurality of possible causes the difficulty is due to careless and avoidable inaccuracy; but, on the other hand, I think it cannot be doubted that in most cases the difficulty is due to the fact that in this practical world we learn a great many things vaguely before we learn anything accurately; so that when any particular question, such as the cause of indigestion, arises we cannot expect anybody to know everything

else so well that he can distinguish the relevant antecedents and consequents from the irrelevant. In a purely hypothetical case a person might be supposed to know that his indigestion after breakfast had absolutely nothing to do with the supper he ate the evening before, or with the soundness of his sleep during the night, or with what he read in the morning newspaper, and so on; and yet not to know what really did cause it. But in most actual cases the facts are reversed and we have to know what caused the trouble before we can find out what circumstances were irrelevant—we cannot state our problem with refinement until we have solved it. And so 'the characteristic imperfection' of the method of agreement remains. We can only say that the more we know to begin with and the more carefully we distinguish slight differences, the less trouble it will give.

Let us now compare inferences—whether drawn by the method of difference or by that of agreement—which are based upon the comparison of two changeless situations with those which are based upon the observation of changes taking place in one.

Two kinds of data compared.

One advantage in having a changing situation when we are seeking for causal relations is that it sometimes enables us, though not always, to distinguish between causes and effects. If we find one place where the soil is moist and vegetation luxurious, and another where moisture and vegetation are both absent, we cannot say whether the moisture causes the vegetation or the vegetation the moisture; but when we notice things begin to grow after a rain we can be quite sure that in this case at least the growth of the vegetation is not the cause and the moisture the effect. If either of them is the cause of the other, it is the moisture. In comparing two changeless situations we talk of antecedents and consequents just as we do in speaking of changes; but here the words are used rather metaphorically, 'antecedent' meaning one of a group of causes, and 'consequent' one of a group of effects; and we cannot tell which of two circumstances

should be called the 'antecedent' and which the 'consequent' until we have found out in some indirect way which of the two is cause and which effect. Where there is a change we can often see directly which event is antecedent, and therefore cause, and which is consequent, and therefore effect.

Yet this advantage in reasoning from a changing situation is rather dubious, so long at least as we are mere observers. Often we notice a situation changing without being able to tell which element in it changes first. We feel the breeze arising and see the aspen leaves begin to quiver or the trees begin to sway. But do we really observe which came first? Can we prove by direct observation alone that the breeze really came first and was therefore the cause—and not the effect, as children sometimes suppose—of the fan-like swaying of the trees? It is only when there are a number of intermediate links in the chain that the interval in time between causes and effects is really perceptible. Then, again, the observation of the apparent order in time often positively leads us astray. Is the sound of thunder caused by the flash of light because we hear it afterwards? Does the falling of the barometer cause rain because it precedes it? The fact is that observation of the order in time does not help us to tell which of two events is the cause and which is the effect nearly so much as does a knowledge of other causal relations. We do not believe that the wind moves the trees, and not the trees the wind, because we see that the wind comes first, but rather because we do not know anything but wind that is likely to move the trees, while we do know something besides the trees that can stir up the wind. One theory leaves a broken causal chain and the other does not. In other words, a state of affairs in which the wind moves the trees fits into the rest of the world as we know it better than one in which the movement of the trees causes the wind; and so we assume that the former state of affairs and not the latter is the real one. But such reasoning as this is quite as applicable to cases in which there is not any change as to those in which

there is. Thus this advantage which reasoning about change seems to possess over reasoning about situations which do not change is often only apparent.

Post hoc ergo propter hoc.

When we assume that one event must be the effect of another merely because it follows it—'*post hoc ergo propter hoc*' —we commit a very common blunder, which is sometimes called the fallacy of 'False Cause', but is more commonly designated by the Latin phrase which describes the reasoning.

Advantages of experiment.

Whether the cases compared compel us to draw our inferences by the method of agreement or by the method of difference, and whether we are comparing two different situations or the changes in one, are not nearly so important questions as one not yet mentioned. This is the question whether we merely observe and compare situations as we happen to find them or whether we deliberately create them for the purpose of answering specific questions: the question whether our inference is based upon mere 'observation' or upon 'experiment'.

Mere observation often raises problems; but when they are once raised, experiment, where it can be tried properly, gives the more satisfactory answers.

In the first place, the very object of experiment is to produce conditions which are thoroughly understood and from which all disturbing factors are, so far as possible, removed. Hence when we experiment there is much less chance that some real cause has been overlooked than when we merely observe some of the occurrences that take place amidst the great confusion of natural conditions.

A second point in favor of experiment is that it enables us to tell, as observation does not, not merely that a pair of circumstances have some direct or indirect causal relation, but that one is the actual cause of the other, and which one that is.

We have just seen (p. 271) how these questions can often be settled indirectly, through our knowledge of the rest of the world. Experiment often enables us to settle them directly.

If I blow air against the branches of a tree, they will begin to move; and from this I conclude that in this case at least the movement of the air is the cause and the movement of the tree the effect. On the other hand, if I sway a tree, a little breeze will arise; and from this I conclude that in this case the movement of the tree is the cause, and the movement of the air the effect. So I say that either may cause the other. Again, if I heat the air around a thermometer, I will find the mercury rise; but if I raise the mercury in some other way, I will not find any noticeable change in the temperature of the surrounding air; so I conclude that the temperature affects the height of the mercury, but not *vice versa*. Still again, if I forcibly lower the mercury in a barometer, a storm will not follow; and if I make a little rainstorm around it with a watering-pot without lowering its temperature, the mercury will not fall. So I conclude that the fall of the mercury does not cause the rain, or the rain the fall of the mercury, but that when the two are found together they must be joint effects of the same cause; and if I guess what that cause is, I may be able to verify my guess by a new set of experiments designed to test the relative weights of air and watery vapor, and the effect produced upon the mercury in a barometer by the pressure of such different weights.

We can reach all these conclusions by experiment because we take for granted the relative spontaneity of our own voluntary acts. It may be that we ourselves are as much a part of nature as anything else and that every one of our acts is the necessary effect of some preceding cause. But we assume that we know enough about ourselves to be sure that the immediate causes of our acts are very different from the immediate causes of such things as wind, temperature, and rain. If we did not make this assumption, we should have to assume that our own acts in various experiments and the results which seem to follow from them might be merely joint effects of the same causes. Hence we could not be sure that our blowing of the air made

Assumption in experiment.

the tree move, or that our squeezing of the bulb or our heating of the air around it raised the mercury in the thermometer. For unless we assume our own relative independence it might well be that there was some general state of affairs which made the tree sway and at the same time made our brain-cells discharge and our muscles contract as though we were swaying it; and so with the other examples. On the other hand, if our voluntary acts are spontaneous so far as the events which we are investigating are concerned,—if they are never mere effects of preceding events in the series under consideration,—then, though these voluntary acts and the changes around us which follow them may sometimes coincide through mere chance, they can never be joint effects. If there is any causal relation between them at all, the voluntary acts must be the causes, and the changes that follow must be their effects.*

* A third advantage connected with experiment is that inferences based upon it are accompanied with a greater feeling of satisfaction. The feeling of muscular exertion which comes with our own acts is associated very closely in the minds of most people with the idea of cause and effect or is really a part of it. Consequently the actual production of an effect by our own exertion seems to give an immediate feeling of the causal connection that nothing else can give.

CHAPTER XXVII.

THE JOINT METHOD OF AGREEMENT AND DIFFERENCE.

For a perfect application of the Method of Difference, as we have seen, there must be two cases alike in every respect except that in one a certain effect and its cause are both present; and for a perfect application of the Method of Agreement there must be two cases different in every respect except that the causally related circumstances are present (or, it might be, absent) in both. Moreover there is always danger that the method in question has not been applied perfectly, because of the possible presence of other points of difference or resemblance which have not been noticed. But suppose that other points of difference or resemblance are present and are noticed, and that we cannot find any two cases in which they are not; is inference no longer possible?

Its function.

It is not possible if there are only two such cases; but if there are more and they differ widely from each other, it may be. Suppose, for example, that we have all the following combinations, the letters from A to M standing for known antecedents, the letters from N to Z for known consequents, and the blanks for other possible but unknown antecedents and consequents.

1) ABCDEF–NOPQRS–
2) ABGHI–NOTUV–
3) ACGKL–NPTXY–
4) BCDE–OPQR–
5) CFGH–PSTU–
6) EHIJKL–RUVWXY–

We notice that where A is present N is present, and that where A is absent N is absent (which is the same thing as saying that where A is present N is present, and where N is present A is present); and this suggests that A is the cause of N. But how shall we prove it? The Method of Difference is inapplicable, because there are no two cases (not even the first and the fourth) that differ in no respect except the presence of A and N in one case and their absence in the other. In the same way there are no two cases from which we can prove the connection by the Method of Agreement.

Yet there is a way in which the Method of Agreement can be applied, and applied doubly. *Assuming that the cause of N is to be found amongst the observed antecedents and that it is always the same,* we can prove from the first two cases that it is either A or B or the combination AB, for A and B are the only conditions present in both. From the first and third cases we can prove in the same way that the cause is either A or C or AC; and from the second and third that it is either A or G or AG. And if, as we have assumed, the cause is the same in all the cases, it must be A. Thus, though the presence of A and N is not the only respect in which any two cases agree, it is the only known respect in which all the three positive cases agree, and A is therefore the cause of N, if the cause of N is always the same observed circumstance. This is a perfectly legitimate application of the Method of Agreement; but, like all applications of that method, it is subject to the objections that the cause of N may not be the same in all cases, and then even if it is, the real cause may be some unknown circumstance, represented by one of the blanks.

The Method of Agreement can be applied, again, to the negative cases 4, 5, and 6. These all agree in the absence of A and N, and if instead of only three relatively simple cases we had a great number that were quite complex (as we often have in Nature), we might be inclined to say that the absence of A and N was the only respect, or rather the only

respect worth considering, in which all these cases do agree. In this way we might have a double assurance, one from the positive cases and one from the negative, that A was the cause of N. The negative cases have this advantage: inference based upon them is not subject to the objection arising from the plurality of possible causes. If the absence of A is always accompanied by the absence of N whether any other antecedent is absent or not, then A not only may be the cause of N, but must be; and it must also be the only possible cause. On the other hand the negative cases are subject to the tremendous disadvantage that it is practically impossible to prove a negative, to show that a pair of given circumstances are the only ones which can *not* be found in any of the cases. Consequently, when negative cases are taken by themselves they are much more dangerous to work with than positive. Yet so far as they go they tend to confirm the results based upon the positive cases, and they suggest at least that in all the cases observed there was only one cause. Because results based upon negative instances of this sort can be regarded as confirming those based on the positive, the employment of the two is called by Fowler* the Double Method of Agreement.

But these positive and negative instances can be looked at from another standpoint, and regarded as data for an *indirect application of the method of difference.* We can say: 'When N is present and every condition but A can be varied without causing its disappearance, these conditions are necessarily immaterial. In the same way when every condition but the absence of A can be varied without causing its appearance, these conditions also are immaterial. But the distinction between immaterial conditions may be ignored; therefore the only important distinction between the two sets of cases is that in the one A and N are both present, and in the other they are both absent.' And this is all we need for the method of

* "Inductive Logic", Macmillan, 1889.

difference. Hence the double method is called by Mill the Indirect Method of Difference. Because the method of agreement and the method of difference are both involved it is also called by him the Joint Method of Agreement and Difference.

Mill's canon for the method is this:

"*If two or more instances in which the phenomenon occurs have only one circumstance in common, while two or more instances in which it does not occur have nothing in common save the absence of that circumstance, the circumstance in which alone the two sets of instances differ is the effect, or the cause, or an indispensable part of the cause, of the phenomenon.*"

"This method", Mill says, "may be called the Indirect Method of Difference, or the Joint Method of Agreement and Difference; and consists in a double employment of the Method of Agreement, each proof being independent of the other and corroborating it. But it is not equivalent to a proof by the direct Method of Difference. For the requisitions of the Method of Difference are not satisfied unless we can be quite sure either that the instances affirmative of *a* [*i.e.*, N, the consequent in question] agree in no antecedent whatever but A [the antecedent in question], or that the instances negative of *a* agree in nothing but the negation of A. Now, if it were possible, which it never is, to have this assurance, we should not need the joint method; for either of the two sets of instances separately would then be sufficient to prove causation. This indirect method, therefore, can only be regarded as a great extension and improvement of the Method of Agreement, but not as participating in the more cogent nature of the Method of Difference." *

Compared with simple method of difference.

Mill is right in saying that the Joint Method will not give absolute certainty. But then neither will the Method of Difference, applied in our common human ignorance of a vast number of surrounding conditions. A theoretically perfect

* "Logic", Bk. III, Chap. VIII, Sec. 4.

application of the Method of Difference will give a theoretically perfect proof; but none of our applications of the method are theoretically perfect, and a good many of them are not practically perfect either. Mill and other writers on logic tell us that experiments are usually based upon the Method of Difference and that that is one reason why experiment is better than mere observation. It is true that they are based on that method in any single experiment,—we add one new circumstance and see if the effect in question follows. But why are the conclusions based on experiment sometimes erroneous? Why do scientists all over the world try to repeat and thus 'verify' each others' experiments, if any one could be sure that the method was rigorously applied the first time? The fact of the matter is that in all cases where experiment is possible, whether in common life or in science, the final appeal is not usually to the Method of Difference, but to the Joint Method. I destroy a frog's brain, suspend the creature by the nose, and dip its foot into a solution of acid; and a second or two later I see the foot lifted out of the acid just as if the brainless frog knew what it was doing. By the Method of Difference I reason that the contact with the acid was what made the frog lift the foot up. But am I satisfied with this one experiment? If such an occurrence happens to be unfamiliar and I am really interested in the question, will I not try the experiment again under as many different conditions as I can think of, and ask others to do the same? And if everybody gets the same result, will they not then have two groups of cases to compare, the members of each group varying in as many respects as possible except the two in question? In one group there are all the cases in which the foot has not yet touched the acid and has not been drawn up; in the other are all the cases in which the foot has touched the acid and has been drawn up. And these are the kind of data to which we apply the Joint Method.

In conditions known to be theoretically perfect one experiment based on the Method of Difference would be

sufficient to give absolute proof of a causal relation; but because there are always a great many possible sources of error, we can always feel surer of conclusions when the experiments upon which they are based have been performed many times under different conditions than when they have been performed only once. And thus even where the Method of Difference is most applicable we appeal from it to the Joint Method.

CHAPTER XXVIII.

COUNTERACTING AND COMPLEX CAUSES.

We have reasoned thus far on the assumption that an adequate cause is invariably accompanied by its effect. We have virtually said: 'The effect may be present without this particular cause, because the same effect may be due to any one of several causes; but that does not imply that the cause can be present without its effect, and if what we supposed to be a cause of a given effect is ever found to be present without the effect, we were mistaken in supposing it was the cause (though it might have been part of the cause). If A causes N, N may sometimes be present without A, but A can never be present without N.' This is the principle on which we have been reasoning up to the present, and if we were in a world in which nothing else could 'come between' A and N, or affect their relations to each other, reasoning based on this principle would always be correct. As it is, it is not. It may be that A is a perfectly adequate cause of N and yet that it is sometimes present without N. In the presence of a 'Counteracting Cause' a cause perfectly adequate in itself will fail of its effect. The swift current of a river causes things floating in it to drift down the stream; and yet if there is a hurricane blowing in the opposite direction, the things may drift up and not down. The working of the engines makes the ship move; but now she is fast on the rocks and for all their work the engines cannot move her.

Counteracting causes.

Without going into any theoretical discussion, the practical lesson to draw from such cases is this: There is a difference between saying that a cause always produces its natural effects and saying that it always tends to produce them; and this latter is all we have a right to say. Put more concretely, this rule means that if we are searching for the cause of a given effect N and find that A is sometimes present when N is not, we must not conclude from this that A is not the cause of N, until we are sure that there is nothing present which can counteract A's effect.

If we know what can counteract the effect of A, or, what amounts in this case to the same thing, what can prevent the production of N, our task of discovering the relation between A and N will be easy enough. But if we do not know enough about either A or N to say what would prevent the one from causing the other, then our task will be very much more difficult. If A and N occur together often enough to make us suspect that A is really a cause of N, though sometimes counteracted in its working by G, H, or J, we must simply leave the matter doubtful until we can make or find conditions simple enough or varied enough to let us infer something about the real nature of some of these influences.

Sometimes a situation may be so complicated that we have to deal not only with several kinds of causes and counteracting causes, but with still other antecedents that counteract the counteracting causes. But however complicated our problem may be, the principle of exclusion upon which we must depend for its solution remains the same.

The possibility of counteracting causes makes it possible to commit a blunder of precisely the opposite kind from that made possible by the plurality of possible causes. If we forget that practically the same effect can be produced by any one of several causes, we may assert that something is the cause of this effect, when it is not the cause at all, simply because it happens to be the only one thing present in all

the cases we have observed. On the other hand, if we forget that a cause is sometimes counteracted, we may deny that something is the cause, when it really is, because it is sometimes present without the effect. Thus, if we are careless, the presence of a plurality of causes may make us find false causes, and the presence of a counteracting cause may make us overlook true ones.

Methods of investigating causal relations have been discussed thus far as though we assumed that every effect had some one simple cause and every cause some one simple effect. But it often happens that several causes act together to produce a given effect and that there is some reason why we should not regard them as one.

Causes 'compounded' or 'combined'.

There are two ways in which causes can act together to produce a joint effect. Sometimes the effect of each one of them separately is like that of each of the others and like that of the group as a whole. Sometimes the separate effect of each is unlike the effect of each of the others, and the effect of all together is unlike the effect of any one. Here is a case of the first sort. The amount of money that a man has at the end of the year depends upon how much he had to start with, what he made or lost each day in his regular business, what he made or lost in other ways, what he spent for regular household purposes, what he spent for amusement, what he gave away. In his cash account he sets down all the expenditures on one page and all the receipts on another, adds all the items on the same page together, and subtracts them from the total of the other page. In his balance it makes absolutely no difference what the money that he spent was spent for. If he wants a certain balance and finds before the end of the year that he is spending too much for rent and groceries, he may make up for it by cutting down his expenditures for recreation and charity. All the forces dealt with in mechanics are causes of this sort. When one is 'added to' or 'subtracted from' another the

result is precisely the same as it would have been if the body on which they are acting had been acted upon by only one force equal to the sum or the difference. In mechanics we speak of the 'Composition of Forces'; and Mill paraphrases the term and speaks of the 'Composition of Causes' when the effect of them all can be considered in this way as the algebraic sum of the effects of each of them. The separate causes and effects which are thus added together he speaks of as 'Compounded'.

Now for the second kind of joint effects. To raise a crop of onions there must be seeds, air, moisture, warmth, and soil. If any one of these is left out, the result is, not smaller onions or fewer onions, but no onions at all. Moreover if a farmer finds that his onions are getting too much heat from the sun, he cannot even things up by giving them so much less water. In the same way if a cook finds that she has put too much sugar into her cake, she cannot improve matters by leaving out the flour. Cooking is a matter of chemistry, and chemistry is full of examples of this kind of joint effects. Oxygen and hydrogen 'combine' to form water, whose appearance and action are quite different in almost every respect from those of either of them. In the same way the green poisonous gas chlorine 'combines' with the very different yellowish metal sodium to form common salt, which again is very different from either of them. Joint effects which differ in this way from the effects of any of the causes separately are called 'Heteropathic', and since the effect of uniting the causes is like that of making a chemical 'combination' the causes are said to be 'Combined'.

To produce heteropathic or combined effects it is necessary that the causes concerned should all be present at the same time or follow each other in some fixed order. If the onion seed is to grow, it must be warmed and moistened after it is put in the soil, not before; if the cake is to taste right, its various ingredients must be mixed before it is cooked, not afterwards. To produce compound effects this is not

necessary. The onions weigh as much whether they are all thrown into the basket at once or one after the other; the ingredients of the cake cost as much whether they are all purchased at the same time or at different times. The 'parallelogram of forces' in physics is a graphic way of explaining that when a body that can move freely is acted upon by several forces at once it reaches the same point (though it travels along a diagonal) as it would have reached if it had been acted upon by the same forces one after the other.

Because mathematics can be applied freely in calculating the joint effect when causes are 'compounded' but cannot be so applied when they are 'combined', we have made a distinct advance in knowledge when we can say beforehand in which way they will be conjoined. For example, it is a great advantage if we can be sure that the weight of a compound, however formed, is always equal to the compounded weight—*i.e.*, to the sum of the weights—of its ingredients. According to an old story the Royal Society was once tricked into discussing the question why it was that nothing is added to the weight of a vessel of water when a live fish is put into it; and the discussion of one explanation after another went on for a long time before any one suggested that they try the experiment and see whether what they were trying to explain was really the case. The moral naturally attached to the story is that it is wise to find out whether a fact exists before you try to explain it; but here it is used to illustrate something else. If any one were quite sure that the weight of any body is a compound effect made up of the weight of all its separate parts, he would not think it necessary to try the experiment at all. He could be sure beforehand that an addition of any kind to the contents of the vessel would increase its weight, and he would know that whether the fish put into it were alive or dead could make no possible difference. If the Royal Society ever did seriously discuss such a question as this, it must have been when physicists were

not all certain that weight is a compound effect and never under any circumstances or to any extent heteropathic.

The distinction between 'compounded' and 'combined' causes can be applied quite as well when some of the causes tend to counteract the effects of the others as when they all assist each other.

A counteracting cause of the 'combined' sort simply breaks up or nullifies the combination that would otherwise have produced the effect in question. If somebody puts out the chemist's fire and does it soon enough, the combination which he expected will not occur. If a man quarrels with his employer and loses his place, or if the employer fails and cannot pay the man for his work, the quarrel or the failure is not a kind of expenditure that tends according to its amount to counteract the effect of the man's earnings. It puts an end to his earnings altogether. The effect of these counteracting causes is 'heteropathic'.

A counteracting cause of the 'compounded' sort simply adds a negative result to the positive one produced by the causes that it is said more or less to counteract; and consequently in calculating the net result we have merely to subtract one set of results from the other. Thus the spending or giving away of money tends according to its amount to counteract the effect of earning it, and if we wish to find the gain or loss during the year we have only to subtract the expenditures from the receipts or *vice versa*. Here the effects are not 'heteropathic', but 'compounded'.

CHAPTER XXIX.

THE METHODS OF RESIDUES AND CONCOMITANT VARIATIONS.

When we explain any state of affairs, such as a man's financial balance at the end of the year, as the 'compounded' effect of a number of different causes, and when we give figures to show the amount of the total effect contributed by each, it is evident that we are dealing with the question of causation from the quantitative standpoint. It is no longer a mere question of whether a certain kind of cause was present or not, but it is a question also of its precise amount. If a man's actual balance and the balance shown by his cash account do not agree, there is something which has not been accounted for, and the causal explanation of his financial standing is not complete. This quantitative treatment of a question of cause and effect evidently rests on the assumption that the cause of every event must be capable of producing precisely that amount of effect, no more and no less.

Quantitative treatment of causes.

Because we have a right to demand an explanation for the precise amount of every effect as well as for its quality two other methods of causal inquiry can be added to the three already considered: the Method of Residues and the Method of Concomitant Variations. They both depend upon a quantitative application of the Method of Difference.

For the Method of Residues Mill lays down the following canon: "*Subtract from any phenomenon such part as is known*

by previous inductions to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents.'' If we know that a man has an annual income of $3200, and if we know that his salary is $2000 and that the annual dividend on his railroad stock is $500, then we can infer that he has some other source or sources of income that produce $700 a year. And if we happen to know that his only other source of income is his share in an iron company, we can infer still further that the iron company pays him a dividend of $700 a year, no more and no less. Similarly, if a man in a position of trust, with a salary of $1000 a year and no private fortune, is spending $5000 a year, his employer will have reason to suspect that the man is stealing or has stolen from him enough to make the difference. To take still another illustration: Suppose we know that the planet Uranus is acted upon by the attraction of the sun and of the planets nearest to it, Jupiter and Saturn, in such a way as to bring it to a certain place at a certain time; and suppose that when the time comes the planet is not there: then we can be sure that if our previous calculation was correct, there is some other force acting on the planet and that this force is just strong enough to drag it from the place where the calculation showed that it ought to be to the place where it actually is. By calculating the strength and direction of such an additional force acting on Uranus the planet Neptune was actually looked for and discovered.

Method of Residues.

The difference between the Method of Residues and the ordinary Method of Difference does not lie merely in the fact that the Method of Residues considers questions of quantity; for the next variation of the Method of Difference that we are about to consider does that also. It lies rather in the fact that when we use the Method of Difference our knowledge of what happens when the residual cause is not present is gained from direct observation,—we *see* what happens when all the causes except the one under investiga-

tion are present together. With the Method of Residues we do not directly observe what happens when all the causes except the one in question are present together. We only calculate it from what we know of the way in which they act when they are present separately.

The Method of Residues is attended in practice by three dangers.

The first danger is that in making our subtraction we may overlook the 'combined' or heteropathic effect of some of the causes which we subtract, and thus attribute too much to the remaining causes. For example, three persons, A, B, and C, are in a room from which we hear the sounds of a violent disturbance. We know by previous inductions that A's disposition is quiet and peaceable; we know the same about B; and so we conclude that C is responsible for the disturbance. And yet we may be wrong, for however quiet and peaceable A and B may be in themselves, there may be something about them—some trait of disposition or some old misunderstanding—that makes a conflict almost inevitable when the two are together.

The second danger to which we are exposed in using this Method of Residues is that of overlooking some circumstance that is really present, and of thus attributing an entirely false value to the presence of something else. It may be that the man whose income we were inquiring about a little while ago has an allowance from his grandfather, of which he says nothing, and that the iron company is a source not of income but of expense; and it may be that the man suspected of peculation is getting an immense royalty, that his employer knows nothing about, from a patent.

The third danger is that even when the data are all correct there may be a blunder somewhere in our calculations. If the bookkeeper has made a serious mistake in his addition or subtraction, all the reasoning by which we prove that a certain transaction 'must' have been responsible for a gain or loss of such an amount is worse than useless.

Of course the way to make sure that we have not been misled by any of these blunders is to try the residual cause by itself and see if it really does produce the precise effect indicated by the calculations. Often it does not, and where this is the case and the calculations are all correct it indicates that there is still another residual cause or group of causes to be looked for. Often, of course, direct experiment is impossible, and then we have to get along as best we can with the abstract calculations.

In spite of its difficulties "the Method of Residues is one of the most important among our instruments of discovery. Of all the methods of investigating laws of nature, this is the most fertile in unexpected results: often informing us of sequences in which neither the cause nor the effect were sufficiently conspicuous to attract of themselves the attention of observers. The agent C [*i.e.*, the residual cause] may be an obscure circumstance, not likely to have been perceived unless sought for, nor likely to have been sought for until attention had been awakened by the insufficiency of the obvious causes to account for the whole of the effect. And *c* [the residual effect] may be so disguised by its intermixture with *a* and *b* [the effects whose causes are already known] that it would scarcely have presented itself spontaneously as a subject of separate study." *

The Method of Residues is, like all the rest, a method of exhaustion; for we cannot be sure that a given residual effect is due, or at least partly due, to a certain residual antecedent until we are sure that that residual antecedent is the only one present that could have any influence on the effect.

Method of Concomitant Variations.

Often it is impossible to use any of the methods already discussed, at least without modification, simply because it is impossible to find or to make cases in which all of several possible causes are not present. Suppose that we want to know what makes a wheel stop turning after a while, or a pendulum stop swing-

* Mill: Bk. III, Chap. VIII, Sec. 5.

ing, or a sleigh stop sliding along a smooth and level road. It may be the nature of all material things to stop moving and come to rest, or it may be the presence of the earth that makes them do so, or it may be friction or some resisting influence exerted by the air. How are we to tell? We cannot experiment for the sake of comparison with things that are not material; we cannot get away from the earth; we cannot create conditions in which there is absolutely no friction and absolutely no air or other surrounding medium. We cannot eliminate any of these possible causes. How then can we choose between them?

Though we cannot wholly eliminate any of them, we can introduce changes in some of them, and if we find that the variation of any condition is accompanied by a variation in the result in question, then we can be sure that that condition has some causal connection with the result. We know that the presence of the air has something to do with the movements of bodies, because we know that things stop moving sooner when the wind is against them than when there is no wind at all, and sooner when there is no wind at all than when it is moving with them. If the air exerted no influence, the direction in which it moved could make no difference either. This proves that air exerts an influence, at least when it moves. That stationary air also exerts an influence and that this influence tends to make a thing stop moving can also be proved; for a pendulum will swing or a wheel will turn longer in a box from which the air has been partially exhausted than in one from which it has not, and the more nearly the air is exhausted the longer the motion will continue. So much for the influence of the air; but how about friction? We cannot make any contrivance in which there is no friction at all; but every one knows what happens when we diminish it. The more slippery we make a surface the further things will slide upon it, and the more we diminish friction in wheels and pendulums by lubricants or special bearings the longer they will keep on moving.

If friction did not affect the continuance of a movement, a vast number of influences that agree in nothing else but the diminution of friction would not all agree in prolonging the movement. Thus we can use the Method of Concomitant Variations to prove that the resistance of a surrounding medium and friction help at least to make things stop moving.

To take another illustration of the same method. Suppose that things weighed on an extremely delicate spring balance seem lighter and lighter, or that pendulums swing more and more slowly, the farther we take them on a mountain or in a balloon, away from the centre of the earth, no matter which side of the earth we may be on or where the earth may be in its orbit. If the experiments are tried with proper precautions, the distance from the centre of the earth is the only condition that is varied alike in all of them, and the continual variation in this one respect is invariably accompanied by a corresponding variation in the downward pull of everything we try, whether we measure that pull by its effect upon the spring in the balances or by its effect upon the rate at which the pendulum swings. From this we have a right to conclude that the nearness of the earth, and therefore the earth itself, has at least something to do with the tension exerted by a weight upon a spring and with the swinging of a pendulum—and perhaps with the general tendency of things to fall.

Mill's canon for the method is as follows:

"*Whatever phenomenon varies in any manner whenever some other phenomenon varies in some particular manner is either a cause or an effect of that phenomenon, or is connected with it through some fact of causation.*"

The last clause in the canon is intended to cover cases where two phenomena have corresponding variations, not because one causes the other, but because they both depend, at least to some extent, upon some third variable which perhaps has not been observed at all. If we could be sure

that there were only two variables, we could be sure that one of them was the cause, or at least a part of the cause, of the other. Because we cannot be sure that one of the observed variables is the cause of the other until we are sure that there is no third variable to play this part the Method of Concomitant Variations is at bottom, like all the other inductive methods, one of exhaustion.

So much we can infer according to the Method of Concomitant Variations if we take account of only the mere fact of change. If we know and take account of its amount also, we can often go further and find out with more or less certainty how much the one variable has to do with the other: whether it is the complete cause or only a part of the cause, and what is the mathematical relation between the cause and the effect. For example, if we know that the income of a certain agent always increases as his sales increase and diminishes as they diminish, we may infer from this that at least a part of his income is derived from a commission on sales; but that is all. On the other hand, if our information is more definite, and we know that when the sales are doubled or quadrupled the income is also doubled or quadrupled, we shall probably be correct in inferring that the whole income is derived from a commission on sales, and of course we can tell what the commission is. If we know that when the monthly sales amount to $10,000 the income is $280, when they amount to $20,000 it is $480, and when they amount to $40,000 it is $880, we shall be able to guess that the agent has a salary of $80 a month and a commission of two per cent on his sales.

Cautions.

In reasoning of this sort, however, there are two things about which we must be very careful. In the first place, where there are only a few data they are often consistent with any one of several laws of relation, and we cannot be absolutely certain that the one we happen to hit upon is correct. To take the case just given: the figures would be explained just as well if we supposed

that instead of receiving any salary the agent paid his principals $120 a month for his office or his 'outfit' or the use of their name, and received in return a commission of four per cent on all sales up to $10,000 a month, and two per cent on the balance; and it might be that if we thought about the matter long enough we could find still a third arrangement that would explain our data just as well as either of these.

The other thing to be careful about is not to assume that the law found to hold within certain limits will necessarily hold when we get very far beyond them. If we had enough more data which made it practically certain in the case we have been considering that the agent had a salary of $80 a month and a commission of two per cent, this would not prove that he would still continue to get a salary if he made no sales at all, nor yet that he would continue to get two per cent on all his sales if their number were very largely increased. It might very well have been agreed that on all sales over, let us say, $50,000 a month he should get a commission of only one per cent. In the same way, we may have noticed that the more wind there is the more work it is possible to get out of a windmill; but that is no reason for believing that the best results of all would be had in a hurricane, where the mill might break down altogether. We can never tell how far we are away from a point at which some new relation enters in to upset all our calculations.

In spite of the necessity for these two precautions, the fact that there are many cases in which the amount of variation can be measured makes this method particularly useful. "Although the most striking applications of the Method of Concomitant Variations takes place in the cases in which the Method of Difference, strictly so called, is impossible, its use is not confined to those cases; it may often usefully follow after the Method of Difference, to give additional precision to a solution which that has found. When by the Method of Difference it has first been ascertained that a certain

object produces a certain effect, the Method of Concomitant Variations may be usefully called in, to determine according to what law the quantity or the different relations of the effect follow those of the cause.'' *

When the Method of Concomitant Variations is used in this way to give precision to causal laws, it is obvious that it rests no longer upon the bare assumption that every event has a cause, but upon the more refined assumption that the amount of the cause is related to the amount of the effect by some law of definite proportion.

* Mill: Bk. III, Chap. VIII, Sec. 6.

CHAPTER XXX.

GROUP COMPARISONS, OR THE METHOD OF STATISTICS.

In the examples already given, the Method of Concomitant Variations was applied in the following way. We noticed the state or action of some individual thing, then introduced a change into the surroundings and noticed what change followed in the state or action of the same individual. We compared the time which it takes for a pendulum to make a complete oscillation (or a hundred continuous oscillations) when it is swung at the level of the sea and when it is swung at various distances above it, or we compared the distance that a spring is stretched or twisted by some heavy object attached to it at the level of the sea and at various distances above it. In all the experiments necessary for such comparisons we swung the same pendulum or weighed against our spring the same lump of lead. This, however, was not absolutely necessary. If we were quite certain that two pendulums or two lumps of lead were exactly alike in every essential respect, we could use one at the level of the sea and the other at the mountain-top, and still draw our conclusion from the difference in the results. But whether the individuals compared were identical or not, in either case we compared the states or actions of single individuals. Moreover it will be remembered that to draw any conclusion from the results of such a comparison we had to be sure that the change in elevation was the only change made that might affect the downward pull in

Principle and uses.

question. If that downward pull had been subject to variation from a hundred other causes known and unknown, whose influence we could not possibly estimate, then the experiment could have told us nothing about the effect upon that pull of the changed distance from the centre of the earth.

In cases of this latter sort inferences that cannot be based upon what happens in the case of a single individual can be based upon the total or average change produced in a very large number. We know, for example, that the growth of a child as measured by its weight depends not only upon its age, but also upon a vast number of other influences: its own natural vitality, the size of its parents, the nature and amount of its food, the amount of sunlight and fresh air to which it is exposed, its freedom from disease, its exercise, its happiness, and so on; and we cannot possibly tell how much influence any one of these has exerted upon the growth of an individual child. Consequently we can tell very little about the growth due to age by weighing a child when it is six and again when it is seven; much less by comparing the weight of one child of six with that of another child of seven. Yet if we compare the average weight of several thousand children of six with that of several thousand children of seven selected in the same way from the same neighborhood, then we shall have a right to infer that the difference in the average weights is due, to a fraction of one per cent, to the difference in age. If one child of seven has attained its growth under better conditions than some child of six, there is undoubtedly some child of six that has attained its growth under better conditions than some child of seven; so that on the whole the favorable and the unfavorable influences in the two groups are balanced evenly enough to be disregarded. Thus by comparing large enough groups we can often eliminate the effects of all the causes but those under consideration and get a very accurate measure of the effect exerted by the latter.

This Method of Group Comparison is used very commonly at the present time, especially in physiology, psychology, and economics, to estimate the effect of various influences that cannot possibly be isolated from a great many others, and whose results cannot be estimated with accuracy in any other way. To take a practical example.

Is there any connection between a child's size, as distinguished from its age, and its mental development? We should be able to answer this question if we can find out whether large children are better developed mentally than smaller children of the same age, or, what amounts to the same thing, whether children of better mental development are larger on the whole than children of the same age of poorer mental development. To find this out an investigator takes through the teachers of a large city the ages and weights of some thirty thousand pupils in the public schools. He then finds the average weight of all the pupils of the same sex and age throughout the schools, as well as the average weight of all the pupils of the same sex and age in each 'grade'. Comparing these averages together he finds that in practically every case children in higher grades weigh more than children of the same age and sex in lower grades. Accepting the progress which a child makes in school as a fair enough test of its mental development, and the grade in which it is found as a fair enough test of its progress, he concludes from this that children with greater mental development are on the whole larger than children of the same age with less mental development.* From such a conclusion, rightly established, we should have a right to infer that a child's mental development depends not merely upon its age, but upon the development of its body, or, conversely, that the development of the body depends not merely upon age, but upon the development of the mind, or else that the development of the mind and the develop-

* See Transactions of the Academy of Science of St. Louis, Vol. VI, No. 7.

ment of the body both depend largely upon the same conditions; in short, that there is a close causal relation between them.

Here is an example of the measurements:

Average weight of			2188	boys	nine	years	old	57.41 lbs.
"	"	" the	570	of these	in	Grade	I	55.52 "
"	"	" "	1195	"	"	"	II	57.56 "
"	"	" "	357	"	"	"	III	59.26 "
"	"	" "	44	"	"	"	IV	61.91 "

Twenty-two boys out of the total of 2188 who were measured are not accounted for in this table. These twenty-two boys must have been in the Kindergarten and the grades higher than the fourth. Their measurements are not averaged because the investigator thought, very properly, that an average could not be depended upon unless it were based upon at least twenty individual measurements.

The table shows a difference in weight of about two pounds between boys of nine in one grade and those in the grade above it; so that between the boys of nine in the first grade and those in the fourth there is the very decided difference in the average of over six pounds. The very considerable size of these differences, the almost unbroken regularity with which they appear from one grade to the next, not only with boys of nine, but with both boys and girls of all ages between six and sixteen or seventeen, and the large number of total measurements, to say nothing of certain other relations brought out in the original article,—all these prove that the different average weights in different grades cannot be the result of mere chance—that there must be some real cause at work which tends to make the boys or girls of a given age in a higher class heavier than those in a lower class. In this way, by comparing the average measurements of several large groups we can often prove the existence of causal relations which we could never prove by merely comparing a few individuals.

The errors to which we are liable in such investigations as this are very serious. In the first place we must make sure that enough measurements are made to eliminate the effects of purely individual idiosyncrasies. One boy with twenty pounds of extra fat would make a difference of two pounds in the average when he is one of only ten; but he makes a difference of only one fiftieth of a pound when he is one of a thousand. In general, the larger the groups from which we get our averages, the less chance there is that the distribution of peculiar individuals will be uneven enough to make a difference worth considering in the results. In the table it will be noticed that the difference between nine-year-old boys in the third grade and those in the fourth is slightly greater than that between any other two successive grades. This extra difference might well disappear if instead of forty-four boys in the fourth grade to examine there had been a thousand. The accuracy with which we should read our averages always depends upon the number of measurements from which the average is computed.

Number of data.

A second blunder to be avoided in statistical investigations where the data are supplied by different observers arises from what is called the 'Personal Equation' of the observers. Where there is any doubt about a quantity some people constantly tend to overestimate it, others to underestimate it. If one object or set of objects happens to be measured by a person who habitually overestimates, and another by one who habitually underestimates, it is evident that the difference between the two will appear a little larger or a little smaller than it really is. Personal equation of this general sort appears in many different forms. If each of two people has to press a button the moment he sees a certain sight or hears a certain sound, the chances are that they will not both press it at exactly the same time. One will nearly always act a fraction of a second later than the other. The more prac-

Personal equation, etc.

tice each of the two has had in this sort of reaction the more likely he is, not to be more accurate, but to be more constant in his error; and therefore the more likely it is that the interval between the reactions of the two observers or experimenters will always be about the same, so that proper allowance can be made for it. In this way two astronomers may always differ by perhaps a tenth of a second of time in the records which they make of the movements of the heavenly bodies; and before they come to put their results together they always find out what the personal equation of each observer is and make due allowance for it. Another example of this general kind of personal equation is found in our different estimates of distance. In trying to find the centre of a horizontal line, some people habitually go too far to the right, others too far to the left; and when we are asked to estimate the relative lengths of a horizontal line and a vertical line which is actually precisely equal to it, almost everybody will say that the vertical line seems a little longer, but the error will be greater with some people than with others.

There is a similar difference between people in the use of words whose meaning is rather vague. If we are comparing the amount of *drunkenness* in different cities, we must be sure that the persons who collected the statistics all defined the term in the same way. It may be that in one city a man was called drunk when he had imbibed enough alcohol to make him noisy, in a second when he staggered, and in a third not until he fell down. The terms 'sick', 'typhoid fever', 'pauper', 'criminal', 'morbid', and a host of others are subject to variations of this sort in their definition.

In comparing the official statistics of different places or periods we must be careful to see not only that the terms are defined in the same way in them all, but also to see that about the same proportion of actual cases are reported. In some places a much larger per cent of the actual births, contagious diseases, or crimes is reported than in others.

Much the easiest way for a police department to diminish disorderliness—on paper—is to instruct the patrolmen to report as few cases as possible.

Another source of error, very much like personal equation, is due to preconceived opinions on the part of the observers as to what the result of the investigation should be. This danger is especially likely to affect statistical inquiries that depend upon data furnished by a set of observers untrained in that kind of work. But it is hard enough for any one to avoid, even in taking what we call exact measurements. No measurements are absolutely accurate; and if it is a question of whether something measures 26.2 units or 26.3 our decision is likely to be influenced more than we realize by the desire to give our theory the benefit of the doubt. Hence it is advisable to take each measurement in such a way that it is impossible to tell which side of the question it will help until it is not only taken but recorded; and after we do know its bearing it should not be subject to revision. When the measurements are taken by assistants untrained in such work it is wise to begin with a small preliminary investigation in order to make sure that the instructions are absolutely full and clear, and in many cases it is also desirable to exclude any possible influence of prejudice by leaving the assistants in complete ignorance as to the question at issue, though this should not be done if they will imagine that the issue is something else in which they are more interested.

Preconceptions and interests.

This effect of prejudice upon our measurements is not confined of course to group comparisons. It is found everywhere. Every teacher knows how much easier it is to grade an examination paper fairly when he does not know who wrote it than when he does. A phrase used by a good student may suggest all that he probably knows and thus get a high mark, while the very same phrase from a poor one suggests all his stupidities and gets a low mark. Hence the teacher who really means to be guided in making his grades

by the face of the returns will try to read and mark the papers before he knows whose they are, and even to mark the answer to each separate question before he knows precisely what effect that particular mark will have upon the unknown student's standing as a whole, *e.g.*, whether it is or is not barely enough to pass him or to give him honors. The less strongly we feel the issues at stake the more accurately we are likely to judge.

The effect of preconceptions and personal interests appears again in the selection of cases. If a teacher wishes to show how bad his class is, he will put more doubtful cases in his list of those who are disorderly or deficient than if he wishes to show how good it is. So with purely theoretical questions. To go back to our old example. Suppose the question to be settled by weighing the children in the various grades had been hotly discussed by the teachers who afterwards took the weights. How natural it would be for some of them, unless they had most explicit instructions to the contrary, to omit the weight of this or that individual pupil on the ground that to include it in the set from which an average is made would be manifestly unfair! 'Johnny is very bright and very small; but then he is a cripple, and it is certainly not fair to include him'; 'Mary is a great big girl in the second grade, but they really should have let her into the third, so I will not include her'. If we begin in this way to include only the cases that seem to us reasonable, it is perfectly evident that our views of what is reasonable in any particular case will depend very largely upon what we think beforehand about the question at issue; and as the result of the whole investigation will be influenced by our decision in these particular cases, there is always a strong tendency for the investigation to merely confirm the opinion we had to begin with. For this reason it is always desirable to lay down rules of procedure beforehand that leave no room whatever for selective judgment during the course of the investigation. Undoubtedly it often is unfair in making

up an average to include exceptional cases. But the only safe way to overcome the unfairness is to base the average on so many cases that it will make practically no difference in that average whether a few exceptions are included or not. If any such large number of cases is not available, then the statistical method is not reliable. We appeal to statistics because we wish impartial witnesses instead of mere individual opinions, and it is obviously absurd to run the slightest chance of selecting some of the witnesses because they will testify in our favor and excluding others because they will testify against us.

Accidental selection.

However successful we may be in avoiding the influence of our own personal equation or that of others in the selection and measurement of cases, we may still choose our cases according to some plan that does involve an unfair selection though we do not realize it. The method of group comparisons is based on the assumption that the force under investigation is the only one which does not act about as much upon the members of one of the groups compared as upon the members of the other. But if this assumption is not correct, if some other force is present which really does act more upon the members of one group than upon the members of the other, and if we overlook its presence, it is clear enough that we shall attribute too much or too little to the force we are investigating; we supposed we had excluded all other constant causes, and we had not.

Suppose, for example, whether it be true or not, that the further a child gets in school the better able he is either to help his parents at home or to earn money for them outside. It is evident that the poorest and most shiftless parents will be tempted to take their children out of school as soon as their labor is worth a very few cents a day; and unless the law is enforced very rigorously they will find means of doing so. Others less poor or less improvident will allow their children to go a little farther. This process would continue

from grade to grade, until in the high school nearly all the children represented homes that are fairly comfortable and well-ordered. It is evident that causes of this sort would produce a process of partial selection on some basis other than that of the children's individual mental development. Many of the children in the lowest grades would be paupers; those in the highest would not. In such a case it might well be that the difference in weight between children of the same age in lower and in higher grades is due, at least in part, to the fact that the latter come on the whole from more prosperous families, and are therefore better fed and cared for.

If it should turn out upon investigation that there is nothing in this suggestion, we might offer another somewhat different. Suppose that amongst the most ignorant classes in a city there is for some reason or other a rather hostile feeling towards the public schools, so that the most ignorant parents keep their children out of them as long as they possibly can; while parents of the better classes, on the other hand, send them just as soon as they can. This would of itself tend to put children from the better homes in higher grades than those of the same age from the worse homes. Moreover, in the better homes children are helped with their lessons and encouraged to keep up with their class, or ahead of it, while in the worse they are not. In this way children who are better cared for at home, and therefore probably larger, would tend, apart altogether from any mental superiority of their own, to be farther along in school than those who are less well cared for at home. But there is still another, more direct, consideration in the case we have supposed. The smaller the child the longer can a parent prejudiced against the schools pass him off as too young to be sent; and the larger the child the easier it is for the intelligent parent who wants him to go to school to persuade the authorities that he is really mature enough to begin, even if he is not quite up to the legal age.

If it is customary in the city in question to promote nearly

all the children at the end of the year whether they have accomplished very much during the year or not, it is evident that a child's grade in the schools of such a city would depend far less upon his mental development than upon the age at which his school career happened to begin. Thus there might be a double or triple reason why large children should be farther along in school than small ones, apart altogether from any difference in their own mental development.

When such objections as these are offered to the conclusions which any one draws from a set of statistics, the way to answer them is to find out by supplementary inquiries whether the causes suggested really are at work in the case in question; and if they are, to estimate the amount of effect which they are likely to produce, and thus see how much of the total effect is left for the causes originally assigned. Until conclusions based upon the method of group comparison have been subjected to much critical examination of this sort, we must not attach to them anything like absolute confidence.

Misplaced accuracy.

Another danger which confronts this method of group comparisons—and indeed all methods that depend upon precise measurements—is that we shall infer the presence of some cause from numerical computations that are far too precise for the data from which they are derived. By this I mean too precise for the least accurate of the data. When mathematicians take two sets of measurements which are to enter into the same problem, and when they can only get a certain proportion of accuracy in one, they realize that the inaccuracy of these data will affect the problem as a whole in the same proportion, and so they make no effort to get a greater degree of accuracy in any of the other data. For example, suppose we know that one side, B, of a triangle is twelve times as long as the base, A, and that we measure A for the sake of finding the length of B. If A is really 101 inches long but we make it 100 inches, that will mean that B is really 101 feet long, though we cal-

culate that it is just 100 feet. In this way, an error of one inch in the length of A corresponds to an error of twelve inches in the length of B. If we measured both A and B for the sake of comparing them and did not try to measure A more accurately than in even inches, it would be a waste of time to measure B more accurately than in even feet. Moreover, it would be a positive blunder to say that A measured 'exactly' 100 inches; that B measured 'exactly' 100 feet and 1 inch; and that B was therefore a little more than twelve times as long as A. If we measure B to an inch, we must measure A to a twelfth of an inch before we institute any such precise comparisons between them, or draw any conclusion from the existence of such slight discrepancies. When we multiply a measurement we multiply the error that we made in taking it; when we divide a measurement we divide the error. So in general we can say that any figure which has to be multiplied before it is added to, or subtracted from, or compared in any way with, another should be reached by more careful measurements than that other; while a figure which has to be divided before it is compared with another may be reached by less careful measurements than that other.

To take another example of this law of proportion in the accuracy with which we should take various measurements. A certain horse trots a mile in about two minutes and five seconds. The stop-watches by which he is timed will register fifths of a second, but nothing less. If we wish to find his speed as accurately as possible, how accurately should we measure the course over which he trots? A horse trotting at the rate given goes more than eight feet in a fifth of a second, and since the watches will not register any time less than a fifth of a second, they are absolutely incapable of measuring the time that it takes the horse to go eight feet or less. It would therefore be a waste of time to measure the course for such a horse to a fraction of an inch.*

* Absurd, I mean, if we are measuring the track merely for the sake

Indeed, the very precision of such measurement might be misleading. Suppose, for example, that horse A trots over the mile track in New York in two minutes and five and one-fifth seconds, and that horse B trots over the mile track in Toronto in the same time. Suppose also that accurate measurements show the Toronto track to be really six feet longer than the New York track. How natural it would be to say that since B went six feet farther than A in the same time, he must have gone faster! But this conclusion is absolutely unwarrantable; for when we say that the two distances were covered in 'the same time' we mean that in each case the time was at least as much as $2.5\frac{1}{5}$ and less than $2.5\frac{2}{5}$. In other words, we mean that the difference between the two was less than a fifth of a second. But with a possible difference in the time of almost a fifth of a second it may be that A really trotted faster than B after all.*

In our example of group comparisons I think we find a blunder of this same sort. The other objections which we made to the conclusion based upon the weights of school children in different grades were largely hypothetical. This objection is real. The investigator's object is to find out what difference there is in the weights of pupils of the same age who are in different grades. In the tables which he gives for comparison the average weights are all calculated to the hundredth part of a pound. How accurate should he have been in finding the average ages? If we take account of a difference in weight of one pound, should we not take account of a difference in age that is sufficient to produce

of timing that particular horse with that particular kind of watch. The accuracy is justified by the fact that some time we may have a better watch or wish to time a slower animal.

* It is assumed for the sake of simplicity in the argument that the stop-watch really will measure with accuracy to the fifth of a second. When we remember that the starting and stopping of the watch depend upon human action in the midst of exciting surroundings, it is evident enough that there is still less accuracy in the measurement of the horse's time.

that difference of one pound? If we take account of a difference in weight of one hundredth of a pound, should we not take account of a difference in age sufficient to produce that difference of one hundredth of a pound? How much is this?

According to the tables the average weight of all the boys examined who are eight at their nearest birthday is 52.39 pounds; the average weight of all the boys who are nine at their nearest birthday is 57.41 pounds; and the average weight of all the boys who are ten at their nearest birthday is 62.38 pounds. This means that the boys gain about five pounds a year, or about a tenth of a pound a week, and the hundredth part of a pound in less than a day.*

This law of average growth means that we cannot draw any conclusion from an average difference in weight of one tenth of a pound between two groups of children 'of the same age', unless we have good reason to believe that the age really is 'the same' not merely to a year but to a week. A difference of a week in age would account for a difference of a tenth of a pound in weight. In the same way a difference of ten weeks in age would account for a difference of a pound in weight, and a difference of twenty weeks for the difference of two pounds which the tables show between the boys of nine in any two successive grades.

And now the question comes: Have we a right to believe that there is no such difference as this in the ages?

In the tables before us the children are grouped according to their age in years at their nearest birthday. No account is taken of months or days. In each group, then, there will be some children who are almost a year older than some others in the same group. But since it is fair to assume that there are about as many children a little under a given age

* We assume here for the sake of simplicity that the growth is uniform throughout the year. If we took account of the fact that it is not it would complicate the argument, but it would not affect the principle on which it is based.

as a little over it, the average age of all the children called nine would really be almost exactly nine; and so with each of the other ages. In this way we have a right to assume that the difference in average weight between all the boys called nine and all the boys called ten corresponds to a very definite difference in age of almost precisely one year. Thus this inference based upon the tables is perfectly correct, and we have a right to say that it really is a difference of one year in age which makes the difference of about five pounds in weight.

This, however, is very different from saying that the boys called ten in any one grade are on the average a year older than the boys called nine in the same grade, or that the boys called nine in one grade are on the average precisely as old as the boys called nine in another.

In fact the presumption is all the other way. A boy exactly eight years and six months of age is quite as likely to be in a grade with the boys of eight at their nearest birthday as with the boys of nine; and a boy of eight years and seven months is almost as likely to be. On the other hand a boy of nine years and six months is quite as likely to be with the boys of ten as with the boys of nine, and a boy of nine years and five months is almost as likely to be. In this way one boy of nine might easily be two grades ahead of another, not because he is any better developed for his age, but merely because he is ten or eleven months older. And thus, in general, there is every reason to believe that the boys of a given age in a higher grade are considerably older on the average than those 'of the same age' in a lower grade; and the difference in age might well account for a large part of the difference in weight. It could not account for a difference of five pounds, of course; for the difference in age must always be less than a year. Hence it could not account for all the difference which is found between boys of nine in the first grade and those in the fourth; but it might account for enough of it to make the conclusion that bright children are

larger than dull ones extremely doubtful. Precisely how much the actual difference in age really will account for we cannot tell until the age of each child is taken accurately enough to show precisely what this difference is.

There is a difference between this example of conclusions too precise for the data upon which they are based and the one given before it. When we compared two individuals and concluded that the horse B was faster than the horse A because it trotted a few feet farther 'in the same time', our conclusion may have been wrong, but it may also have been right, since the times really may have been the same. But in this other case when we compared several groups of individuals and said that the members of one were so much heavier for their age than the members of another, our conclusion was certainly wrong, since there is every reason for believing that the average ages of the members of the different groups were not at all the same. When the investigator compared the average weight of all the boys of one age with that of all the boys of another he had a perfect right to take the age of each individual very much more roughly than he would have done if he had been comparing two individual boys, because he had good reason to believe that the inaccuracies would balance each other. In this the method of group comparisons has a great advantage. But the very fact that the inaccurate measurements were good enough for one set of comparisons made him take for granted that they were good enough for another. Thus the peculiar advantage possessed by this method of group comparisons may conceal a great danger.

CHAPTER XXXI.

MEANS, OR AVERAGES.

In the last chapter we had occasion to show how an inference could be based upon a comparison of averages. Averages are used so much in various kinds of reasoning that a few definite statements should be made about them.

General conception.

"The first vague notion of an average, as we now understand it, seems to me to involve little more than that of a something *intermediate* to a number of objects. The objects must of course resemble each other in certain respects. Otherwise we should not think of classifying them together; and they must also differ in certain respects, otherwise we should not distinguish between them. What the average does for us, under this primitive form, is to enable us conveniently to retain the group together as a whole. That is, it furnishes a sort of representative value of the quantitative aspect of the things in question, which will serve for certain purposes to take the place of any single member of the group."* In this respect an average is somewhat, though not precisely, like a general name. "The ordinary general name rests upon [*i.e.*, is used to mark] a considerable variety of attributes, mostly of a qualitative character, whereas the average, in so far as it serves the same sort of purpose, rests rather upon a single quantitative attribute. It directs attention to a certain kind and degree of magnitude.

* John Venn, "The Logic of Chance", 1888, pp. 436 ff.

"We can easily see that the number of possible kinds of average, in the sense of intermediate values, is very great; is, in fact, indefinitely great. Out of the general conception of an intermediate value, obtained by some treatment of the original magnitudes, we can elicit as many subdivisions as we please, by various modes of treatment. There are, however, only three or four which for our purposes need to be taken into account.

"In the first place there is the arithmetical average or mean. The rule for obtaining this is very simple: add all the magnitudes together, and divide the sum by their number. This is the only kind of average with which the unscientific mind is thoroughly familiar. But we must not let this simplicity and familiarity blind us to the fact that there are definite reasons for the employment of this average, and that it is therefore appropriate only in definite circumstances."

The Arithmetical Mean of a series of quantities is that quantity which can be substituted for each one of them when they are to be added together, and produce the same sum. Six is the arithmetical mean of 4, 5, 7, 8, because the sum of these four numbers and the sum of four sixes is the same. Hence "for many of the ordinary purposes of life, such as purchase and sale, we come to exactly the same result, whether we take account of"* the exact size of each separate quantity and the differences between them, or suppose each one of them to be equal to the average. If we are paying for melons by the pound it makes no difference in the price whether the dealer says that we bought one which weighed 4 pounds, one which weighed 5, one which weighed 7, and one which weighed 8, or whether he says we bought four that weighed about 6 pounds apiece.†

Various kinds.

The next kind of mean, or average, to be considered is the Geometrical. It is that quantity which can be substituted

* John Venn, loc. cit.

† The arithmetical mean is the simplest case of the mean which is obtained by the method of least squares.

for each one of several quantities when they are multiplied together, and give the same product. In this way 4 is the mean between 2 and 8, for we get 16 whether we multiply 2 by 8 or 4 by 4. In the same way 6 is the geometrical mean of 2, 4, 27. The rule for finding the geometrical mean between any number (n) of quantities is to multiply all the quantities together and find the nth root of the product.*

Whether in any case we should use an arithmetical or a geometrical mean depends altogether upon the relations which we are considering between the things in question. If they are merely added together to produce an external result, like weights in the pan of a balance or like the simple interest which different sums of money earn in the same year, then it is clear from the definition of the arithmetical mean that we should use it; but if the quantities are thought of as bearing some fixed ratio to each other and depending upon each other like the earnings of a sum of money from year to year at compound interest, then it is clear from the definition that we should use the geometrical. If it is not certain what the essential relations in question really are, then it is not certain which mean should be chosen. In the ten years from 1890 to 1900 the population of Cleveland increased from 261,353 to 381,768. That is, it gained 120,415 people, or a trifle over 46%. Shall we take the arithmetical mean, and say that this represents an average increase each year of 12,041; or shall we take the geometrical, and say that it represents an average increase each year of 3.86%? The two means are quite different. If we take the arithmetical, we think of an addition of precisely the same number of inhabitants each year; if we

* The geometrical mean of two quantities is often defined as 'a mean proportional between them', or that quantity which bears the same proportion to the one as the other bears to *it*. According to this definition 4 is the mean between 2 and 8, because $2 : 4 :: 4 : 8$,—not because $2 \times 8 = 4 \times 4$. Mathematically the two definitions amount to the same thing; but the one given in the text is better for the purposes of logic.

take the geometrical, we think of a considerably smaller absolute addition (viz., 10,088) in the first year of the ten, when the city is comparatively small, than in the last year of the ten (viz., 14,159), when it is considerably larger. Which mean we should choose is simply a question of which we believe will best represent the facts. If the growth of cities depended altogether upon the birth of children within their boundaries, we should naturally choose the geometrical mean, for the larger the city (other things being equal) the more children will be born in it. If, on the other hand, the population of a city, like that of a prison or a hospital, were made up altogether of certain kinds of people who were sent there from without, there would be no reason why a large city should gain more inhabitants than a small one ; and the more appropriate average would be the arithmetical. With most cities the natural rate of growth is only partly geometrical and only partly arithmetical ; so that neither a series of means of the one sort nor a series of the other would give a wholly satisfactory representation of the mean growth from year to year between one census and another. If in any case or set of cases we have reason to believe that the true mean lies somewhere between the arithmetical and the geometrical, and if we wish to represent the facts as accurately as they can be represented by any mean, we must make a mean that does lie between the two.*

For many purposes the best mean to choose is not an aver-

* In estimating the population for any given time on the basis of a set of census returns mathematicians actually use a set of equations like this: $p = a + bt + ct^2 + dt^3 +$ etc., where p is the population at some given time (different in the different equations), and t is the number of units of time (e.g., decennial periods) from any starting-point to the time of the population p. The values of a, b, c, d, etc., are found from these equations, in which the value of p is known, by the method of least squares; and then the value of p at any other time can easily be calculated.

This formula actually does represent a rate of increase lying somewhere between an arithmetical and a geometrical progression.

age in any mathematical sense of the word at all, but simply the size or kind that occurs most frequently. This is what is generally meant by the 'average man'. When we say that the average man likes a certain kind of newspaper or a certain kind of play, we simply mean that the people who like that kind of newspaper or that kind of play are more numerous than the people who like any other kind. It does not mean that half the people in the community like something more refined or more intellectual and that the other half like something less refined or intellectual. The 'average man' may happen to lie midway between two extremes; but it is the numerousness of such men, not their middle position or any other such relation to other kinds of men, that the editor or the playwright cares about.

Another kind of average, which is often quite good enough to represent a group for the purpose of comparing it with some other group, is what Mr. Galton calls the 'Median'. If we suppose all the objects in a group to be arranged in a row according to their size, the median size is the size of the middle object in the row. If we have to compare the size of the soldiers in an American regiment with that of the soldiers in a Japanese regiment, we can measure all the men in each and take the arithmetical mean, if we wish to; but it would be much easier and it would answer the purpose quite as well to take the middle-sized man from each regiment and compare the two. The men in the middle make a better basis of comparison between the two regiments than those at either end, because the size of the men who happen to be in that position is not affected so much by chance. In two regiments raised in the same place and in the same way the two men of middle size would be almost exactly of the same height; but the men of extreme size might not, for one regiment might happen to contain a giant or a dwarf and the other not.

Averages, or means, can be used for three distinct purposes, some of which have been referred to already.

Averages can be used, in the first place, to settle the conflicting claims of a number of different measurements of the same quantity. When we try to measure a thing very accurately we usually take the measurement at least twice, and when we come to compare the two or more measurements we always find, however carefully they have been taken, that there is some slight difference between them. This is true when we try to measure the side of a room to an eighth of an inch with an ordinary foot-rule, and it is true of the extremely careful measurements made with the best possible apparatus in a physical laboratory. Indeed, the more accurate we try to be, the more of these discrepancies we shall notice. In a case of this sort the chances are that any measurement which we accept as the true one will be wrong; but with this probability of being wrong, it is valuable to have a reasonable assurance that we are not very far wrong. Suppose that there have been ten measurements and that the smallest is 1038 and the largest 1043. If we arbitrarily chose either of these extreme measurements as the one to go by, and if the measurement at the other extreme really happened to be right, then we should have made a blunder of 5 units; if, on the other hand we chose some number about half way between them as the true measurement, then, if either of the extreme measurements were correct, the error would not exceed two or three units, and if the real quantity should lie between the two extremes, then, of course, the error would be still less. Thus, if we are reconciled to an error in our measurement but wish a reasonable assurance that the error is small, it is usually better to choose some kind of a mean measurement than one of the extremes.*

First use of average.

* There are three principles which can be taken for granted when we try to find the true value of a quantity from a series of different measurements: (1) Positive and negative errors are equally probable; (2) There will always be more small errors than large ones; (3) Very large errors do not occur at all. To explain this last point. If I am measuring a wall with a yardstick and the record shows that according to each of the

I say 'usually' because it is only usually that it is better. Choosing the mean is no infallible rule for getting certainty out of uncertainty; and sometimes the blunderer who accepts the first measurement that he takes comes nearer to the truth than the careful man who finds the mean of a large number.

To take the mean of several estimates is a rule that might very well be carried farther than it is into every-day life. Too often we are guided by our last impressions, not because we have any reason to believe that the last is better than any other, but merely because it is the one which is present at the time when the final estimate must be given. A teacher, for example, examines a piece of work and marks it 5. The next day he examines it again and it seems to be worth 7. The chances are that this is the mark which he will hand in. What he should do under the circumstances is to treat himself objectively; to recognize that he had made two conflicting judgments; to ask whether the conditions under which they were made were any more favorable in the one case than in the other; and, if they were not, to split the difference between them and mark the work 6. It is not true that 'first impressions are always the best'; but the saying would never have come into existence if people had not often blundered by ignoring them altogether.

first four measurements it is somewhere between 102 feet 3 inches and 102 feet $4\frac{1}{2}$ inches, while according to the record for the fifth measurement it is 104 feet $3\frac{1}{2}$ inches, it is perfectly evident that this last record is wrong. It could not have been produced by any combination of 'errors' (such as slight variations in the length of the measuring-stick, in the accuracy with which it is placed and read, etc.). It is simply a *mistake*—I wrote down the wrong figure, or I made a mistake in my counting—and if it is not clear how the record can be corrected, the measurement must be disregarded.

It is in accordance with all three of these principles that the mean is not only a safer measurement to accept than either extreme, as I have stated in the text, but also that it is far more likely to be substantially correct, though of course it is not certain to be.

Taking the mean of several measurements is not always the best way to find a quantity. In the first place, it is useless to take the mean of several measurements unless we can put up with the amount of error which such a proceeding will necessarily or probably involve. This error may be the one which is calculated from the differences between the individual measurements; it may be some constant error which probably exists and affects all the measurements, but for which we cannot make proper allowance because we do not know its amount and direction; or a combination of them both. In the second place, we must not take the average of a set of measurements if we wish results which are as accurate as possible and if the measurements were taken carelessly and it is possible to take them over again more accurately. Finally, we must not take the average if we believe that there was some cause at work which prevented the errors from being scattered fairly evenly in both directions, so long, at least, as there is any possibility of judging which direction the errors tended to take. We must not choose a mean between the measurements given by two individuals if we have reason to believe that one of them is dishonest or incapable, and, on the same principle, we must not choose the mean of several measurements if we have any reason to believe that the more skilfully a measurement is taken the more it approaches to one of the extremes. "In endeavoring to obtain a correct estimate of the apparent diameter of the brightest fixed stars, we find a continuous diminution in estimates as the powers of observation increased. Kepler assigned to Sirius an apparent diameter of 240 seconds; Tycho Brahe made it 126; Gassendi, 10 seconds; Galileo, Helvetius, and J. Cassini, 5 or 6 seconds. Halley, Michell, and subsequently Sir W. Herschel came to the conclusion that the brightest stars in the heavens could not have real discs of a second, and were probably much less in diameter. It would of course be absurd to take the mean of quantities which differ more than 240 times; and as the

tendency has always been to smaller estimates, there is a considerable presumption in favor of the smallest." *

The second use of averages is to find the individual which best serves as a type or representative of the species to which it belongs. In the case of averages like those that we have just considered there was one true quantity which we were trying to find. In this case there are many individuals of a given kind, each with a quantity of its own, and we wish to pick out one that will represent them all. A type of this sort is often doubly valuable; for in many cases (as with natural species) an individual lying about half way between the two extremes is not only a fairly good representative of each of them, but is also an excellent representative of a majority of the whole species, because men or horses or cats of medium size are very much more common than the very large ones or the very small ones. If a large number of men be arranged in a row according to their height, a line that just touched the tops of their heads would be about the shape of the black line on the next page.† There would be much more difference between the very tall man or the very short man at one end of the line and the man next to him than between any two successive men near the middle of the line.

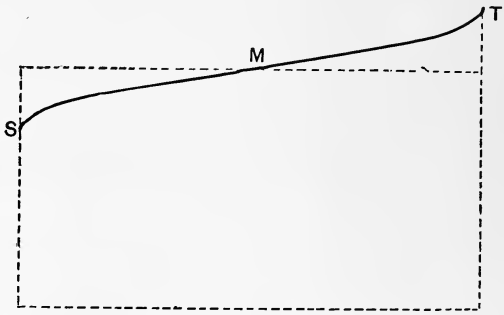
Second use of average.

The reason for this is easily explained. Suppose that there are ten independent variables which together determine a man's height; *e.g.*, the height of his father as compared with the average of the species, that of his mother,

* "Principles of Science", p. 390.—Instead of rejecting the poorer measurements entirely physicists sometimes 'weight' them, or admit them into the final reckoning in such a way that they do not affect it very much. What weights are to be attached to the various measurements—whether A's shall count for half as much as B's or only for a tenth as much—has to be determined arbitrarily, though there is a definite rule for determining the weights when they are made to depend wholly upon the number and probable error of the measurements in each of the series to be combined.

† After Bowditch. "Growth of Children", etc. 22d An. Report of the State Board of Health of Mass.

his own health during the period of growth, his food during that period, the amount of outdoor life which he had, the amount and regularity of his sleep, etc. Let us represent these different conditions by different letters from A to J; let us suppose for the sake of simplicity that each of these conditions is either distinctly favorable or distinctly un-



favorable; that each condition is as likely to be favorable as to be unfavorable; and that one favorable or unfavorable condition counts for as much as another. Condition A will be favorable in one case out of two; and in cases when A is favorable B will be favorable in one out of two. That is to say, A and B will both be favorable in only one case out of four. Similarly they will both be unfavorable in only one case out of four. But in one case out of four A will be favorable and B unfavorable, and in one case out of four A will be unfavorable and B favorable. Hence with only two variables there is one case out of four when both conditions are good; one when both are bad; and two when one is good and the other is bad. Working the problem out in this way we find that all ten conditions are favorable (or unfavorable) in only one case out of 2^{10}, *i.e.*, in 1 out of 1024; but that cases in which some of the conditions are favorable and others unfavorable occur much oftener, and that the more evenly the favorable and unfavorable conditions are divided

the more frequently does the combination occur. Out of 1024 cases

10	conditions	are	favorable	1	time.
9	"	"	"	10	times.
8	"	"	"	45	"
7	"	"	"	120	"
6	"	"	"	210	"
5	"	"	"	252	"
4	"	"	"	210	"
3	"	"	"	120	"
2	"	"	"	45	"
1	"	"	"	10	"
0	"	"	"	1	"

To students acquainted with the binomial theorem these figures and the process by which they are reached will be familiar. The mathematical law on which they are based represents the facts in most cases well enough to show how much more frequently things of medium size or quality are produced than those at either of the extremes. Thus the man of medium size or attainments is usually the 'average man' in two or even three senses of the words. He is the man who stands in about the middle of the line; he is the commonest kind of man; and very likely the number that expresses his size or attainments is nearly the arithmetical mean of the numbers which express the size or attainments of all those in the group.

Before we have a right to expect the arithmetical mean or the median of a group to be the commonest size or kind found in that group we must make sure that we are really dealing with one group of homogeneous things and not with several. If we measure a group of men half of whom are Americans and the other half African pygmies, the average height will be too small for the Americans and too large for the pygmies; and it might well be that not a single man in the whole complex group came anywhere near it. In the same way, if we should find the average size of all the articles

in a given room, from tables and lounges to pins and collar-buttons, there is no reason to think that things the size of the average—even if such things existed—would be any more common than things of any other size that any one might happen to think of. The things in the room are not homogeneous; and neither are the Americans and pygmies. To be homogeneous, in the sense in which the word is used here, things must be of the same general kind,—*i.e.*, produced by essentially similar groups of causes,—and the differences between them must be the 'compounded' result of a large number of relatively independent conditions of approximately equal value. The articles in the room were not of the same kind at all; and in the case of the Americans and the pygmies the one condition of ancestry, different for the two groups, overshadowed all the rest.

Even when the members of a group are perfectly homogeneous it does not always follow that those of medium size or attainments are the most numerous. In a previous paragraph I said that in a regiment of soldiers arranged in line according to their height there might be a giant at one end of the line and a dwarf at the other. But if the soldiers are regular infantrymen recruited in time of peace, the dwarf would not be there, simply because the government refuses to accept recruits under a certain height. The well-marked curve at one end of the line which touches the men's heads is thus cut off, and in a regiment recruited in this way the commonest type of man would therefore be a little to the small side of the median and a little smaller than the arithmetical mean. So in any class at school or college, the students who are laziest and most stupid have been cut off by previous examinations; and consequently at the lower end of the class we do not find one person of extraordinary ineffectiveness, but rather a fairly large group who have barely succeeded in fulfilling the minimum requirements. Here also, therefore, the largest group is towards the lower end and somewhat below the arithmetical average.

Another thing we must be careful about with averages of this sort is not to mistake the average for the ideal. If a child's growth is not up to the average, the physician has a right to suspect, though perhaps not to conclude, that something is wrong; but then the average by which the physician is guided is an average of children in good health, and then again it is only when the child's weekly growth falls below the average—not above it—that the physician is anxious. Thus he regards the average growth as a kind of minimum—not as the maximum to be striven for. So also with matters of conduct, the fact that 'everybody' does a certain kind of thing is no reason in the world for believing that that is an ideal kind of thing to do. In the case of a race perfectly adjusted to its environment and incapable of further improvement it might be; but, as things stand, the commonplace of to-day is the ideal of yesterday, and the ideal of to-day is the commonplace, not of to-day, but of to-morrow. Another reason for striving for something better than the average in the case of conduct is this: The average is made up of good, bad, and indifferent; and if the best people in a community should suddenly cease to keep as far above the average as the bad are below it, the average would necessarily fall, and would keep on falling until the community went to pieces or until some one arose again who was willing to be better than the average of his fellows.

Third use of average.

The third purpose for which we find a mean is convenience in representation—to have "a merely representative number, expressing the general magnitude of a series of quantities, and serving as a convenient mode of comparing them with other series of quantities", as in group comparisons. "Such a number is properly called *the fictitious mean* or *the average result.*" *

The average weight of the players in a football team may

* W. S. Jevons, "Principles of Science" (1887), p. 359. The distinction which Jevons here makes between the use of the words Mean and Average is not always observed, and I have ignored it in the text.

not come anywhere near the weight of any one of them, and it is not a kind of type towards which football-players tend; for there is reason why the quarter-backs should usually be lighter and the centres heavier. There is therefore no one thing in the world which the mean employed in this way represents or attempts to represent, and yet it has a real use when we consider the group as a whole in its relations to something beyond: in this case in relation to some other football team and the chance of beating it. So when we give the mean temperature of Winnipeg we do not mean to say that that is the commonest temperature there, nor yet that that is a kind of type which the temperature of each day naturally tends to approach; for we know that most days are either hotter or colder and that it is natural for days to be much hotter in summer and much colder in winter. But with reference to places and relations that lie beyond, a statement of the average temperature may be full of meaning. If the mean temperature of Winnipeg is lower than that of San Francisco, this means that for some reason or other it receives less heat from the sun in the course of a year, or radiates more away, or perhaps both; and if there is any process of growth or manufacture which depends upon the total amount of heat (regardless of variations from day to day) which Nature gives in the course of a whole year, a knowledge of the mean temperature of each place would tell which of the two would be the more favorable in this respect. To quote again from Jevons:

"Although the average when employed in its proper sense of a fictitious mean represents no really existing quantity, it is yet of the highest scientific importance, as enabling us to conceive in a single result a multitude of details. It enables us to make a hypothetical simplification of a problem, and avoid complexity without committing error. The weight of a body is the sum of the weights of infinitely small particles, each acting at a different place, so that a mechanical problem resolves itself, strictly speaking, into an infinite

number of distinct problems. We owe to Archimedes the first introduction of the beautiful idea that one point may be discovered in a gravitating body such that the weight of all the particles may be regarded as concentrated in that point, and yet the behavior of the whole body will be exactly represented by the behavior of this heavy point. This Centre of Gravity may be within the body, as in the case of a sphere, or it may be in empty space, as in the case of a ring. Any two bodies, whether connected or separate, may be conceived as having a centre of gravity, that of the sun and earth lying within the sun and only 267 miles from its centre." *

While averages of this sort can represent the individuals in a group for certain purposes, it is only as members of the group. The average weight of the men in any athletic team is nothing more than the figure obtained by dividing the total weight by the number of players. The minute any one of them leaves the team that average ceases absolutely to represent him in any way whatever, and ceases at the same instant to represent the others either, whether individually or collectively. An average of this sort need not be in any sense either a representative of a single individual, or of a type towards which the individuals tend, or of an ideal. It is the mere product of an arithmetical process, useful for the estimation of certain outward relations of the things averaged.

The term Expectation of Life as used in insurance is likely to lead to the confusing of the two ideas which we are here trying to distinguish. To the insurance company it means merely the average time that insurable people of a given age and sex continue to live. To the layman it is likely to mean the time that he, as an individual, will probably continue to live—a very different thing, which should be calculated in an entirely different way.

* Jevons, op. cit., pp. 363–4.

Often it is well to add to an average some indication of the accuracy with which the average represents the quantities whose average it is. Ten is the arithmetical mean between 9 and 11. It is also the arithmetical mean between 5 and 15. But in the first case the average comes much nearer to each of the separate quantities than in the second. In the first case the difference between the average and each of the quantities averaged is only 1; in the second it is 5. When the average represents a large number of quantities, the simplest measure of the difference between it and each one of the quantities averaged is the *average variation of the separate quantities from that average.* The arithmetical average of the variations is found by finding the difference between the average and each one of the separate quantities (regardless of whether that quantity be larger than the average or smaller), adding all these differences together, and dividing by the total number of quantities. Thus the average of 5, 6, 7, 11, 13, 8, 6, 20, 10, 14 is 10; the separate variations from the average are respectively 5, 4, 3, 1, 3, 2, 4, 10, 0, 4; the sum of these separate variations is 36; and since there are ten quantities, the average variation is 3.6.

Measures of error.

When we are dealing with a number of separate quantities a knowledge of this average variation enables us to tell to what extent the average may be regarded as representative of each of them, and as thus serving the second purpose of an average, and to what extent, on the contrary, it must be regarded as a purely fictitious quantity serving the third purpose only. Of course the smaller the average variation the more accurately the average represents the separate quantities averaged.

When we are dealing with different measurements of the same quantity the average variation of the separate measurements from the average gives a measure of their accuracy. To be sure it does not tell anything about 'constant' or 'systematic' errors which affect all the measurements in the

same way; but it does tell how much importance must be attached to 'accidental' errors, or those which result from a large number of different causes and are as likely to affect a measurement in one direction as in the other. The larger the average variation, the more important are these 'accidental' errors and the less can we rely upon an average derived from a small number of measurements.

This average variation is easily found, and it is a good enough measure of error for some purposes; but mathematicians do not use it. What they do use is either the Median Error—generally known as the 'Probable Error'—or the Mean Square Error. The former is commonly used in English-speaking countries, the latter in Germany.

The Median Error, or so-called Probable Error, is the variation from the mean that half the separate measurements fall short of and the other half exceed. If we suppose all the separate measurements to be arranged in order of magnitude, the central quantity is the median, and the Median Error is the difference between that central quantity and the quantity half-way between it and the end of the line in either direction. The 'Probable Error' is thus the amount of error that any one of the quantities is as likely to fall short of as to exceed. It is not the amount of error or variation from the mean that will probably be made. If we denote the difference between each individual measurement and the mean (i.e., the 'errors' or 'residuals') by v_1, v_2, v_3, etc., and the total number of measurements by n, the formula for finding the Probable Error (r) of a single observation is this:

$$r = .6745\sqrt{\frac{1}{n-1}(v_1^2 + v_2^2 + v_3^2 + \ldots + v_n^2)},$$

or, more briefly,

$$r = .6745\sqrt{\frac{[vv]}{n-1}}.$$

The probable error of the mean, r_0, is $\frac{r}{\sqrt{n}}$.

The formula for the Mean Square Error of a single measurement, ϵ, or of the mean, ϵ_0, is the same as that for r or r_0, except that the factor .6745 is omitted.

The calculated error of a measurement is usually written after it, thus: 1287 ± 3. Unfortunately, however, this is ambiguous, for sometimes the error indicated in this way is the error of any one measurement out of a series, and sometimes it is the error of the mean. By giving the latter where one expects the former an observer sometimes makes his results appear more accurate than they really are.

The student who wishes a fuller treatment of this subject is referred to Venn's "Logic of Chance", to Jevons' "Principles of Science", or to some one of the many mathematical treatises on the theory of probability, such as Merriman's or Comstock's.

CHAPTER XXXII.

PROBABILITY.

In the last chapter we found that where several measurements or estimates do not agree it is often of practical advantage to assume that the truth lies somewhere between them, and therefore to find out the average. We found, however, that sometimes the nature of the case is such that an average is out of the question. If two people both claim the same piece of land, it would hardly do to say that each of them owns half of it; if one toss of a penny gives heads and another gives tails, there is no practical purpose which will be served by assuming that the natural position of the penny is neither with heads up nor with tails up, but on its edge; if we do not know whether a certain act will please a person or annoy him, it will hardly do to assume that it will do neither one nor the other. In cases of this sort the mean is almost sure to be wrong, and so far wrong as to serve no practical use; it is therefore excluded and we have to choose between the extremes. The theory of probability discusses this choice.

Why needed.

To the ordinary human mind and to all of the brutes this choice between extremes is often more natural than the search for a mean. If we do not know whether to fight a certain enemy as hard as we can or to run away from him as hard as we can, it is usually better to do one or the other—no matter which—than to follow the middle course and sit still and wait to be devoured; and, fashioned as we are for the

world in which we live, it is more likely that we will. If a friend is accused of playing us false, we may believe the charge, we may indignantly reject it, or we may alternate between these two extremes; but we are not likely at the time to assume that he was partly true and partly false or to take an attitude of perfectly neutral doubt. We are naturally partisan, and even when we seem to be calmly halting between two opinions we are generally not halting at all, but uncomfortably oscillating from the one to the other. Thus not only is it often perfectly rational, but it is also perfectly natural, to choose and accept as true some one of several incompatible alternatives, and it is the business of a logical theory of probability to show the kind of ground on which we can justify the choice of any particular one to the exclusion of the rest.

When we toss a penny we say that heads and tails are equally 'probable' or equally 'likely'—that 'the chances' are even; and when we throw dice we say that double sixes is 'improbable' or 'unlikely'—that 'the chances' are against it. What do we mean by these words 'probable' and 'improbable' and their equivalents?

Probability has sometimes been defined as a measure of belief. According to this definition, when we say that the chances are nineteen to one in favor of a certain event, we mean that we have some expectation of its happening and some expectation of its not happening, but that the expectation of its happening is painted nineteen times as vividly upon the mind as the expectation of its not happening, so that when the two pictures come back to us the one has nineteen times the force and vivacity of the other.* But if this were all that we meant by probability, the way to find out in any particular case what the chances or probabilities were would be merely to examine one's own

What it is not.

* See Hume's "Treatise of Human Nature", Bk. I. Pt. III, Sec. XII.

—or some one else's—mind and compare the strength of its various expectations. This, however, is not what we do, or rather it is not what we know we ought to do. When we want to get at the probabilities in a case we should examine the case, not our own feelings about it, and accept the result of this examination whether it agrees with our feelings or not. Of course we cannot examine things without having thoughts; but probability has to do, not with the thoughts, but with the things.

Probability has to do with things; and yet it is not some mysterious inward power that strives to force them into a given course. We often speak of events happening 'by' chance, as though chance were a real cause.* In the same way when we speak of the probabilities as favorable or unfavorable to a certain event, there is a tendency to think of these shadowy probabilities as allies or opponents of the event in question; and consequently when the physician says that 'the probabilities are slightly favorable' we feel more like rejoicing than when he merely says that it looks now as though the patient might recover. The conception of probability as a kind of inward force striving to work itself out is called by Venn "one of the last remaining relics of [Scholastic] Realism, which after being banished elsewhere still manages to linger in the remote province of Probability." †

* Of course Chance is never a cause, nor does the word imply the absence of causes. "We call a coincidence casual, I apprehend, when we mean to imply that no knowledge of one of the two elements, which we can suppose to be practically attainable, would enable us to expect the other, we know of no generalization which covers them both, except of course such as are taken for granted to be inoperative. In such an application it seems that the word 'casual' is not used in antithesis to 'causal' or to 'designed', but rather to that broader conception of order or regularity to which I should apply the term Uniformity. The casual coincidence is one which we cannot bring under any special generalization: certain, probable, or even plausible". (Venn, 246.)

† "Logic of Chance", p. 22.

How then shall we define Probability without making it either a measure of belief or a mysterious force controlling things? The only way is to keep the objective standpoint but get rid of the mystery. When we say that the probabilities are five to one against throwing six with a single die in any one throw, the only clear meaning we can have is this: that when dice are thrown a large number of times we get six in about one-sixth of the cases and something else in the remaining five-sixths. Consequently before we make the throw we can say that the probabilities are against turning up a six; and if when the throw is made we really do turn up a six, we can say that the improbable has occurred, but we ought not to say that our estimate of the probabilities had been erroneous, even though the guess we based upon that estimate was wrong. The only way to prove that the estimate of the probabilities was erroneous would be to show, either by direct experiment or from the nature of the causes involved, that in the long run of cases essentially similar to the one in question sixes are not turned up in about one case out of six. In the same way, when we say that a man of thirty will probably live a year longer, all we mean is that most men of his age and apparent health do live a year longer. We do not mean to say that this particular man will not die the very next day. If he really did die on the next day, we could say once more that the improbable had occurred and that our judgment of the probabilities was good in spite of the event.

What it is.

The statement that a thing may be improbable and yet occur is not absurd. When we throw a die the chances are always five to one against any one of the six faces, and yet we cannot help getting one of them. In the same way it is highly improbable that this, that, or the other individual who goes into a lottery will win the prize; and yet if it is an honest lottery, one of them must. In these cases the improbable has to occur. Doubtless it is often said that if an event occurs, that very fact proves that it was not really improbable,

and that if it does not occur, that proves that it was not really probable, although it may very well have seemed so. But when people speak in this way they use the word 'Improbable' as though it meant 'Impossible', and the word 'Probable' as though it meant 'Inevitable'.

Hence all that we mean by saying that a certain state of affairs is probable is that that state of affairs or something which is bound to bring it about actually exists in most cases of the sort. It does not mean that it will be found in that individual case. The outcome in the individual case is unknown, and all the words in the dictionary will not turn ignorance into knowledge.

But if the 'probability' that we speak of with reference to a particular case really does not belong to that particular case at all, but only to the whole series or class of cases of which it is a member, what good does it do to think anything about it when the question at issue is not concerned with the series or class, but only with the individual case? It may be very interesting to know that most men of thirty live another year; but what good does that do when the question is not about men in general, but about this particular man?

What value?

None whatever!—if the question at issue really does concern this individual case only. If a person is going to do only one risky thing in the whole course of his life, he might as well do it with his eyes shut as after a long calculation of probabilities. Some of the richest strikes in the world have been stumbled upon by fools and tenderfeet where wise men and experts have missed them. It is a pure risk. If we win we win, and if we perish we perish, and in the latter case it will not be much consolation to be assured that the good fortune we missed had once been 'probable'. Generally, however, the question is not of one case only. If we bet on dice, we usually mean to bet more than once and we are willing to lose many individual bets, provided the 'odds' are so arranged that we win at least as much as we lose in

the long run. If a company insures lives or houses or plate-glass windows at all, it tries to insure a great many and it expects a large number of losses. But it expects still more gains, and fixes rates that will make the total gains more than meet the total losses. In this way a business in which every single operation is extremely risky becomes on the whole one of the very safest and surest that a person can engage in. This is as true of professional gambling as it is of insurance—each single operation is as risky to the 'banker' as it is to the chance visitor; but if he only bets often enough and the chances are slightly in his favor, his winnings are bound to be greater than his losses in the long run. Now the conditions of life are so complex that, as all the proverbs tell us, "the best laid schemes o' mice and men gang aft a-gley", and "nothing is certain but death and taxes". We are engaged in operations which are more or less risky all our lives long. Therefore the only thing to do is to act in such a way that the gains will more than make up for the losses in the long run; that is, to take account of the relative frequency of the various different outcomes, and where odds—or the possible gains and losses in a single transaction—are even to act each time as though the most frequent outcome were going to be present then: to act 'on general principles' or 'according to the probabilities'.

This way of acting on general principles is as natural as it is rational. Nature provided for it when she made us creatures of habit and imitation. It is inculcated in every maxim and moral rule. And because it is so natural as well as so rational to act in each particular case with reference to the general principle that will bring us out right in the long run, we come to feel that the general principle—whether it be a principle of morals or of probability—really means something for the individual case *per se;* and consequently it becomes almost impossible to get far enough away from its influence even in thought to realize that if there were only one uncertain situation in the whole world and a solitary

human being who had to venture his life on one alternative or the other, the outcome would be with him a matter of mere unqualified luck, and the word 'probability' would have absolutely no meaning.

If probability belongs to an individual case only as a member of a class, so that an event may be improbable and still occur, why is it that the estimation of probabilities often seems so difficult, and what is the difference between good judgment in estimating them and bad judgment? The difficulty which makes the difference between good and bad judgment so apparent is nothing more nor less than the difficulty of putting each individual case into the best class. The man of experience in any particular line sees distinctions between things that the novice overlooks, and on the basis of these distinctions he can subdivide his classes and estimate the probabilities in any particular case by reference to the subdivision rather than to the larger class to which the case belongs.

Why hard to estimate.

A stranger to the United States who happened to know that the two great political parties were nearly evenly divided would have to say that any American he met was just as likely to belong to the one as to the other. But a person who knew the country better would defer his judgment until he had found out in each case as well as he could what section of the country the person in question came from, what race he belonged to, what was his business, and so on. Every one knows that there is some danger in railroad travel, and, by comparing the total number of accidents in any given country during a given year with the total number of trains run, anybody can roughly estimate the chances of accident on some particular journey. But a man skilled in such matters might say that the road on which this particular journey was to be taken had a double track and block signals and tests for color-blindness and this particular kind of brake and that particular kind of coupler, and that where they had all these things the proportion of accidents to trains

—*i.e.*, the 'chance' of accident—was very much less than in the country at large. If he happened to know the records of the men who had charge of that particular train, he might form a more accurate estimate still. In the same way, a physician examining an applicant for life insurance does not think of him merely as a person of a given sex and age who looks strong or delicate. He regards him as a person with a certain heredity, certain habits, certain lung capacity and heart action, and with or without symptoms of this, that, and the other definite disease.

Such details do not tell how any given case will turn out; but they enable one to classify it with a small group of cases that resemble it quite closely rather than with a larger group that resemble it rather vaguely; and if the proportion of cases in which an event (such as a railroad accident) occurs or (if it is an event like death that must occur in every case) the average interval before its occurrence is something which we know as accurately for the smaller group as for the larger, this is a distinct advantage; for the greater the resemblance between all the individuals in a group the smaller is the variation between the outcome for any particular case and the average outcome for the class. If we could find a past case that we knew to be absolutely similar to the one in question in every particular, and could thus base our judgment upon what we knew about a class of one, probability would come to an end and we should have certainty. The greater our knowledge and skill the nearer can we come to such a class. Hence it is in the subdivision of classes and the knowledge of the outcome for each that the difficulty and the room for special skill come in.

If probability is not a force and therefore exerts no influence on the course of events, it is a mistake to believe that the non-occurrence of a probable event at one time makes it any more likely to occur at another. If a person is tossing a coin and gets heads three times running, he is likely to say that next time it *must* be

Caution.

tails. But this is a blunder. It is conceivable that one toss of a coin will affect another, just as a man's success in one enterprise may give him the confidence that leads to success or the overconfidence that leads to failure in the next. But if there is no such causal relation as this between the events themselves, probability is not going to become a cause and make one toss balance another. Whether we have tossed heads three times running or ten times, the coin will know nothing about it and the result of the next throw will be exactly what it would have been if no throws had preceded it at all. In the same way if a very improbable event occurs at one moment, that is no reason for believing that it will not occur again at the very next. Lightning is quite as likely to strike again in the same place (the proverb to the contrary notwithstanding) as to strike in any other given place of the same size. A second Galveston disaster is quite as likely to come exactly ten years after the first as to come exactly 137 years and 48 days after. We are quite as likely to throw ten heads running as nine heads and then one tail, or any other precisely designated series.*

The *reductio ad absurdum* of this view that events must balance and come out according to the probabilities is found in the story of the physician who said to his patient: 'Madam, you can't help getting well; for the books say that one case out of every hundred does, and I have already lost ninety-nine.'

The mathematical calculation of probabilities is in the main very simple—so far at least as the underlying principles are concerned. If we are tossing a single die, we expect to turn up one face in the long run about as often as any other, or, in other words, to turn up any given face in one-sixth of the total number of throws.

Mathematical principles.

* Of course a series is not precisely designated unless the order of the heads and tails is designated as well as their total number. 'Nine heads and then one tail' does not mean the same thing as 'nine heads and one tail', for this last does not tell which one of all the ten throws the tail is to be.

We therefore say that the chance of getting that face—say the six—is $\frac{1}{6}$. Since the ace and the six each occur in one-sixth of the total number of throws, *one or other* of them will occur in one-third of the total number; and so we say that the chance of getting *either* an ace or a six is $\frac{1}{3}$, and so on. It will be noticed in this case that the throw which gives any one of the six faces cannot by any possibility give any other; and so we can make some such general statement as this: Where two or more events are incompatible the chance of getting either one or the other is found by adding together the fractions which express the chances of each. Of course the chances *against* any given event or alternative are found by subtracting the fraction in favor of it from 1.

When we are tossing two dice (A and B) instead of one, we expect that in the long run each of the following combinations will occur about as often as any other:

A	B	A	B	A	B	A	B	A	B	A	B
1	1	2	1	3	1	4	1	5	1	6	1
1	2	2	2	3	2	4	2	5	2	6	2
1	3	2	3	3	3	4	3	5	3	6	3
1	4	2	4	3	4	4	4	5	4	6	4
1	5	2	5	3	5	4	5	5	5	6	5
1	6	2	6	3	6	4	6	5	6	6	6

With reference to the 6 or any other given face we can summarize these results as follows:

A six and B six,	$\frac{1}{6} \times \frac{1}{6}$ times	$= \frac{1}{36}$
A six and B not six,	$\frac{1}{6} \times \frac{5}{6}$ "	$= \frac{5}{36}$
A not six and B six,	$\frac{5}{6} \times \frac{1}{6}$ "	$= \frac{5}{36}$
A not six and B not six,	$\frac{5}{6} \times \frac{5}{6}$ "	$= \frac{25}{36}$
Total.....		$\frac{36}{36}$

Putting these results into more general form: "If the chances of a thing being p and q are respectively $\frac{1}{m}$ and $\frac{1}{n}$, then the chance of its being both p and q is $\frac{1}{mn}$, p and not

q is $\frac{n-1}{mn}$, q and not p is $\frac{m-1}{mn}$, not p and not q is $\frac{(m-1)(n-1)}{mn}$, where p and q are independent. The sum of these chances is obviously unity; as it ought to be, since one or other of the four alternatives must necessarily exist.'' *

One thing that a non-mathematician is liable to overlook in these figures is this, that the throws in which we get a six with either of two dice are not so common as the throws in which we get either a six or an ace with one die. We turn up as many sixes with the two dice as we turn up sixes and aces with one; but since the two sixes are on different dice and are therefore not incompatible, they come together in one throw out of thirty-six, and we do not turn them up in so many separate throws. This explains the necessity for the word 'incompatible' in the formula which we gave on page 339.

Four more cautions.

A second thing to notice about the table has been already referred to in another connection: namely, that if we add together the numbers on the two dice in each throw, we shall find that one sum is by no means as common as another. Seven is the commonest, for it can be made by six different combinations; 6 and 8 next; then 5 and 9, and so on until we reach 2 and 12, each of which occurs only once. Thus once more the mean is commoner than the extremes.

A third thing about these tables is worth dwelling upon because we are all likely to forget it when the figures are not before us: namely, the extremely small number of cases in which two independent improbable events coincide. Sixes with a single die are thrown in one case out of six, but double sixes with two dice in only one out of thirty-six, and if we should guess double sixes as often as they are thrown (*i.e.*, one time out of thirty-six) the guess would be right

* Venn, p. 174.

(*i.e.*, coincide with the throw) in only one case out of 36×36, *i.e.*, in one out of 1296. To take another example of the same thing, if the chances of taking a certain disease are $\frac{1}{100}$, and if a first attack neither increases nor decreases the liability to a second, the chance of a given person having that disease twice is only $\frac{1}{10000}$. In other words, if one person out of 100 has it once, only one out of 10,000 will have it twice. The difference between these two figures is of course very striking, and any one who sees them is likely to forget about the mathematics and jump to the conclusion that a first attack affords almost complete immunity against a second. As Wallace says in his very ingenious (though by no means conclusive) article against Vaccination: "'Very few people have smallpox a second time.' No doubt. But very few people suffer from any special accident twice—a shipwreck, or railway or coach accident, or a house on fire; yet one of these accidents does not confer immunity against its happening a second time. The taking it for granted that second attacks of smallpox, or of any other zymotic disease, are of that degree of rarity as to *prove* some immunity or protection, indicates the incapacity of the medical mind for dealing with what is a purely statistical and mathematical question." * Unfortunately "the medical mind" is not the only one that is likely to forget how rapidly fractions diminish when they are squared.

The method of ascertaining causal relations by comparing the number of actual coincidences between two events or circumstances with the number that would naturally be produced by mere chance according to the theory of probability is being used more and more as statistics of various sorts become more and more available; and by this method we must expect to reach many conclusions that seem at first, for the reason just given, to be contrary to all experience.

A fourth word of warning about the interpretation of

* Alfred R. Wallace, "The Wonderful Century", N. Y., 1898.

tables of probability. That an event turns out so many times in a given way is no reason why we should act that many times as though we expected it to turn out that way. On the contrary we should act each time as though we expected it to turn out in the most probable way. If we are guessing the total number of spots turned up by two dice and if we guess 7 every time, we will be right in 6 cases out of 36, or 216 out of 1296. But if we should guess each number as many times as that number is actually turned up, we should be right in only 146 cases out of 1296, as the following table shows :

Number.	Times thrown out of 36 throws.	Times guessed out of 36 guesses.	Times guessed right out of 36^2 or 1296 total guesses.
2	1	1	1
3	2	2	4
4	3	3	9
5	4	4	16
6	5	5	25
7	6	6	36
8	5	5	25
9	4	4	16
10	3	3	9
11	2	2	4
12	1	1	1
		Total number of correct guesses	146.

What is true in this case is true in any other: we should act each time as though we expected the most probable outcome to be found then. The figures show how much we can afford in the long run to risk upon each guess. They do not show how many times in the long run each of the possible outcomes should be guessed.

CHAPTER XXXIII.

OBSERVATION AND MEMORY.

INDUCTION tries to weave facts together into a coherent world. But our knowledge of every one of these facts depends sooner or later upon a perception through the senses; and if our 'senses deceive us' and we perceive or think we perceive what is not really present, that false perception will tend to give us a wrong conception of the world. Hence it is necessary to know something about the difference between good and bad perceptions. Moreover many of the perceptions from which we draw inferences took place some time ago, and if we depend upon our memory but do not remember them correctly, we are as badly off as if the perceptions themselves were wrong. Hence we must consider memory also.

Observation and inference.

The first thing to learn about Observation is the vast difference between what one actually perceives and the inference by which he explains it. The word 'Observation' seems to refer to the perception only; but as it is generally used it includes a vast amount of inference also. To explain this difference. A Frenchman makes a flying visit to the United States and then goes home to write a book in which he recounts his observations upon the character of the American people. But the 'observations' he recounts involve at least three successive sets of inferences. What he has really observed is a specific set of words and acts on the part of this, that, and the other specific indi-

vidual. His first inference is that by these words the individuals in question intended to convey certain specific ideas, and that in the acts they were guided by certain specific purposes; but he may have wholly misunderstood them both. His second inference is that people wishing to convey such ideas or acting for such purposes must have such and such characteristic conceptions and feelings ; but again he may be mistaken. His third inference is that what is characteristic of this score or more of individuals whom he happens to have met is characteristic also of the seventy-six millions whom he has not. Once more he may be wrong. Yet he calls it all observation.

So with scientists. Their inferences are more careful; but still they often use the word Observation to include them. Astronomers, for example, may speak of the observed course of such and such a comet, when they have only observed a few of its positions and have calculated all the rest. Indeed in the strictest sense of the word they can hardly be said to have observed even a single position. They have perceived a speck of light at a certain apparent position in the field of a telescope; at about the same time they pressed an electric key connected with a clock ; and they have afterwards read off certain figures from various parts of the telescope and its attachments. That is all. The position of the comet even at one moment is obtained from these data and others like them by elaborate calculations. Thus the word 'observation' is used in science as well as in common life in a very loose sense that is likely to deceive because it seems to imply a closer contact with immediate experience than it really does.

Even when we realize all this and do not regard anything as an Observation except a direct perception we are not yet free from inference. Perceptions are not bare sensations. They are sensations interpreted, and the interpretation, however rapid and involuntary, is an inference, and may be wrong. I *see* my brother across the road, but when I cross to speak to him it turns out to be some one else ; I *hear* some one say 'Be honorable', but he really said 'It's in an enve-

lope', and so on. These are what psychologists call Illusions. Then again we have Hallucinations, when the interpreted sensation itself arises wholly from within. I hear my name called, see a flash of light, or feel a drop of rain or the crawling of ants, when there is nothing there at all. But because I say 'There is' a sound, a light, or an insect, instead of merely saying 'I have' such and such a feeling, or that such and such a feeling exists, I am drawing an inference once more, and once more it is or may be wrong. The only way to avoid all chance of Illusions and Hallucinations is not to interpret any sensation. But a live person cannot do this; and if he could and did, he would die.

Thus the paradox: induction sets out to base its inferences upon the observation of facts, but the observation is itself a matter of inference. What then shall we do: reduce the element of inference in our observations to the very minimum mentally possible, or let it reach a maximum? What we really do in most cases is to infer without scruple until something makes us suspect that we have been deceived or that we are dealing with a class of facts in which we are likely to be deceived; and then, if it is not too late, we turn back and examine the phenomena more carefully and critically. No one ever thinks of distrusting his 'senses' so much when he is watching a farmer or a carpenter at work as when he is watching a conjurer or a 'medium'. In the one case we include a great many spontaneous inferences in our 'observations' and say that we *saw* him do thus and so; in the other case we only say that he *seemed* to. What we do naturally in this respect is perfectly logical; for no apparent perception can be tested except by the surrounding conditions as we know them.

All the difference between the absurd credulity or incredulity of ignorant peasants and the reasonable judgment of the educated depends upon the different extent of their knowledge about phenomena like those in question or the wider world in which they occur. A scientist

Credulity.

never thinks of doubting the existence of other men, and thus when he 'sees' a colleague in the room with him he believes that the colleague is there. An unsophisticated peasant never thinks of doubting the existence of ghosts, and so when he 'sees' a ghost in the room with him he believes that it is there. Logically the scientist and the peasant are in precisely the same position, and unless we are willing to say that the scientist should not be so credulous when he believes that he sees something, we have no right to say it of the peasant. Incredulity in general is no better than credulity. The scientist would probably distrust his 'observation' of the ghost, but he does not distrust his observation in general. Indeed he distrusts his 'observation' of the ghost only because he trusts his other observations and the inferences he has drawn from them enough to believe that ghosts probably do not exist. Thus in observation, as in everything else, general faith precedes and has a logical right to precede specific doubt. The doubt does not come spontaneously or for its own sake, but it is forced upon us by our faith in our other observations and the larger system of things which they seem to have revealed.

If the field of our observations is a new one and we cannot tell where it is that the spontaneous inferences which we naturally include in our 'observations' are most likely to be wrong, the only thing to do is to go ahead bravely yet cautiously, placing provisional confidence in our observations everywhere, yet always ready to turn back and re-examine any point more carefully. The scientist cannot avoid blunders; for he sees and hears as other men do, and he draws all their spontaneous inferences; but unlike them he knows how much of what he seems to see and hear is really inference, and how likely it therefore is that some of his 'observations' are erroneous. Consequently he generally takes pains to verify his observations before he announces them, he states them modestly when he does, he expects others to verify them for themselves before accepting them, and he is

willing to be corrected when he has made a mistake. In all of which respects he is very different from most of those who have not received his training.

Since what we 'observe' and the faith we put in it depend upon what we already believe, it is perfectly evident that a wrong belief to begin with will lead to wrong 'observations', so that when we are once started on the wrong track we keep going further and further, and generally find 'sufficient proofs' for our false convictions. **Errors cumulative.**

> Trifles light as air
> Are to the jealous confirmations strong
> As proofs of holy writ.

We pile proof upon proof until at last we stumble across some fact so obvious that we cannot ignore or distort it, or until we discover that the general conception of things that led to so many bad 'observations' is inconsistent with some other general conception just as well 'established'. Then comes the doubt, the true testing of the 'observations', and the better general standpoint. The only point in favor of the scientist as contrasted with other people is that he is on the watch for such inconsistencies, and therefore corrects his blunders of theory and observation sooner.

This danger of 'observing' what we expect to observe and ignoring what we do not is inevitable. Often we wish to submit a fact to an unprejudiced observer. If we mean by an 'unprejudiced' observer one who has absolutely no convictions that can possibly affect his observations, then the only unprejudiced observers in the world are newly born babies who do not even believe that there are things and people. But their freedom from prejudice makes it impossible for them to 'observe' anything at all. When we demand the testimony of an 'unprejudiced observer' the most that we can really wish is that the observer in question shall have no more personal interest in one side of the question at issue than in the other, that he **Unprejudiced observers.**

shall have conceived of both sides as distinctly as possible, and that he shall then have made his observations dispassionately, calmly, and deliberately for the sake of deciding as fairly as possible between the two sides. We do not ask that his mind shall be free from all preliminary convictions, but only from those special convictions the truth of which is disputed by one side or the other. He is not, and cannot be, without prejudice in general, but only without prejudice on the questions involved in this particular dispute. For the rest, we must expect him to take something for granted. Hence if the question changes after the observations are made and now the dispute turns upon some point that our unprejudiced witness never seriously questioned, his testimony is no longer of any special value. And so, in general, an observation made for the sake of settling one question has very little value for the settling of another. When the question is changed the observations should be repeated.

Errors of observation are divided by logicians into those of Mal-Observation, where we perceive things wrongly, as has just been explained, and those of Non-Observation, where we fail to perceive or take account of certain important facts at all. Here are some examples of the latter.

Two classes of errors.

"Most of the books do not give us a psychology, but rather a *eulogy*, of animals. They have all been about animal *intelligence*, never about animal *stupidity*. . . . Human folk are as a matter of fact eager to find intelligence in animals. They like to. And when the animal observed is a pet belonging to them or to their friends, or when the story is one that has been told as a story to entertain, further complications are introduced. Nor is this all. Besides commonly misstating what facts they report, they report only such facts as show the animal at his best. Dogs get lost hundreds of times and no one ever notices it or sends an account of it to a scientific magazine. But let one find his way from Brooklyn to Yonkers and the fact immediately

becomes a circulating anecdote. Thousands of cats on thousands of occasions sit helplessly yowling, and no one takes thought of it or writes to his friend, the professor; but let one cat claw at the knob of a door, supposedly as a signal to be let out, and straightway this cat becomes the representative of the cat-mind in all the books. The unconscious distortion of the facts is almost harmless compared to the unconscious neglect of an animal's mental life until it verges on the unusual and marvellous." *

Again, during a thunder-storm a timid woman often tries to justify her fears by recounting all the cases she can remember of persons who were struck or nearly struck by lightning in previous storms, and forgets the thousands who lived through them unscathed. Many a man buys a ticket for a lottery because he thinks his 'chances ought to be as good' as those of some acquaintance who once won a prize, but forgets that they are also as bad as those of unnamed hundreds who invested but never won. "Bacon quotes the case of the sceptic in the temple of Poseidon, who, when shown the offerings of those who had made vows in danger and been delivered, and asked whether he did not now acknowledge the power of the god, replied: 'But where are they who made vows and yet perished'? This man answered rightly, says Bacon. In dreams, omens, retributions, and such like, we are apt to remember when they come true and to forget the cases when they fail." †

Errors of observation are sometimes spoken of as Fallacies. This does not mean that it is a fallacy not to see something that is there. The fallacy comes in when we infer that it was not there because we did not see it. I may not notice the number of times that dreams fail to come true, but I commit no fallacy unless this makes me infer that they come true much oftener in proportion than they really do. In the same way it would be rather a straining of language to call an

* E. L. Thorndike, "Animal Intelligence", pp. 3–5. Macmillan.
† Minto's "Logic", p. 24.

erroneous observation a fallacy. The fallacy or blunder in reasoning comes in when we do not make proper allowance for the possibility of such errors, and insist that something must be true because 'we saw it'.

Memory and its dangers.

When an observation is not fully recorded at the time of making, the danger of error is twofold; for we are quite as likely to be wrong in our memory of the observation as in the observation itself. The longer the time between the observation and the recollection the greater in general is the danger of error. "Col. Nicolay is reported to have said that he and Mr. Hay received very little aid from contemporary memories in writing their history of Abraham Lincoln, and that they came to the conclusion that mere memory unassisted by documentary evidence was 'utterly unreliable after a lapse of fifteen years'." *

The dangers in the case of Memory are like those in the case of observation. We may forget, and if we insist unreasonably that we never had a certain experience because we cannot remember it, we commit precisely the same kind of blunder as when we insist unduly that a thing was not present because we did not observe it.

When we do 'remember' we are more likely than not to get some of the details wrong, and sometimes we take 'recollections' for true when they are really complete inventions. Errors of the first kind correspond to illusions in perception; those of the second kind to hallucinations.

What we remember.

In memory as in perception we constantly tend to find what we think we ought to find. When we tell a story of some past conduct of our own we almost inevitably make it more logical and coherent than it really was. We had the motives that we ought to have had, we said the clever things that we ought to have said, and the mere blind impulses and incoherent acts and meaningless speeches are forgotten altogether or twisted into

* W. H. Burnham, "Memory", Am. Jour. of Psychology, II, 235.

shape.* When things are simply incoherent and meaningless we cannot remember them any more than we can observe them. We may remember that there was something incoherent, but we cannot remember or describe it without giving it a certain coherence, even if it be a coherence of absurdity. When some one makes a meaningless speech we either give it a reasonable meaning, forget it altogether, or make it a kind of monstrosity far worse than it really was. If it does not make a definite impression of some kind or other, we forget it. If it does, we fill in all the details to fit our notion of the whole. It is the same way in our recollection of an argument or a quarrel. We were right and our opponent was wrong; and we remember our good sayings or acts and his bad ones because they fit in with the impression of our rightness, and forget our bad sayings or acts and his good ones because they do not fit in with this impression and are therefore in a sense purely irrelevant. Or it may be that we are impressed with the wrongness of our case, and then we go to the other extreme and remember our own bad sayings and acts and his good ones and forget our own good and his bad. In either

* "Mere illusions of memory suggested by present impressions are common in normal life. As we apperceive any object or event through the media of the feelings and ideas in consciousness at the moment, and thus no two of us apperceive the same thing in the same way, so in recollection each apperceives the past from the standpoint of his present state of consciousness, and the latter bears its part in determining what the resulting recollections shall be. We remember only main features of an event anyway, and the imagination fills in the gaps. Thus remembrance is never a true reproduction of reality. It is always more or less an illusion. At best it is an approximation to the truth. How near an approximation depends largely upon the apperceptive mood of the moment." (W. H. Burnham, "Memory", Am. Jour. of Psychology, II, 449–50.)

Consistency according to the laws of nature is the only test of truth; but consistency in conduct is consistency according to a purpose, and this is only an ideal. When a person tells a story that makes all his acts or all the acts of his hero rational and consistent, we can be quite sure that it is not true.

case the recollection is distorted by the almost inevitable tendency to remember things as coherent wholes capable of brief and definite description and congruent with the emotion of the moment.

As memory distorts the inner content of an experience itself, so it may easily distort its relation to other experiences; and then we get the dates and places wrong. We feel that experience A must have taken place in connection with B, possibly because that is the logical order, possibly because we have often thought of them together; and yet as a matter of fact they may have been miles or months apart.

Serious consequences often result from this erroneous 'recollection' of the connections between experiences and the corresponding forgetfulness of their real connections. If we remember a dream or a fancy with such vividness that it has the feeling of reality and do not remember the outward relations that would clearly distinguish it as a dream or fancy, it will seem to us that what we are remembering is not fancy but fact, and we easily fill in the connections that facts like those 'remembered' ought to have. Thus there are people who lie, and lie habitually, with the very best faith. The only possible remedy for this unconscious lying is to distrust one's own memory and deliberately test or verify one's 'recollections' in every case of importance; and if one does not wish to be deceived by others, he must distrust theirs too. Lawyers are proverbially unsatisfactory witnesses, simply because they know how uncertain memory is, and only say 'I think so', when others far less accurate and careful say 'I know'; and of course the confident assertions of the man who 'knows' carry far more weight with the ordinary juryman than the hesitating beliefs of the one who only 'thinks so'.

Honest lies.

"The medico-legal aspect of this subject is of the most practical importance. The more common forms of paramnesia [or false memory] . . . show that it is not impossible to manufacture testimony. A member of the bar tells me

that this is actually done in some cases, the method employed being somewhat as follows: The witness is a person of deficient memory. It is desirable that he should testify to the occurrence of a certain event. The lawyer asks the witness if he remembers this event. The reply is, No; and nothing more is said. But the idea of the event has been suggested to the mind of the witness. In a few weeks the lawyer repeats the same question, and again receives a negative answer. But after a few similar experiments the witness becomes uncertain whether he remembers the event in question or not. He begins to think that he does. The images of the imagination suggested by the lawyer's questions loom up vaguely in the mind, the memory is confused, and in a few months the lawyer, if skilful, may develop a pseudo-reminiscence so strong that the witness will give the desired testimony with complete sincerity. Of course this cannot succeed with persons of strong memory and critical judgment, but with children and aged people it may not be difficult. . . .

"Nothing, as Motet says, is more effective than a child's story of the details of a crime of which he pretends to have been a witness or a victim. The child's *naïveté* adds to the interest and elicits confidence. His hearers urge him on by their sympathy. Parents, friends, and neighbors accept the account, true or false. They suggest new details and fill up the gaps in the story. The child's uncritical mind assimilates these details, repeats the story without variation, and makes his accusation before the magistrate with an apparent accuracy that is most telling. . . .

"The uncertainty of human testimony was notably illustrated a few years ago in the case of the Bell Telephone Co. vs. the People's Telephone Co. The chief point at issue was whether Daniel Drawbaugh had a telephone in his shop prior to 1876. Several hundred witnesses gave testimony bearing directly or indirectly upon this point. The honesty of most of the witnesses seems to have been admitted, yet

evidence offered by one side was generally refuted by testimony from the other. The Supreme Court divided upon the case, and the seven thousand printed pages of evidence in the suit seem rather to prove the fallibility of human testimony than anything else. See article on Daniel Drawbaugh, by H. C. Merwin, *Atlantic Monthly*, Sept., 1888." *

Since memory is always unreliable, almost the first thing for one who is doing scientific work of any sort, no matter how humble, to learn is the importance of keeping full and clear records of every detail of his experiments or observations that may have the slightest bearing on the question at issue. If a detail is to be preserved at all, it must be taken down at once; it is usually almost as easy to take down a point of doubtful value as to neglect it, and the best and most accurate of experimenters are only too liable to find their work less valuable than it might otherwise have been because there was some small detail of which they did not make a note. Most beginners need to be warned, too, not to keep notes on loose scraps of paper, not to use unfamiliar abbreviations without writing down their meaning, and to make their writing very legible. If they themselves are to be sure of its meaning, the record should be so clear and unambiguous that it could be easily understood by any one. In other words, it should be a true record, and not a mere series of suggestions for the memory. It is important, too, to number the pages (unless the record is in a book) and to leave plenty of blank space on each of them. There is a strong tendency to get things crowded some time or other before the results are finally computed, and a crowded record is very confusing. Finally, the notes should be indexed and put away in such order that they can be found at a moment's notice for years afterwards. To observe these simple precautions is to save much time and annoyance for everybody concerned; and what is said here

Resulting commonplaces.

* W. H. Burnham, loc. cit.

about scientific work is just as true, *mutatis mutandis*, of a farmer trying to remember what his fields have done each year, or of business, school-teaching, housekeeping, or anything else where it is worth while to remember transactions accurately.

CHAPTER XXXIV.

THE DISCOVERY OF PAST AND FUTURE EVENTS IN GENERAL.

When we wish to ascertain some specific fact that we have not been able to observe for ourselves there are only two ways of doing it. One is to depend upon the testimony of others; and the other is to draw an inference from what we know of the general laws of nature and the specific facts that we or others have observed. The latter method can be described very briefly, and so we shall speak of it first. This is the method used in tracing the history of the solar system from, or rather back to, the vapor and the star-dust from which the planets were made and in prophesying the condition of cold and darkness and lifelessness to which they may be destined. It is the method pursued by geology in tracing the changes which have taken place on the earth's crust and prophesying those which will take place. And it is the method pursued by evolutionary biology in tracing the history of life in the world as it has developed from one form to another. In all these sciences the starting-point is the present, and the question is always this: Granting the truth of the general laws assumed or ascertained by various sciences, what is the only concrete state of affairs that could have preceded the one which we observe to exist at the present, and what is the only concrete state of affairs that can succeed it?

The first thing to notice about this method is that we

always start from the present. In telling about history of any sort we may often begin 'at the beginning'. In investigating it we never can. Moreover, if we are mistaken about some of the general laws or if we are not quite accurate about some of the concrete facts with which we start, the consequences of our error will affect all our history and all our prophecy; and since there is a chance of overlooking some essential fact or making some miscalculation at each stage of our regress into the past or progress into the future, the chances are that the farther we go the less accurate our account of things can be. The possibility of such accumulation of errors will always make a definite and detailed description of the world more doubtful the farther the described state of affairs is removed from the present data with which we have to start. But indeed what we know about the laws of nature and present concrete conditions is so slight in comparison with what we do not know, and even those things that we do know are so enormously complex, that no one really attempts to work out the problem in all its details, and the most that any scientist attempts to tell about either the distant past or the distant future is the broad outlines of things, which would remain substantially the same no matter what were true about any one of countless smaller details. A geologist can tell with perfect confidence that where there is now a certain group of hills there was once a fairly level plateau, and he can tell that the change from one to the other was due in the main to the action of water running down to the valley below, but he would never attempt to tell the exact location of every stream or the amount of earth that one of them carried down on some particular day ten thousand years ago.

The starting-point.

Even what a geologist does tell about the past and the future is not based upon the most ultimate laws of matter known. If it were, he would have deduced the history of the world from the known laws of chemistry and molecular physics; and such deduction is impossible because the situ-

ations that these sciences deal with are exceedingly simple, and from the relations that are found to exist in these simple situations no one could possibly calculate what would happen under the vastly more complex conditions that are dealt with in geology. The geologist starts rather with 'empirical laws' which are much less precise (so far as particular molecules are concerned) than the laws of molecular physics, but which give a much better idea of what happens when things are arranged as he supposes them to be in the large. He sees, for example, that streams actually do wear away earth and rock from their beds and carry the débris away, and he determines by actual measurement the amount of earth of a given kind that a stream of a given size and swiftness carries off in a given length of time; and then he applies the 'empirical law' which he derives from such measurements directly to the problem in hand. He knows, of course, that the facts in the case are consistent with molecular physics, but he knows also that his data are much too crude and complex to be dealt with by that science; and so he works away with his 'empirical laws' in comparative oblivion of it. Almost all of our history of the world and our prophecy of its future is based upon such 'empirical laws' as these, derived from a view of things in the large; and of course any history or prophecy which is based altogether upon such broad rough laws cannot attempt to describe small details.

Another thing to notice about scientific history and prophecy is that there is nothing in the laws and concrete facts upon which they are based to tell the scientist when the whole world-process began or when it will come to an end. For all we know or ever could know, God may annihilate the whole world to-morrow; but our prophecy based on laws and concrete facts of the present looks forward without limit towards a whole eternity. In the same way we trace what we suppose to be the broad outlines of history further and further into the past, and we never reach and never can reach a point at which we can say: Here everything

The limit.

must have begun. If the world ever was made and wound up like a watch, whether five thousand years ago, as theologians used to suppose, or twenty-four hours ago, we can never see anything on the face of it to indicate when that took place. All we can say is that if this world-watch really was going six thousand years ago as it is going now, then at precisely that time the hands were in such and such a position.

It seems much easier to believe that the world started five thousand or five billion years ago than twenty-four hours ago, merely because our whole conception of so distant a past is vaguer. But if it started five thousand years ago, it started with fossils in the rocks and all the other absolutely definite conditions that must have preceded the present and that lead scientists to trace its history beyond the five thousand years of its actual existence. And if it started five billion years ago, every detail must have been just as definite; however hazy our idea of it may be. It might just as well have been created twenty-four hours ago with mines half empty and cities all built and ships in the harbors, and adult human beings busy at their work, with brains so fashioned that they look back towards an imaginary past and believe they remember it. Thus, starting from the present as science does and must, and assuming as it does that things acted in the past and will act in the future according to the same laws that we find now, it is quite impossible for us to find any point at which the world-process must have begun or must come to an end, even if there really was such a beginning and really will be such an end.

One might think that the uncertainty due to the fragmentary nature of our knowledge and this possible accumulation of errors should affect our knowledge of the past quite as much as our knowledge of the future; but as a matter of fact it does not. We know a great deal more about the past; and the reason is that many conditions, when they are once produced, remain practically unaltered for a great many years. No geologist can tell whether the

place where I am now writing will ever be covered with water or not; but from the layer of gravel beneath the surface of the ground I can infer with reasonable certainty that once it was. No one can tell whether Vesuvius will ever destroy a city again, but from the buried remains of Pompeii and from written records we can infer that once it did. This is merely because we have reason to believe that such things as beds of gravel, the stones of buried cities and forgotten documents remain comparatively unchanged for years or centuries. Thus when we find such things they take us at one leap beyond all the intervening years, and each of them shows us one fragment of the past as it existed at the time.

Accordingly, if we are right in assuming that such relics of the past have remained unaltered in any given respect, and if we have any means of finding out their age, we can use each one of them as a starting-point in the construction of the past, or at least as something fixed by which to test our inferences about it. In such cases we might almost say that we really do start with the past. This is why we can tell more about some things in the past than in the future; why there is such a thing as serious history where there is no corresponding prophecy.

It is because there are such absolutely or relatively permanent 'monuments' of the past as these that it is possible for us to distinguish between relatively *direct* evidence concerning past events and *indirect*. The most direct evidence concerning any event is, of course, the present and personal perception of the event itself. This is something independent of 'monuments' of any sort, but if the event is past, this evidence is unattainable. The most direct evidence then possible is the perception of something which we can assume to be a direct effect of the event, and after that the supposed effects of effects of the event or the supposed effects of something that would have caused the event; until the chain is as long or crooked as you please. The most direct evidence of the destruction of Pompeii by a volcanic eruption would be

the perception of the event itself. The most direct evidence possible for us is the personal perception first of the unbroken lava and then of the houses being dug out from beneath it. Photographs of such lava-beds and houses give less direct evidence, and the tales of persons who claim to have seen such photographs less direct still.

It is hardly necessary to add that the most reliable evidence about past events is afforded by monuments which we know to have undergone the least possible change, and which are connected with the events under investigation by the shortest and surest series of causal relations. The value of written records as monuments of the past will be spoken of in the next chapter.

Circumstantial evidence.

One hears a great deal in discussions of criminal trials about *circumstantial evidence*. Evidence is called circumstantial when the witnesses have not observed, and therefore cannot tell about, the very fact at issue (*e.g.*, A murdering B), but can and do tell about various other facts so connected with the fact at issue by the law of causation that from them the jury can infer what that fact really was. Thus 'circumstantial evidence' is only the common name for indirect evidence of the sort that may be given in a court of law. If A and B were heard talking loudly as though they were in a violent dispute and if A was afterwards seen leaving the house covered with blood, while B was found stabbed to the heart with A's knife, there would be good circumstantial evidence that A had killed him, and he might very well be convicted and hanged for a crime that no one had seen him commit.

Circumstantial evidence is conclusive only if a supposed state of affairs is the only one that will fit in with the ascertained facts according to the general laws of nature. In a simple case of astronomy or chemistry a conclusion of this sort can be drawn with practical certainty; but with human affairs the case is somewhat different; for human life and the conditions that determine it are so exceedingly complex that it

is very rarely possible to say with anything like certainty: This supposed state of affairs and this alone is consistent with all the established facts. In the case given, for example, it is quite possible that B was seized with a sudden mania during which he first picked a quarrel with A and then seized A's knife and stabbed himself, while the blood on A was due to his efforts to prevent B from doing the mad deed. Or it might be that A and B were not quarrelling with each other but with some third person, who had afterwards picked up the knife, committed the murder and then made his escape. Or the real truth of the matter might be something entirely different that nobody happens to think of at all. A person convicted of some crime on circumstantial evidence is thus often convicted on a degree of probability which falls considerably short of practical certainty; which means, of course,* that amongst a large number of such cases there are a few in which the person convicted and punished is really innocent. This accounts for the general feeling of uneasiness or dissatisfaction when a trial of vital importance is decided altogether upon circumstantal evidence. But we must remember that the uncertainty of a conclusion based upon such evidence is due to the complexity of the situation dealt with and our ignorance of many of its details, not to a defect in the general principle; for the principle is that involved in all indirect evidence, and it is the only one in virtue of which we can gain any knowledge whatever of either the past or the future.

* See p. 334.

CHAPTER XXXV.

TESTIMONY.

AMONGST the most important monuments of past events are the impressions made by them on the minds of persons who were present at the time. From these impressions, if we can find out what they are, we can often judge the nature of the events that made them. Yet the impressions in the minds of others can never be observed directly, but only inferred from what they say and do. Hence there is always a double inference: from what a witness says to his thoughts, and from his thoughts to the events that caused them. The necessity for this double inference often makes the correct estimation of testimony very difficult; and yet in all matters of history—whether they be the events of last week told in a police court or the events of two hundred years ago set forth in a formal treatise—its proper estimation is most important. If we estimate it wrongly we are bound to reach false conclusions; and if we ignore it altogether we can hardly reach any conclusions at all, for apart from the memories of men and what they have written we have very few unaltered monuments of the past life of individuals, and our human environment is so complex and our knowledge of human nature so slight that an investigator who tried to find out about the past deeds of anybody else merely by going back step by step from his own personal experiences would hardly get started before he had to stop.

Its importance.

Even if there were plenty of other evidence, reliable testi-

mony would still have a great and very unique value. It sets the whole scene before us, presumably as we ourselves would have seen it if we had been there, with the important details accented and the meaningless trivialities omitted or properly subordinated; and thus without much trouble on our part it gives us a warm, human, unified account of the whole matter at a single sitting, instead of leaving us coldly and laboriously to construct it all bit by bit from various remnants of impersonal evidence that give the trivial and the important all together and leave us to find out what are the points of essential human interest for ourselves. In this way testimony enables us to reap the fruits of some one else's perceptions and inferences. But this has its dangers, for it tends to put us at his mercy if he chooses (as mere inanimate relics of the past cannot) to deceive us, and to make us share his errors if his perceptions, inferences, or memories were wrong. How then can we be sure in any particular case that there is no mistake and no falsehood, and that the words of the witness have therefore given us anything like the idea of the matter in question that we should have had if we had observed it for ourselves; and if we cannot be sure of this, how can we make use of testimony at all?

Its unique value and danger.

It is easier to answer this question now than it would have been while we were discussing the inductive methods by which we learn the laws of nature; for throughout the whole discussion we simply took for granted that when people are working together for a common end they trust each other, and have a logical right to do so. Now that the question is raised we shall find that in every specific case the answer depends upon our knowledge of these very laws that we learned largely through the aid of our spontaneous faith in one another.

The weighing of testimony involves some departure from the attitude that we take most naturally and spontaneously towards the stories that are told to us. Our first natural attitude towards testimony is one of trust; not because we have reasoned that it is trust-

Accepting, rejecting, and weighing.

worthy, but merely because we cannot help it. If it had not always been natural to accept the statements of those about us as substantially true, we should probably not be alive now to discuss the matter; for the telling of the truth on the one hand and confidence in the story told on the other are very important means for the preservation of the race. Each of us is born into a larger or smaller community with common interests: a common living to gain, common diseases to avoid, and common enemies to overcome. We can attain these common aims only by co-operation, and we cannot co-operate intelligently unless we all have approximately the same ideas. In warfare the chief must rely upon the scouts for knowledge of the position and numbers of the enemy, and, on peril of its existence, the whole tribe must act upon that knowledge as the chief commands. A sceptic who refused to believe and obey would be likely to sacrifice his own life and perhaps that of the tribe as well to his rebellious incredulity; and so likewise with the chase, sanitation, and other relations of life.

Since we are all fitted for this life in common it is almost as natural for us to share the ideas of our friends as to share their interests, and to accept the beliefs of those in authority as to obey their commands. Indeed it is no accident that the words Yes and No, which are used to express obedience or consent and their reverse, are also used to express intellectual assent and dissent, and that such words as Ally, Opponent, Our Side, Their Side, Attack, Rescue, Beaten, Victor, are used as much with reference to a 'conflict' about some matter of opinion as to any other, however deadly.*

* This need for a community of thought within the tribe as well as of action is expressed in many ways. In the heat of a military or political campaign a man who says that his country or his party is in the wrong is branded as a traitor. In religion the demand for uniformity of belief has shown itself in state churches with their old-time persecution of heretics and the modern prosecutions for heresy, in the maintenance of our various Protestant sects on a community of creeds rather than of aspirations, and in the feeling we often find that even though a belief be true it is wrong to accept it if the Church says otherwise. Even social

Along with this tendency to accept without question the statements of those whom we love and honor there soon grows up the tendency to reject the statements (whether we believe them to be sincere or not) of those whom we hate or despise. 'The truth for friends and lies for enemies.' This is a good rule of tribal policy and one so often followed that even in modern warfare, and often, too, in modern diplomacy, we expect the enemy to deceive us if he can. Even when we are sure that we know the real beliefs and institutions of another tribe, we feel that they cannot be accepted without disloyalty to our own. If any one is a foreigner or an 'Outlander', that is enough to make his language, his clothing, his customs, and his religion all 'outlandish'. We do not weigh his opinions any more than we consider his interests; we simply reject them with contempt. In the same way with private individuals within the family, the tribe, the nation, or the church, if they antagonize us it is almost a point of honor with us to disprove, or at least to disbelieve, everything they say. Thus we tend to accept the statements and the real beliefs of our friends and to reject those of our enemies; but not to weigh either the one or the other.

This rough instinctive way of accepting and rejecting testimony is good enough in the main for children and for the plain men and women who are content or compelled all their life to follow the lead of others, if the leaders are already given. But it is not sufficient when there is a conflict of authorities and one has to choose his leader. It is not sufficient either when a person wishes to go without a leader and perhaps to become a leader himself; that is to say, when he differentiates himself from the rest of the set, whether it

classes tend to have their own characteristic beliefs, and one rejects those of another without argument as visionary, aristocratic, or vulgar. Science itself is not free from the same influence, for there are different 'schools' of philosophy and medicine as well as of theology, and an *odium medicum* as well as an *odium theologicum*. The whole tendency can be summed up in the common use of the phrase *Vox populi vox Dei*.

be family, tribe, nation, church, or social clique, and feels that he has interests which the others do not share, or feels that he knows more than the rest or observes more carefully or has better natural judgment or a better standpoint from which to judge. A person in either of these situations who has testimony to deal with and who wishes to act rationally must try to settle his problem for himself in the light of what he already knows about the world and its laws.

In the first situation he must weigh as best he can the merits of the rival authorities. That is to say, he must try to find out what causes would naturally make a person an authority on such matters and then inquire how many of these were present in the case of each of the claimants. In the second situation, where he is trying to find the truth through testimony about some matter without the appeal to any authority, he must 'weigh' the testimony itself. This simply means that he must treat all the testimony on the matter as an effect and inquire what causes could have produced it. In either case he must settle his question by an appeal to the known uniformities of Nature; and if he does not assume that Nature is uniform, he cannot settle it rationally at all.*

Expert evidence. Caution.

When the question at issue is one about which we cannot hope to judge altogether for ourselves because a wholly independent judgment requires technical knowledge of a sort that we do not possess, we must appeal for aid to those who do possess such knowledge; but we must take care to avoid the fallacy known as the 'Argument from Authority' (*Argumentum ad Verecundiam*). This expression does not mean that every appeal to authority is fallacious. The fallacy arises only when, awed perhaps

* By finding the truth in a case of this sort we mean finding what we ourselves should have perceived if we had been present, or what would have been perceived by some one else who shares our interests and has at least as good natural ability and training. We do not mean finding the absolute truth, as God might perceive it; for very likely that would not fit into our general conception of the world at all.

by the greatness of a name,* we accept as expert opinion the dictum of some one, however great in some other direction, who is not an expert on the matter in question. A minister's training may fit him to speak with authority about the world to come, but not about the value of a patent medicine. A barber knows how to cut hair, but that gives him no special skill in preparing a hair-tonic. A great scientist is not necessarily an authority on metaphysics; a great philosopher (such as Kant or Hegel), on the history of philosophy; a great soldier, on politics; or a successful evangelist on Church history or Biblical criticism. 'Practical business men' often speak with scorn of the 'theorists' who differ from them on economic questions, though the very necessity for knowing the details of their own business may keep such men from seeing the general relations of all a country's industries. A telescopic survey of the world is called for, and they claim to speak with authority because they have a microscopic knowledge of a corner in a city lot!

Similarly a scientist who is great because of his power to collect facts may be a poor interpreter of them; just as a skilful census-enumerator might be utterly unable to explain a single figure in the returns.

This is a fallacy which one can never be sure of avoiding, unless he sets out to believe nothing at all. The great thing is to select one's experts with proper care and then trust them

* "Even proper names themselves do not seem always spoken with a design to bring into our view the ideas of those individuals that are supposed to be marked by them. For example, when a schoolman tells me 'Aristotle hath said it', all I conceive he means by it is to dispose me to embrace his opinion with the deference and submission which custom has annexed to that name. And this effect is often so instantly produced in the minds of those that are accustomed to resign their judgment to authority of that philosopher, as it is impossible any idea either of his person, writings, or reputation should go before. So close and immediate a connection may custom establish between the very *word* Aristotle and the motions of assent and reverence in the minds of some men." (Berkeley, "Principles", Introduction.)

only where they are strong. When selected the best of them will often be found to have marked tendencies in one direction or another—"personal equations"—for which due allowance must be made. We see best the side of things with which we are most familiar and in which we are most interested. It is thus natural enough that a physician, for example, should find nothing but physical disease behind some act that a clergyman might attribute altogether to sin. Indeed physicists and biologists often tend to ignore such a thing as conscious thought altogether or to treat it as a mere by-product of bodily life, while idealistic philosophers return the compliment by ignoring or denying the existence or the influence of the body. So, turning from training to temperament, a cruel man skilled in statecraft might advocate a measure of which a humane man equally skilled might wholly disapprove. Then, too, personal interest often affects our judgment, not because we wish it to, but because it keeps us constantly thinking of our own side of the case, which consequently gets well thought out, while the other sides does not. In the Southern States, where economic conditions made slavery profitable, it was regarded as a divine institution; in the North, where economic conditions had crowded it out, it was regarded as diabolical. Such interests are likely to affect experts as well as others.

Three specially flagrant kinds of fallacious arguments from authority are what Whately calls the Fallacy of References, the appeal to what Minto calls the Abstractly Denominated Principle, and what we may call the Appeal to an Imaginary Expert.

The first of these, which, according to Whately, is "particularly common in popular theological works", consists in making a great show of scriptural or other authority for some particular doctrine by, not quoting, but merely *giving references* to a large number of passages that have some bearing or other upon the subject, though few or none of them "distinctly and decidedly" favor the opinion in

question; "trusting that nineteen out of twenty readers will never take the trouble of turning to the passages, but, taking for granted that they afford, each, some degree of confirmation to what is maintained, will be overawed by seeing every assertion supported, as they suppose, by five or six Scripture-texts".

In the appeal to the Abstractly Denominated Principle, "A conclusion is declared to be at variance with the principles of Political Economy, or contrary to the Doctrine of Evolution, or inconsistent with Heredity, or a violation of the sacred principle of Freedom of Contract" (Minto); or appeal is made to the Monroe Doctrine or the Rights of Man or the Law of Nature or the Nature of God, or anything else that is vague, but high-sounding and terrifying. "It is assumed that the hearer is familiar with the principle referred to", though it may well be that neither hearer nor speaker really knows anything about it. The only thing for the hearer to do in such a case is to stand his ground and frankly confess his ignorance, if he is ignorant, and demand an explanation of the principle and of its precise bearing on the question at issue. But this often requires great courage.

As an example of an appeal to an abstractly denominated principle we may perhaps take the argument of Cardinal Manning against vivisection. It runs somewhat as follows: 'Truth of Nature must be sought only by methods in harmony with the perfection of Nature's God. Mercy is one of the perfections of God. Vivisection is not in harmony with perfect mercy. Therefore truth must not be sought by vivisection.' To all of which Professor Hodge replies: "How the worthy cardinal knows that vivisection is not in harmony with God's perfect mercy he nowhere explains." (*Popular Science Monthly*, Sept. 1896.)

By the appeal to an Imaginary Expert I mean the stating of one's own opinions in such a way that they seem to carry with them the weight of expert testimony, though really no expert is quoted at all. Benjamin Kidd's "Social Evolu-

tion", for example, is full of passages in which the author's opinion is backed up by that of an imaginary *future* expert, thus: "Yet nothing can be clearer to the evolutionist when he comes to understand the nature of the process in progress throughout our history, than that those ideals have been and are quite foreign to our civilization" (p. 142). "It has been the custom to attribute the success of the Revolution to the decay, misrule, and corruption of these classes; but history, while recognizing these causes, will probably regard them as but incidental. Its calmer verdict must be", etc. "A fuller and franker recognition of the true position . . . must apparently be one of the features of the work of the future historian who would do justice to the Revolution" (pp. 185–6). "At a future time, when the history of the nineteenth century comes to be written with that sense of proportion which distance alone can give, it will be perceived that", etc. (p. 299; see also pp. 301, 310, edition of 1895).

Kidd is not the only sinner in this respect. I have just run across the following sentences in *Harper's Magazine*: "There were to follow many more desperate encounters. . . . But in all probability the careful historian will yet decide that in shaping events which, step by step, wrought the downfall of the Southern coalition, Fort Donelson stands pre-eminent."

Leaving this question of how a layman should deal with the testimony of real or alleged experts on matters about which he cannot pass an independent judgment, let us turn again to the principles according to which we should deal with testimony as to matters of fact concerning which we are not wholly incompetent to judge for ourselves.

Inferring without trusting.

To find the whole truth which lies behind a statement will always be impossible for a human being; for that means discovering and distinguishing between all the innumerable causes that may have contributed more or less to produce it. Our knowledge is far too slight, our perceptions are far too inaccurate, and

our powers of calculation are far too limited for this. To find a part of the truth is a very different matter. We can often do this by causal inquiries without raising the question of truthfulness—which seems so important when we are merely accepting or rejecting statements—at all.

If a seedy-looking stranger rings your door-bell and says that he has not had anything to eat for three days, you need not inquire into his truthfulness in order to infer that he wants you to give him something and does not expect you to hand him over to the police. In the same way, if some one wrote a book that is obviously intended to help a certain cause, it is evidence—whether he believed what he said or not—that at the time he wrote, such a cause existed and at least one person thought it important enough to write about. If he told a marvellous tale to amuse his readers, we can tell that he expected readers and expected them to be amused by a tale of that kind. If he is hopelessly prejudiced, we can conclude that in his time it was possible for that particular kind of prejudice to exist. When he tells about some event of which he was a witness he may be both careless and untruthful, and yet there is always something that he reveals incidentally. An eye-witness may get many of the details of a great battle wrong, but he is not likely to discuss the way in which the combatants managed their bows and arrows and the manœuvres of the triremes if the battle was really fought with Maxim guns and Mauser rifles a hundred miles from any navigable water; and it is certain that he will not talk of Maxims and Mausers and ironclads if he lived in an age of bows and arrows, slings, and triremes. However dishonest or prejudiced or stupid a writer may be, he cannot make any statement whatever about things that no one thought of in his time and country, he cannot give his story a general background absolutely different from that of his own direct or indirect experience, and he cannot write in a language that he never heard or read.

To take a more concrete illustration of the same point.

Suppose we have reason to believe that Paul really made the speech before Agrippa recorded in Acts xxvi., but have no reason to believe that he told the truth, we can still infer that Paul and Agrippa were contemporaries; that Paul was a prisoner wearing some kind of 'bonds' (v. 29); that he had been accused by certain Jews (v. 2) of some offence connected, or capable of being plausibly explained as connected, with their peculiar customs (v. 3) and religious beliefs (v. 7), and more particularly with his own real or assumed belief in the resurrection of the dead (v. 8) and with what he represented as the divine mission (vv. 15–22) of one Jesus of Nazareth (v. 9). If we have no reason to believe that this speech was ever made, but do know the time and place at which the story of the speech was written, then we can still infer that at that time and place people knew the names Paul, Agrippa, Festus, Jesus, Christ, Satan, Jerusalem, Damascus, etc.; that they had either heard of the Jews and their religious sects (v. 5) or had invented the idea; that it seemed to them credible that a Jew might be prosecuted and even put to death (vv. 10, 31) for a religious belief, and that a king would listen to a prisoner in bonds who talked of prophets, visions, repentance, forgiveness of sins, and the resurrection of the dead; that at least some of the people of the time and place where the story was written knew the meaning of the common terms used to denote such things, and that at least one of them was interested enough in these things to write down a long story that turns upon them—a story, too, that shows a strong sympathy for Paul, his hero, who is supposed to believe in them.

More refined methods.

In the example just given the method of causal inquiry by which we discovered some of the facts behind a story was very crude, and the results attained, though perhaps important, were correspondingly meagre. The method was crude because it made use of no sources of information outside of the story itself—*i.e.*, it proceeded wholly from internal evidence—and because it

took no account of the difference between different individuals and the intimate surroundings in which they are placed. It took account of the fact that certain things, such as using a language that he had never learned, would be impossible for any individual whatever; but it did not take account of the fact that for this particular individual there must be other things which are equally impossible. Moreover, where each of several alternatives was possible—*e.g.*, Paul's sincerity or insincerity in the speech before Agrippa—it did not ask which was the most probable.

Another thing to notice about the conclusions drawn in the example is that most of them have nothing whatever to do with the question or questions which we probably had in mind when we read the story. If we are trying to find out something about the character or real beliefs of Paul, it does us little or no good to know that he was the hero of a story that may have been written hundreds of years after his death by a person who knew nothing about him but his name, and that the audience for which this person wrote was supposed to be interested in discussions of sin, forgiveness, and resurrection. Every story reveals something about the person who tells it, but it does not necessarily reveal anything—except that the author knew the name—about the persons or events of whom it is told.

To learn anything from it about these persons or events we must find out who the author is and what relation he bears (or has borne) to them. If he knows nothing whatever about them (except their names) we can learn nothing about them (except that their names were known in his time) from the story that he tells, however plausible it may be. If he has been in a position to know about them, so far as his external circumstances are concerned, but is too prejudiced in some particular direction to perceive or tell the exact truth, we may be able to find it from what he says if we can find out what his prejudices were and make proper allowance for them. If he really knows the truth but has

such interests at stake that he will not tell it, we may get it from him in spite of all his efforts if we only know what these interests really are.

In such cases of prejudice or falsehood we may get the truth from what the witness says, and if we have patience and ingenuity enough and if he talks enough, or if we know enough about him and about the surrounding circumstances, we undoubtedly shall. But in most cases this is too much to expect, and our knowledge or belief that the witness is prejudiced or untruthful only leaves us in doubt as to what the facts really are. On the other hand, if a witness who is in a position to know the facts is both unprejudiced and truthful, and if we know it, we are vastly better off. We expect him to make occasional blunders and so we cannot abandon the critical attitude altogether; but in the main our causal inquiry is greatly simplified. Instead of having continually to make uncertain allowances for some prevailing perversity that may (or may not) have affected this, that, and the other statement, we know that the vast majority of his statements are substantially true. Any given one of them may be more or less false, to be sure, and therefore if the matter about which it is made has intrinsic importance enough to justify the trouble, we must inquire further before we accept it. But if the witness has really been in a position to know and if he is really truthful and unprejudiced, there is no reason why all his blunders should lie in the same direction; and therefore when we care nothing about mere individual details as such, but only about the larger whole into which they are all combined, we may accept his statements as true enough on the whole for our purposes. We thus avoid the continual correction of details which is necessary before we can get a true conception of the events as a whole from the story of one who is prejudiced or deliberately untruthful. The wall of a building may be plumb although every brick in it is a little out of plumb in

one direction or another, but it will not be plumb if all the bricks incline in the same direction.

Who is the witness?

To learn anything from a story about the facts of which it tells, our first task, as we have just seen, is to identify the witness. This does not mean merely to find his name. It does not tell us anything about the truth of the Iliad to know that the author's name was Homer. Knowing who the witness is means knowing all that we possibly can about his personality, his interests, and the circumstances under which he gained or expressed his ideas. If the Iliad is a work of pure fiction written centuries after the siege of Troy, it may reveal very much about some other time and place, but it can tell us absolutely nothing about what transpired at that siege. Many of the stories that we read and hear are precisely such pieces of fiction. Lawyers and historians know this, and therefore when a person is about to give testimony in a court the lawyers begin by asking his name, his residence, his business, his relations to the parties concerned, and so on; and careful historians do not try to extract information from a writing until they have asked similar questions about the author.

In this matter of identity we must not take too much for granted. It is often in the interests of one individual to personate another or to forge his signature. The law recognizes this and guards against such deceptions as well as it can, not only by imposing heavy penalties upon those who commit them, but also by requiring that deeds be signed in the presence of a specified number of witnesses each of whom makes oath that the deed was signed in his presence and that he knows the persons who signed it. On the same principle, bankers require the identification of those who present checks for payment; society demands the proper introduction of all newcomers; and manufacturers are continually warning their customers against 'spurious imitations'. Where there are no penalties for deception and

where no precautions against it were taken beforehand the need for caution is especially great. The historian in particular should realize that the writings with which he has to deal often bear the names of some one who did not write them; for it is easy to ascribe manuscripts and works of art of unknown origin to some famous maker, just as it is easy to say that our rugs were made in the Orient or that our furniture and our ancestors 'came over in the Mayflower'. It increases their value, and our friends are not usually very critical about such matters. But that does not make such happy fictions true. The older any manuscript or other possession appears to be, the greater is the danger that it has been assigned to the wrong person.

One of the most famous examples of false authorship is the "Donation of Constantine". This remarkable document purporting to come from the early part of the fourth century was fabricated some time between 750 and 840, and imposed upon the world for six hundred years. In the document Constantine is made to tell how he was miraculously cured of leprosy by Silvester, the Bishop of Rome, and then to hand over his "palace, the city of Rome, and all the provinces, places, and cities of Italy and [*sive*] the western regions to the most blessed Pontiff and universal Pope, Silvester . . . and his successors". But the whole style of the document is that of the eighth or ninth century, not of the fourth; Constantine is made to describe himself as conqueror of the Huns, though they did not appear in Europe until at least fifty years later; Silvester is described as Summus Pontifex, though 'this title was still borne by the chief of the pagan college of priests'; the date written in the document is impossible, for no such joint consulate as that mentioned ever existed; and there are many other evidences in the document itself that it could not possibly have been written in the time of Constantine.*

* I have used part of the translation and criticism by Thomas Hodgkin: "Italy and her Invaders", Vol. VII, pp. 149 ff.

In seeking the authorship of a writing we cannot even take for granted that the whole piece was the work of a single author. Sometimes a report which some official signs is really the work of half-a-dozen different secretaries or departmental managers. Sometimes the principal author had a collaborator whose name does not appear. Sometimes, again, the original piece has been garbled by a copyist or an editor or a whole series of them, each one of whom made some stupid blunder or tried to improve the story.

Joint authorship.

We cannot detect the work of different authors and editors in the same piece unless we assume that human beings are subject to the general uniformity of Nature, and that therefore a person's vocabulary, style, knowledge, and interests do not change suddenly or without some adequate cause. To give an example. The play *Henry VIII.* was assigned by the editors who first published it to Shakspere; but recent critics tend to divide it between Shakspere and Fletcher. The evidence for the division that they make is found partly in the form of the verse. Fletcher was very fond of lines ending with an extra, unaccented syllable; Shakspere was not. Fletcher generally made his phrases and sentences end at the end of a line; Shakspere not so often. Quite recently Professor Ashley Thorndike has discovered another test by which he confirms the division made on such grounds as these. He has noticed that, in the works about whose authorship there is no doubt, Fletcher almost always contracts the word 'them' to ''em', using the latter fourteen or fifteen times as often as he uses the former, while Shakspere uses ''em' rarely and 'them' frequently; and he finds that the use of these words in each part of the play agrees with that of the author to whom the part is assigned.*

A more complex example of the method by which a document is shown to be the work of several different

* "The Influence of Beaumont and Fletcher on Shakspere", O. B. Wood, Worcester, Mass., 1901.

authors and by which his own part is assigned to each is found in recent researches into the origin of the first six books of the Bible. According to the critics, or some of them, the composite character of these books is proved by such facts as these: (A) The "many unnecessary repetitions" which they contain, *e.g.*, the creation of beasts and birds in Genesis i. 21–25, and again in ii. 19, and of man in i. 27 and ii. 7; the story of the Manna and the quails in Ex. xvi and again in Nu. xi; and the frequent repetition of similar laws in the legislative portions of the books. (B) "Frequent discrepancies and inconsistencies", *e.g.*, the account of creation in Gen. ii. 4*b* and i. 1 to ii. 4*a*; different statements as to the duration of the flood; Abraham's incredulity about the birth of a son, Isaac, on account of his own advanced age, and yet his subsequent marriage after Sarai's death; the law that altars shall be of unhewn stone, unpolluted by the use of any tool upon them (Ex. xx. 24), and the directions for ornamenting an altar of acacia-wood (Ex. xxvii. 1–8). (C) "The want of continuity and order in the narratives, *e.g.*, in the stories of Abraham, Noah, and Lot. The story as it stands in Ex. xix makes Moses, an old man of eighty, ascend and descend the mountain more than four times. (D) The "differences in style and conception". According to the first chapter of Genesis man and woman are made together, apparently out of nothing, at the end of creation. According to the second, Adam is made first out of the dust of the earth and Eve is made last out of his rib, when no helpmeet for him could be found among the brutes who were created in the meantime. "The first account is in form artificial and rhythmical, and the second graphic and picturesque."

So much for the kind of evidence by which critics believe it can be proved that these books are not all the work of the same author.

When it comes to the identification of the different authors whose work has been combined, it is necessary to study the

documents much more closely. Throughout almost all of Leviticus and Numbers the writing is characterized by a 'love of ceremonial law, fondness for statistical details, tendency to symmetry and similarity of phraseology, the insertion of the same or similar headings, and a style that in general is stiff and formal. When narratives occur they are little more than a collection of dry annals'. Moreover, the language of the original Hebrew shows certain peculiarities and uniformities. Since passages with these same peculiarities are found scattered throughout other parts of the Hexateuch they are all assigned to the same writer or group of writers, called P on account of his priestly tendencies. Since some of these passages written by P refer to what could not possibly have existed before the centralization of worship in the time of Solomon's temple, critics assume that they could not have been written before the building of that temple; and the relative maturity of many of the conceptions points to a much later date. From the minute directions as to the order and ceremony of worship critics infer further that P wrote at a time when such directions were necessary—probably when worship in the temple had been interrupted by a period of exile—and so they infer that P's work is probably "the result of the religious movement which began with Ezekiel in Babylon and found its completion with Ezra".

The Book of Deuteronomy is as marked in its characteristics as Numbers and Leviticus, but very different. Its style is 'smooth and flowing, redundant, pleasant to the ear, but sometimes tedious from the accumulation of synonyms'. Its aim is monotheistic and its character hortatory. It is remarkable for "its tone of gentle pleading, its spirituality as regards both God and man. God has no outward and visible form; God is near man, and His law within man's heart". It is "the Gospel of the Hexateuch". From the presence of these and other characteristics so different from those of P, critics infer that Deuteronomy was written by

another author, D; and they attribute to him or to others of his time and spirit the other passages scattered throughout the Hexateuch which are marked by the same peculiarities. Since D refers to the monarchy, the prophetic order and the priesthood of the Levites, it is inferred that this writer or group of writers could not have lived and written before these institutions were established. From the similarity between the spirit of D and that of the prophetic movement in the reign of Josiah and from other evidence it is inferred that Deuteronomy or some of it is the book of the law discovered in that reign, and written not many years earlier by some one who shared in the prophetic movement.

When the writings of P and D are removed other passages still remain, and these seem to have been written at an earlier period. The institutions to which they refer are more primitive, and the language seems like that of one who is thoroughly familiar with them. These passages are supposed to constitute or to be taken from the old book of the Covenant, and are therefore denoted by the letter C; but when they are examined closely it is seen that they are not all alike, and the differences between them and the way in which they are combined seem to show that they are the blended product of at least two different writers in different periods.

In one set of these remaining passages the name Jehovah (translated The Lord in our English Bible) is almost always used for God; and God is given very human characteristics: He makes Adam out of the dust, walks with him in the cool of the evening, sends confusion of tongues to prevent the human race from becoming too strong, and has to go down to Sodom to find out what is taking place there. In this same set of passages the name Israel is used for Jacob, and Sinai for Horeb, Israel and other names are used collectively to denote the Sons of Israel and of other tribal ancestors, and there are certain peculiar phrases, such as "to call upon the name of Jehovah". In these passages,

too, the narratives are always vivid and interesting and the conceptions are very *naïve*. Miracles, for example, are recorded without any insistence upon their miraculous character, as though they were perfectly natural. These passages are therefore assigned to one writer, or group of writers, known from his name for God as J. In the other set of passages in C the name Elohim (translated God) is used instead of Jehovah, unless there is some special reason for using the latter; and God is conceived of as more separate from man than in the passages assigned to J, and not so human. Amorites are spoken of instead of Canaanites, Horeb instead of Sinai, Jacob instead of Israel (though not always), the word 'lord' is used to mean husband, and there are other linguistic peculiarities. The style is more stiff and formal than J's and not so interesting, and the general conception of things is more sophisticated. When miracles are mentioned, for example, their miraculous character is recognized and perhaps insisted upon. This second set of passages is therefore assigned like the first to a single author or group of authors, E. The greater *naïveté* of the conceptions expressed by J indicate that he was an older writer than E, though the stories told by each of them probably originated at different dates, and some "were perhaps centuries old".*

For historical purposes it is often much more essential to distinguish the work of a given author from that of his editors than from that of his collaborators, for the latter at least were his contemporaries; and even though their statements may not be so trustworthy as those of the author himself, the ideas which they express and the words in which they express them must have existed in the author's time

* See Article Hexateuch, in the "Dictionary of the Bible", published by T. & T. Clark in 1899. The account of Hexateuchal analysis which I have given in the text is, I hope, sufficient to give some idea of the method involved in such investigations. It is far too fragmentary and superficial to be accepted as an accurate account of their results.

and country. If we are fortunate enough to possess several different manuscripts which have descended through different editors from the same original, the agreements and differences between them will make it relatively easy to 'restore' the original, or find what it really did (and did not) contain. If there is only one text, the task is harder, for we must depend altogether upon the same kind of data as those which must be used for the discovery of collaborators. But however difficult this work of restoration may be, it cannot be dispensed with if we wish to prove anything by the writing in question; for a subsequent addition to a book proves nothing about the times in which the book itself was written, and the firmest possible faith in a given individual gives no ground for believing something that he never said. A writing whose authorship has been determined is said to be 'authentic'. But this word does not imply anything about the truth of the statements that it contains.

When the witness is identified, the author found, his text restored, what then? If he has not been in a position to know anything whatever about the events of which he speaks, except perhaps a few names—that is to say, if his conceptions have not been caused in any way by these events—it may be worth while to account for his knowledge of the names, but otherwise we can learn nothing from him. If, on the other hand, he has been in a position to know about the events of which he speaks—if his story is in some way or other the result of an impression made upon him by these events—we must examine his account of them in the light of all that we can find out about the witness himself, his circumstances, the interests he had at stake, his prejudices and his veracity, and thus try to discover the events themselves by finding the cause that would be most likely to produce the story that he tells.

First the question of how the witness got his information. Witnesses do not as a rule discuss this question; they simply speak with an air of authority which leads the unsuspecting

hearer to assume that they know. If they do mention their sources of information, they generally make them appear better than they really are. A person may describe himself, for example, as an 'eye-witness' of some great battle in which he took part. But a battle is much too large and complex an affair for any single individual to see the whole of it, much less one who is busy fighting in his own part of the field. Nine-tenths or more of what the 'eye-witness' probably tells about is therefore based upon conjecture or the confused tales of other combatants often heard at fourth or fifth hand.

How does he know?

If a witness got his information through somebody else and if he is not himself a trained investigator, his statements must be taken with the greatest caution. This second-hand evidence, or "evidence which does not derive its value solely from the credit to be given to the witness himself, but rests also, in part, on the veracity and competency of some other person", is what is known to the law as 'Hearsay'; and except in a few specified classes of cases our courts refuse even to listen to it. "The law requires . . . the testimony of those who can speak from their own personal knowledge. It is not requisite that the witness should have personal knowledge of the main fact in controversy; for this may not be provable by direct testimony, but only by inference from other facts shown to exist. But it is requisite that, whatever facts the witness may speak to, he should be confined to those lying in his own knowledge, whether they be things said or done, and should not testify from information given by others, however worthy of credit they may be."

Hearsay.

We do not reject hearsay altogether in common life, and there is no reason why we should. The courts can afford to do so because they give power to the litigants to compel the attendance of witnesses (if they are alive and within the court's jurisdiction) who can speak from their own personal knowledge. Moreover, in the courts the only matters which have to he proved by witnesses are those which one party or

other is not prepared to admit. Thus the procedure of the courts does not imply that nothing can be proved by hearsay outside of them. It does show, however, that in the opinion of jurists hearsay is not in general a very satisfactory kind of evidence. The objections to it are the probability that the statement of the person quoted "was imperfectly heard, or was misunderstood, or is not accurately remembered, or has been perverted"; the ease with which a lying witness can shield himself when he says 'he was told'; the impossibility of cross-examining the person quoted, "that it may appear what were his powers of perception, his opportunities for observation, his attentiveness in observing, the strength of his recollection, and his disposition to speak the truth"; and the frequent impossibility of telling "through whom, or how many persons, the narrative has been transmitted, from the original witness of the fact." *

If we cannot tell anything about the origin and history of a story, if we have no reason to believe that the person who tells it to us would sift it thoroughly before telling it, and if it does not bear some very strong internal evidence of truth,

* Greenleaf, "The Law of Evidence", §§ 98, 99. The rule against the admission of hearsay does not of course exclude everything that a witness claims to have *heard said*. Evidence about the statements of another person is not hearsay when "*the very fact in controversy is whether such things were written or spoken, and not whether they were true*". If a person is being tried for slander a witness would be allowed to testify that he heard him say the slanderous words; but he would not be allowed to testify that some one else had told him that *he* had heard him say them. Again, evidence is not hearsay when the statements, whether written or spoken, which the witness repeats are "natural or inseparable concomitants of the principal fact in controversy". In a murder trial, for example, a witness may testify that he heard the prisoner threaten to kill the deceased, not because there was necessarily any truth in the threat, but because it indicates a state of mind in which such a deed might be done. The threat is thus a part of the *res gestæ;* it has, or may have, a direct or indirect causal connection with the act in question; and it is thus so much circumstantial evidence, not hearsay.

we have no right to assume that it is true. This statement applies to the ordinary newspaper anecdotes about public men, to much of the gossip that we hear about our neighbors, and to almost all of the legends of ancient heroes and mediæval saints.

Now comes the question whether a witness is truthful. If we are really weighing testimony, this question must be answered according to some rational principle. To depend upon our impulses for an answer is not to weigh it at all, but merely to return to the primitive condition in which we were before we discovered the need for weighing it. Hence we must not assume that a witness is truthful merely because the story he tells is pleasant or flattering, or because it is interesting or pathetic; because he is polite or intelligent, his voice and accent winning, or his manner confidential; because he was hard to find, knows so much about the subject, or is perhaps the only person whom we can find that knows anything about it at all. We must not trust him either merely because he was doubted once before and vindicated, or even because 'he told us himself he that was telling the truth'. So, *vice versa*, we must not assume that a witness is untruthful or ignore what he says merely because he is uninteresting or disagreeable.

Is he truthful?

In the effort to escape the influence of our impulses where impulses are liable to mislead, and to find some fixed test of veracity, there is danger of choosing something purely arbitrary; of assuming, for example, that there must always be 'truth on both sides' when witnesses disagree, and trying to split the difference between them; or of assuming that the story supported by the larger number of witnesses must be the true one, regardless of the character of the individual witnesses and their sources of information.

Arbitrary tests.

Because truthfulness is right and lying wrong, people have often invented arbitrary tests based upon the supposition that God would intervene in some particular manner to vindicate the right and punish the wrong. Thus a person sometimes

says, 'May God strike me dead if what I say is not true', and in a case of this sort if the calamity named does happen to occur it is often accepted by those who see it as a divine judgment. Such tests are easily systematized and adopted by a people as a whole. Consequently the Hebrews cast lots to discover a culprit, and our own ancestors in the middle ages appealed to the ordeal or to trial by combat. Such tests still survive, although the definite appeal to God may be lost; and schoolboys assume that one of their number has 'defended his word' or proved his truthfulness if he has won a fight with the boy who impeached it. In the same way when boys play the disputed part of some game over again, the winner announces that 'that proves it'.

If such simple methods as these for settling the rights of a case and the veracity of the witnesses had been found effective, the state and the church would certainly still continue to use them. Nowadays, however, we recognize that God or Nature gives no arbitrary sign by which we can detect a lie and vindicate the truth. We are not left wholly without a test—far from it; but the test which we have is hard to apply and, like the oracles of old, it often leaves us in doubt. There is no ultimate test of truthfulness but fact, and the test of fact is nothing less than consistency with the whole course of Nature.

When the fact itself is unknown and we are trying to estimate a witness's veracity in order to find it, the most important things to consider are his personality; his special relations to the question about which he speaks; the consistency and probability of the story he tells; and its relations to the stories of other witnesses. We must speak of each.

Character.

First the witness himself. We must not judge everybody by ourselves. Whether a person has a strong disposition to speak the truth is largely a matter of his own individual character. This is very different in different people, and it is something about which we cannot always know. We often can know, however, about a per-

son's race, the age in which he lived, his occupation, and the set to which he belonged; and all these help to determine his veracity. An Oriental is generally less likely to speak the truth than a European, a slave than a master, a promoter than a military officer. If Othello had stopped to think, he might have known that an Italian of Iago's time, even though an officer, would not hesitate to deceive a man of alien race if he had anything to gain by it.

In considering the effects of character we must not assume that the most innocent persons are necessarily the most truthful. A child may tell the most harmful lies in all innocence simply because the thought has come (or been put) into its head, and he has not critical judgment enough to distinguish clearly between truth and falsehood or to recognize the wrongness of the latter. A child once more may lie from pure nervousness. Ask him in a threatening tone whether he has done some perfectly innocent thing which you have seen him doing, and very likely he will deny it.

Next the circumstances. We must not divide people once for all into sheep and goats, those who lie or prevaricate and those who do not. "Falsus in uno falsus in omnibus" does not mean that if a person lies once he will lie always, but only that he is likely to lie again on the same occasion and about the same matter. Conversely, though a person has never yet told a lie, he may tell one in some new situation where the strain is greater. There always must be first offences. Therefore, when we have estimated a witness's general character, truthfulness and characteristic motives as well as we can, we must try to find out what special circumstances are present to influence him. Much depends, for example, upon the solemnity of the occasion on which the statement is made. A person is not likely to lie when he is confessing to his priest or when he knows he is at the point of death. The law recognizes the influence of this solemnity, and therefore it demands that every witness who comes into court shall swear to tell

Circumstances.

the truth;* but on the same principle it regards a dying declaration, even when the witness was not sworn, as equivalent to an oath.

Interests.

One of the most important circumstances which we have to consider is the effect which the witness expects his story to have upon his interests. If the story which he deliberately tells is against his own interests or the interests of those whom he would like to help, and if he must realize this, it is much more likely to be true than if it were one which he probably believed to be favorable; for people often lie to further their interests, but not to injure them. Here again the law recognizes our natural tendencies and attaches great importance to confessions of guilt or other statements against one's own interest, while on the other hand it has generally refused to listen to the testimony of any one who has a pecuniary interest in the result of a suit or to a prisoner who is on trial, regarding the latter's plea of 'not guilty' as a plea only, or a demand that the charge be proved, and not as evidence.†

While we may learn something from the procedure of the courts, we must be careful in this case, as in that of hearsay, not to infer too much from it. A court of law cannot administer justice effectively (especially when the work is divided

* The oath is intended to direct the attention of the witness to God—not to direct the attention of God to the witness.

† Dying declarations and statements against interest, including confessions of guilt, are two of the very few kinds of hearsay that the law admits as evidence. Of course the witness who tells of a person's admissions or confessions must have heard them himself and not merely heard some third person tell about them.

The refusal to listen to an accused person is partly in his own interest. His testimony cannot help him very much in any case, for he would not be on trial if his word were not suspected; and if he is really innocent, it may do him much harm, on account of the embarrassing circumstances under which it is given. Hence in States where the prisoner is allowed to speak in his own defence the jury is warned that if he does not choose to do so, it must not be taken as any evidence of guilt.

as it is in our system between a judge and a jury) unless it acts according to general rules; and evidence which the rule excludes might sometimes be recognized as perfectly good if it could be considered on its individual merits. Again, as we said concerning hearsay, the questions of fact that juries have to settle are always disputed; and rules which may be perfectly appropriate in settling disputes that the disputants cannot settle for themselves may be quite absurd when applied to matters that were never disputed. Then, too, it must be remembered that a person is not usually brought to trial for an alleged crime unless there is at least some *prima facie* evidence against him, and that in every civil suit that turns on some question of fact, one of the two litigants must have a bad case and be trying wittingly or unwittingly to 'make the worse appear the better reason.' When both parties to a transaction are clear-headed and honest they are not likely to get into court about it. Even where misunderstandings arise people generally succeed in keeping out of court when they have confidence in each other's honesty and fair-mindedness. Under these circumstances it is not unreasonable that the court should say: Often half of the parties interested cannot be trusted; the jury—untrained as they are—cannot always tell from the way in which the parties give their evidence which half this is; we will therefore exclude the testimony of them all, and depend upon that of disinterested persons, whom we can nearly always summon, and whose average honesty and intelligence will probably approach nearer to that of the community as a whole. This is very far from meaning that a person should never trust his neighbor's truthfulness when the story told by the latter happens to be in his own interest.

Though we need not always go so far as the courts of law and reject the evidence of interested parties, we must not neglect to make proper allowance for their interests. Most people who tell a story, whether it be written or spoken, have some object in telling it, even if it be nothing more

than their own temporary amusement or the entertainment of their hearers; and if the story is about some important matter, the chances are that the narrator has some personal or party interest which leads him, perhaps insensibly, to distort the facts a little for the benefit of his own 'side'. If he is writing a biography of a deceased friend for circulation amongst his admirers, it is moderately certain that he will dwell upon his good deeds and leave as many as he can of the doubtful transactions out. His very purpose or prejudice, however, may make some of his statements all the more trustworthy. If he mentions a few 'regrettable incidents, due not so much to the fault of the deceased as to the very unfortunate position in which he was placed', or incidents for which he feels compelled to offer any other kind of apology, his reader can feel moderately certain that the incidents occurred. The reader can be sure at least that the author believed they did, and that he would probably not have believed it if the evidence had not been very strong.

Special confidence in statements against the interests of those who make them is not deserved unless the speaker really knew or believed that they were against his interests. If a prisoner had been told that 'it would be better for him if he confessed' or that if he did not confess he would be arrested (and if he had not been subsequently warned that anything he said might be used against him), a court of law would refuse to listen to the story of his confession. In the same way the disgusting story which some one tells of his own villainy must not be accepted with the confidence which naturally belongs to a confession unless it is disgusting to the narrator as well as to the hearer. It may be that he is really boasting. Again, a person may confess some fault that he never committed in order to gain sympathy by the very fact of confession or to show how good he is now, since he is willing to confess it, or how much he has changed. In a sentimental age confessions are not always hurtful. Once more, if a man takes sides against 'his own country'

and attacks its leaders, what he says is quoted all over the world with the weight of a confession; and yet the man who does it may have divested himself of all special interest in his country, or this interest may be outweighed completely by the hostility which he feels towards the leaders whom he attacks, or by the pride which he takes in being thought 'broad-minded'. These same considerations are applicable to the alleged confessions of 'converted' nuns, priests, freemasons, and pagans. Finally, a confession should not carry much weight unless the person who makes it is in his right mind. It is not unusual for weak-minded persons to falsely confess hideous crimes of which they have read or which have taken hold in some other way of their imagination.

Consistency and general probability.

The only direct proof of a witness's truthfulness or untruthfulness in any particular case is the agreement of his story with the facts themselves. We may reject his story because we know beforehand that his word is not to be depended upon; but we could not know this unless we had been able to compare the stories which he told on other occasions with facts.

A story contrary to fact must be false, and unless we are willing to abandon absolutely all tests of truth and truthfulness we must be prepared to admit this, no matter how trustworthy the author of the story may have previously seemed to be.

Of course a direct comparison with the facts in the case is impossible when these are unknown and it is through the story itself that we are trying to find them. But there are certain more general facts which we do know and with which every true story must agree. A story 'inconsistent with itself' cannot be true because the alleged facts which it asserts cannot all coexist in a world ordered as we believe ours to be. On the same principle a story cannot be true which asserts the existence of any single state of affairs which is incompatible with the world's general arrangement. On

the same principle, finally, a story is probably false if it is like other stories which often grow out of the interests, prejudices, or vanity of such narrators as the witness, under circumstances like the present, but which are seldom true. This is the justification for incredulity towards tales of extraordinary luck in fishing, of wonderful hands in cards, and of remarkable coincidences that the narrator attributes to occult causes. By saying that stories of this third sort are probably false we do not mean that any given one of them is false, for we expect the most improbable of possible events to happen some time or other. We only mean that a person who never believes such stories in the absence of exceptionally strong evidence will be right oftener in the long run than one who generally believes them.

If there is any reason why a witness should not wish to tell the truth, the falsity of a part of his story may be sufficient to show that he is probably untruthful and therefore to discredit all the rest. But the falsity of one part of a story is not sufficient in itself to prove the falsity of the rest. A person may have some prejudice which prevents him from seeing some one part of the facts and yet be perfectly able and willing to tell the truth about other parts; or he may be willing to tell a 'little' lie, but not to tell a 'big' one. Even when a witness is perfectly unprejudiced and perfectly honest a certain amount of error and consequent inconsistency in his story is almost inevitable,—so much so indeed that a story which is absolutely glib and consistent often seems too studied to be perfectly true.

Confirmation and contradiction.

It is much easier to judge of the veracity of a witness if we are able to compare his story with that of others. When several independent witnesses agree in their stories they 'confirm' or strengthen each other; and the more of such witnesses there are the more likely is the story to be true; simply because each new witness makes it so much harder to account for the agreement on any other hypothesis. We must be quite sure,

however, that the witnesses really are independent. If they tell precisely the same story in much the same words—if they use the same strange phrases or make the same improbable blunders, like schoolboys who copy each other's examination-papers—it is practically certain that they are not independent. In that case they no longer confirm each other: there is really only one story, which they have all learned to repeat. In matters of this kind, however, some writers of history are very careless; for it is not at all uncommon to find one citing a whole series of ancient authors or 'authorities' in support of some statement, when there is not the slightest doubt that they all got the story from the same source.

Discrepancies, when they are found between the stories of several witnesses who speak about the same affair, may or may not indicate that at least one of the witnesses is probably dishonest. If they do and if the witnesses are evidently working together, this may discredit them all; for a perfectly honest man with a true story will not ask or accept the assistance of a dishonest man with a false story. But if there is no evidence of collusion between the witnesses themselves, the dishonesty of one ought not to discredit the others, though it might perhaps discredit the person who summoned them all. It would be a gross fallacy to use Peter's story (on the assumption that it is true) to discredit Paul's and then use Paul's to discredit Peter's.

As we can judge something about the truth or falsity of a story from its confirmation or contradiction by other stories, so likewise can we often infer something from the lack of such confirmation or contradiction. The importance of some events is so great and their consequences are so apparent that, if they happen at all, they are certain to be known by a vast number of people and referred to in one way or other by many of them. If an alleged event of this character is mentioned by only one out of a large number of contemporary writers, it is a fair conclusion that he is

romancing. So with contradiction. If a story that almost certainly would be contradicted if false is not contradicted, that affords some evidence of its truth. We must be careful, however, not to rely upon these tests where they are not applicable. People do not usually contradict newspaper anecdotes, or libels that they think beneath their notice. They certainly cannot contradict charges of which they are not told. Sometimes, moreover, a story is contradicted but we do not hear of the contradiction. Much the same is true, *mutatis mutandis*, of confirmations.

Sometimes the nature of an alleged event makes confirmation or contradiction of the story told about it absolutely impossible. If Elijah was alone at Horeb, as the story seems to imply, a hundred different documents could give no confirmation to the statements in I. Kings xix about what happened to him there. The most they could possibly prove would be that the writer in Kings had correctly stated Elijah's own account of it. In the same way, a hundred other 'accounts' of the event could not prove that Elijah's story was false, though they might prove that he did not tell the story as it is given in Kings.

Before one story can confirm or discredit another, it must be clear that the events to which the two refer are the same. This cannot always be taken for granted. Because A and B both say they saw a woman fall as she was getting off a street-car in the city yesterday we have no right to assume that they must have seen the same accident, for events of this sort are common enough and often very similar. In the same way, we have no right to assume without further investigation that there is any connection between the story of a flood told by some tribe of North American Indians and that told in Genesis; for there are certain types of story that we expect to find everywhere—stories of a creation, of a great flood, of the origin of speech and of fire, of a very strong man, a very wise man, a very faithful friend or lover, of talking animals, ghosts, giants, and the good old days.

Such stories are developed by a common mental process out of the common elements of human experience. The only way to prove an historical connection between two similar stories of this sort from the stories themselves is to show that there is more similarity between their details than the general similarity of human thought and experience will account for. It is not remarkable that the Hebrews and the Greeks should both have stories of a wise man and of a faithful friend; but it is curious that the names Solomon and Solon, David and Damon, should be so similar, and a few more coincidences of this sort would suggest very strongly some historical connection between two sets of stories or the events on which they are based.

So much for the tests by which we judge of the veracity of a witness and the probable truth or falseness of his story. If these tests give us reason to believe that he was in a position to know about the matters of which he speaks and that in the main he is unprejudiced and truthful, the task which remains for us is comparatively easy. But if we have found that he is prejudiced or untruthful, it is not necessarily hopeless. In this case (as in the other) we are far better off when we have the accounts of several different witnesses than when we have only that of one, no matter how prejudiced or untruthful we may believe each one of these different witnesses to be, or how conflicting their stories are. For every lie must be made to fit somehow or other into a background of truth. The person who tells it lived in a real world and he is bound to express his ordinary conceptions of that world up to the point at which he thinks it necessary to falsify things in order to make his principal lie seem consistent with itself and with admitted facts. Different persons lying about the same event (however much they may agree about the main point at issue) are bound to connect the false state of affairs which they allege with the true which they take for granted, in different ways; and if each of them is cross-examined carefully enough, they will probably reveal enough

between them to make it perfectly plain what the truth must have been. In this case, as in every other, the more effects of a given cause we know the more likely we are to find out what the cause itself really was. When a person merely wishes to absorb the truth from others a contradiction between them is necessarily very embarrassing; but when he is trying to find the cause that lay back of their statements it may be very helpful.

To what has been said about the principles by which evidence should be weighed I should like to add a few words about the attitude of the judge who has to apply them.

The judge must judge impartially.

If anybody wishes to have a question settled on its merits, he must make sure, to begin with, that the judge or jury to which he submits it is not so incompetent, dependent, interested, or prejudiced that the investigation will be a mere mockery. The State tries to provide for this by appointing judges who are learned in the law, and often by appointing them in such a way and for such a period that they will be independent of those whose cases they have to judge and of popular whims in general; by refusing to accept as a juryman any one who has any personal interest in the outcome of a case or who has already formed an opinion upon its merits; by providing that lawyers shall not ask 'leading questions', or those which indicate to the witnesses by their form what answer is desired; and by visiting severe penalties upon any one who attempts to 'tamper' in any way with judge, jury, or witnesses. In the settlement of our own problems we must not forget to take similar precautions. If the question is one which demands special skill (whether it be in medicine or in morals), we should choose our judge with reference to his skill, not with reference to the decision that we think he will give. If we desire an estimate of our own conduct, we should not go for it to our admirers or dependents, to those who have not the courage to form an opinion of their own and tell us what it is, or to those who cannot condemn or

approve of what we have done without also passing judgment upon themselves. When we have chosen our judge we must put our questions fairly, in such a way as to bring out the opinion held by him, and not in such a way as to tempt him merely to echo the opinion held or desired by us. We must not therefore say to him, 'Isn't this true?' or 'Don't you think so?' and we must not beg him not to be 'too unfavorable' or 'too hard on us'. When all that we seek is an opinion it is absurd to bribe the judge and then congratulate ourselves upon his decision.

When we ourselves are called upon to act as judges we must strive hard to resist not only the importunities of those who appeal to us, but the influence of our own desires and prejudices as well. Like good old Locke we must be able to say in all sincerity, "It is truth alone I seek"; and we must be prepared to judge of the truth by the evidence, not, like sentimentalists, to ignore the evidence or discredit or distort it because it is not what we should like. If we have interests, prejudices, or cherished beliefs which are affected by the issue, we must deliberately force them aside until after the decision has been rendered. We must not heed the "appeal to consequences", which points out that if we believe one thing we shall be forced to believe something else also which is unpleasant, though not necessarily untrue. If the affair touches us and those we love and honor so deeply that we cannot cast aside these interests and prejudices until the decision is given, then we must recognize that so far as this question is concerned we are not proper judges, and leave the decision to some one else.

According to evidence.

Again, when we ourselves have been called upon to decide a case (whether it be to determine some fact itself or to discover what authority is best fitted to determine it) we must not forget that for the moment at least we have assumed a position of independence, and we have no right to quietly abandon this position

Must judge for himself.

until our task is finished or definitely given up. The decision which we reach must be based upon facts, and the burden of drawing the inference from the facts rests upon us and not upon anybody else. To find the facts from which an inference can be drawn it is our right and our duty to get all the testimony we can; but when a witness has given us all the facts that he knows we have no right to ask what inference he draws from them, and then merely absorb his opinions, any more than a lawyer has the right to ask a witness whether he believes the prisoner is guilty instead of merely asking him what he saw the prisoner do, and leaving the question of guilt to the jury. If it is we who are called upon to decide, even an expert cannot relieve us of our responsibility. A lawyer has a right to ask an expert on gunshot wounds how far the muzzle of a gun must have been from the body of the deceased when he was shot; but he has no right to ask whether the witness believes that the prisoner killed him. This is the question which the jury must settle—and settle for itself. So, likewise, if you happen to be a director in a company, it is your business to examine the accounts yourself and see that everything is right. If there is anything which you do not understand, you must insist upon having it explained, not merely to the limit of the manager's patience, but to your own full satisfaction. It is your business to judge for yourself, and you are not doing your duty if you merely accept his assurances that everything is satisfactory. The tendency to ask, 'What do you think?' and then chime in with the answer, no matter how little it is worth, is well illustrated by the story of the Frenchman who thought he would play a trick on his fellow townsmen, and went up the street telling everybody he met that there was a whale in the harbor; but when the crowd going down to see the whale became very large he concluded that the report must have some truth in it, and went himself.

We can absorb real or pretended opinions, but we cannot

weigh evidence, without a definite problem. Therefore whenever we assume the independent position of a judge we must begin by finding out precisely what it is that we have to decide. This is often difficult; for we cannot make our questions definite until we know many things about the matter with which they have to do. We must therefore keep reframing the vague questions with which we started and making them more and more definite as our knowledge of the subject increases. The more light we have already obtained upon a subject the more we can break our questions up, and the more we break them up the more light we can obtain. Hence the saying that it is as hard to state a question properly as it is to answer it. In law there is a whole system of 'pleadings' which must be conformed to in every civil suit and which has been devised for the express purpose of compelling the parties to come at last to some one definite 'issue' upon which the whole case turns, so that it all may be settled upon the strength of argument * (if it be a question of law) or of testimony (if it be a question of fact) upon this one single issue, and not in a loose way upon the case as a whole.† In history or in science we cannot always

Must have a problem and make it definite.

* This distinction between 'argument' upon a 'question of law' and 'testimony' upon a 'question of fact' is purely technical. The true meaning of the law, as intended by the lawmakers or as decided in previous cases, is as much a question of fact as anything else, and the 'argument' addressed to the judge is a presentation and explanation of the evidence as to what this meaning is, precisely as the examination of witnesses and the address to the jury is a presentation and explanation of the evidence concerning other facts.

† Civil suits and criminal proceedings must always be based upon something definite. No court will entertain a suit for general injuries or a charge of general misconduct. When the plaintiff—to take a civil suit—has made his definite 'declaration' he must serve notice of it upon the defendant. If the latter makes no reply within a specified period, the case goes against him by default. If he makes one, it must be as definite as the charge. (1 *a*) He may 'demur for matter of form', or say that the charge has not been made in legal form, perhaps, for ex-

reduce everything to one single question, for our interests are manifold and one aspect of a case may be quite as interesting as another. Yet we can separate our questions and discuss them one at a time. Often, too, we can find some one 'crucial test', like the evidence which settles the vital issue in a lawsuit, by which each one of them can be settled. If we have too many questions in mind at once, whether they be relatively independent or whether several of them be subsidiary to some other, we are almost certain to become confused or to neglect some aspects of the case without knowing it, and thus to do bad work. The only safe method is to follow the example of the courts and hear only one case at a time, reduce this case to as simple an issue as possible, and dismiss as 'irrelevant' every alleged fact that does not bear directly upon this issue, however interesting the fact may be in itself and however important for the settlement of some other issue.

The lack of a definite problem reduced by successive analyses to one or more definite issues, and the lack of the consequent sense of relevance and irrelevance, is particularly

ample, that it is too indefinite. (1 *b*) He may 'demur for matter of substance', or say that there is no reason known to the law why he should not do the very thing charged against him. (2) He may 'plead by way of traverse', or say that he did not do the very thing charged. (3) Finally, he may 'plead by way of confession and avoidance', or say that he did do it; but that there were certain other facts, which he specifies and stands ready to prove, which change the legal aspect of the case. By compelling the defendant to make some one of these answers to the charge and then compelling both sides to stand by the question which the pleadings have developed, on pain of losing the case, the court succeeds in reaching an issue of law or of fact about which the two sides differ, and which is definite and simple enough to be settled reasonably. If the final issue be one of law—raised by a demurrer—it is settled by the judge after he has listened to argument. If it be one of fact, it is settled by the jury after they have listened to the evidence. In either case the argument or the evidence (as it may be) is addressed to the particular point at issue, not to the case as a whole, and upon it the whole case depends.

apparent in most discussions of politics and history. When it comes to these subjects the emotions connected with our party interests and party traditions make analysis especially difficult. There is nothing in a chemist's emotions to keep him from regarding a pail of water as really consisting of so many separate atoms of oxygen and hydrogen; but if a person has been brought up to revere the Roman Catholic Church or to hate the Democratic party, there is much in his emotions to prevent him from regarding the one or the other as nothing but a number of separate individuals who differ greatly from each other in many ways and are determined in their acts by all sorts of different motives and external influences. What is true of groups of persons is true also of groups of events. If we are accustomed to feel strongly about some great historical movement, like the Reformation or the Civil War, our emotions make it difficult for us to analyze it into several long series of separate acts, and to realize that while each individual act has its own particular moral relations the sum-total can hardly be said to have any whatever.

Issue before evidence.

Having made our question definite we must not attempt to answer it from the impression that the evidence made before we did so; for we only attend to the parts of a story that interest us, and details which may be of the highest importance for the settlement of some particular point may be very uninteresting in themselves, and therefore pass unnoticed when they are told before the point is raised. The only thing to do is to go over every bit of the evidence again with each new question and pick out the parts that have a bearing on it. Moreover, we should do this again every time an issue is split up into others more definite, or abandoned for others more appropriate. A story, for example, which was rejected as false in the preliminary stages of an inquiry may contain some phrase or some allusion that will explain the whole matter when the problem is reduced to its last analysis. This process of going over

all the evidence afresh with each new question is not so tedious as one might suppose, for the new question always makes us see the evidence in a new light.

The strength of our natural tendency to decide a case by our first impressions and to neglect the later evidence is well recognized by those persons who take care to have their own version of some quarrel told first. Here again the law recognizes our weakness, and not only provides that the judge and jury shall always hear both sides of a case, but also determines the order in which they shall be heard, and provides that after the evidence has all been given each side shall have an opportunity to review it and point out its bearings.

Must not be managed by the witnesses.

When we are determining the precise nature of our problem and sifting the evidence by which it is to be settled, the witnesses with whom we have to deal will often try to mislead us. They will appeal to our emotions. They will try to make us substitute some other issue for the real one. They will introduce all kinds of irrelevant matter to distract our attention, and when they do they will make it as interesting as they possibly can. At the same time they will pass as lightly and indifferently as possible over the points which are of real value for the decision of the case. If they are compelled to dwell upon them, they may try hard to make what they have to say so tedious that we will not listen to it, or so obscure that we shall stop trying to understand it. But in spite of all these artifices we must never let a witness determine what issue or what part of the evidence we shall attend to. To be sure we cannot put him on the rack to make him tell the truth; but we must always remember that it is we and not he who is judging, and therefore we must not say 'Yes' when we do not understand; we must not be afraid to cross-examine him on the points that we think essential (instead of those on which he invites us to examine him); and we must not stop the examination when the witness seems to

think it has gone far enough, for fear that if we do not he will think us hostile or distrustful or stupid.

Finally, if we really mean to settle our questions rationally, we must never settle them while we are confused or excited or while there is reason to believe that we have not yet found or sifted all the evidence worth considering. The impulse to have done with deliberation and get things settled one way or the other is very strong, and like every other impulse it has its value; but when we set out to decide a question on rational grounds it is out of place and we must resist it. Strangely enough, when we are trying to discover new truths the very presence of this impulse is the best possible sign that we should act against it, for it only arises when all the evidence will not fit together easily according to our preconceived notions, and that is the only circumstance under which there can be anything essentially new to discover. We must resist also the tendency to be hurried by the impatience of others. We cannot yield to it without sacrificing the independence which it was necessary to assume when we set out to judge for ourselves instead of blindly following the leader or the crowd. Indeed the attempts which others make to hurry us are sometimes only a device to keep us from finding the truth. If in any particular case there is not enough evidence to settle a question rationally, we should leave it unsettled—not settle it irrationally. A good way to guard ourselves against the tendency to jump to conclusions when the evidence is only half weighed is always to ask whether the evidence on which we act would satisfy some cooler critic to whom we might submit it. The best judges are those whose decisions are reversed least often by the higher courts.

Must not decide in a hurry.

CHAPTER XXXVI.

THE THREE ULTIMATE TESTS OF TRUTH.

Much has been said in the foregoing chapters about ways of testing truth. Something must be said now about the ultimate tests to which all others can be reduced. Of these there are three: Consistency, Conceivability, and Uniformity including Simplicity. We apply the test of consistency because we are rational beings who recognize that two incompatible states of affairs cannot both exist; we apply the test of conceivability because we have the gift of imagination as well as reason, and we believe that if certain relations are not imaginable, they cannot exist; and we apply the test of uniformity and simplicity because as active beings it is easier for us to do something old and simple than something new and complex. These three tests may not be equally decisive, but each has its own sphere, and in that sphere it is the best that we can get.

Consistency.

The test of Consistency was considered at length in the discussion of deduction. We saw that statements may be regarded as inconsistent not merely when they directly contradict each other, but also whenever one of them asserts the existence of a state of affairs that is incompatible for any reason whatever with the state of affairs asserted by the other. We saw, too, that in order to tell whether one supposed state of affairs really is incompatible with another we must know something about how the world is actually constituted. This appeal to the actual constitution

of things vastly increases the number of cases to which the test of consistency can be applied ; but it cannot help weakening it, for there is always a question of whether things really are constituted as we suppose, and before this question can be answered we are often thrown back upon one or both of the other tests, and that, too, perhaps by some process that is vague, indirect, or complicated.

Conceivability.

The test of Conceivability, or rather of Inconceivability, for it is the negative relation that is decisive, is illustrated best in geometry. We believe that two straight lines cannot enclose a space, because we cannot possibly imagine or picture two lines which look straight—*i.e.*, look as though they ran evenly in one direction throughout their whole course—and at the same time look as though they enclosed a space. We believe that the circumference of a circle is curved in one place precisely as in another because we cannot possibly picture a figure that looks as though it conformed with the ordinary definition of a circle—*i.e.*, looks as though every part of the circumference were exactly the same distance from some one point within it—and at the same time looks as though the curvature of the circumference were not the same throughout ; and because the more unequal the curvature of a circumference appears to be the more unequal also appear to be the radii.

What is true of the definitions, axioms, and postulates of geometry is true also of the demonstrations. They too appeal to the fact that we cannot picture it otherwise. Suppose it is a question of proving that the three angles of a plane triangle are together equal to two right angles. The student goes through the demonstration with reference to the figure given in the book, and then if he is not convinced he tries to draw some kind of triangle to which the demonstration will not apply, and he feels sure of the demonstration only when he has convinced himself that no matter how hard he tries he can never imagine a plane triangle so drawn. Thus the first principles and the demonstrations of geometry both depend

upon certain limitations of our imagination. We believe them to be true because we cannot picture the contrary.

There are two objections that may be raised against geometry as a demonstrative science. The first is that there may not be such a thing in the whole world as an absolutely straight line, an absolutely round circle, an absolutely perfect triangle, and so on. How can we say, the objector asks, that things must be so when in reality they may never be so? The second objection is this: Even if our geometrical conceptions do agree with things as they happen to exist in this world, how can we say that anything *must* agree with these conceptions when for all we know there may be some other world in which they do not agree at all—some world in which straight lines can enclose spaces and in which plane triangles do not have their three angles together equal to two right angles?

The answer to the first of these objections is that geometry is not primarily concerned with the question of whether things have certain forms or not, but only with the mutual relations of those forms themselves as we have to conceive of them. It does not say that such things as perfectly straight lines exist in the real world, but only that if they do, no one or two of them can enclose a space; and to find out what a straight line is we go to our own imaginations, and not to the world of reality. It is true that geometry uses real diagrams, but the force of a demonstration does not lie merely in the fact that we cannot put a certain combination of relations on a blackboard or a piece of paper without also putting in a second or leaving out a third; but rather in the further fact that we cannot imagine what the impossible combination would be like if we could put it there. It is not the physical impossibility, but the inconceivability, that appeals to us. If it were a mere question of physical fact, the force of every demonstration would depend upon the accuracy with which the figures are ruled and measured; but it does not. Whether perfectly straight lines and

perfectly round circles are ever found in the world is a question to be settled outside of geometry and wholly irrelevant to it.

As to the other objection—that in some other world the fundamental relations of figures in space might all be turned topsy-turvy—the answer is that if it were so and if we were moved there, unless we also were changed we could never see it. We should undoubtedly find that things did not come out as we expected, and it might be that we could stumble across some algebraic formulæ (like those applicable to the "fourth dimension") by which we could learn to adjust ourselves to their vagaries; but unless we ourselves were transformed or enlarged along with our world we should be no more able to see and imagine the new relations than a musician can hear and feel perfect harmony and melody in the chaotic strumming of a child. If we picture things at all, we have to picture them as we can. When we talk about lines and size and shape and direction our words have no meaning unless they refer to something which is at least partly picturable; and if we cannot even begin to picture a kind of shape or direction that meets a given description, we have a right to conclude that it is not picturable and therefore is not really a shape or direction at all. And this is just as true whether the shape and direction spoken of are supposed to be only imaginary or to be real. So that the test of conceivability which is applicable in the first instance to mere images as images can be applied also to the relations of things which are assumed to be imaginable.

The danger attending the test of conceivability is that we may apply it where we should not. Geometry deals only with the arrangement of points and lines in space, and if some relation has nothing to do with space, we must not deny its existence because we cannot make a drawing of it. Nobody denies the existence of a song because the tones cannot be arranged in triangles and circles, or of a feeling of remorse because we cannot picture it as lying to the right or to the left of the knowledge that goes along with it. In

the same way no one has any right to deny the existence of a soul merely because we cannot represent it as round or square, yellow or green, loud or soft, or literally sweet or sour.

Our third ultimate test of truth depends upon our animal organization as creatures who form habits and tend in other ways also to do whatever must be done for our preservation and welfare, in the shortest and easiest way possible. **The third test.**

We have seen how the tendency to make uniform reactions leads to the judgment that what we react upon is uniform, and how this uniformity that we find because we look for it is thought of as involving a vast number of permanent Things, each of which belongs to some definite Kind, and acts in the same Way as every other member of the Kind under the same Conditions. We have seen, too, how some apparent anomaly is 'explained' when it is shown that in spite of appearances it is still a case of these same uniformities; and how an interpretation of any particular experience which cannot show how that experience grows out of these uniformities is branded as false and unhesitatingly abandoned for one which can.

But the test of truth growing out of our organization as active creatures does not end here; for it may be that each of two or more explanations of an experience is consistent with these uniformities, and still we are able to choose between them, even though it be without quite so much certainty. **Uniformity in the mass.**

It will be remembered that we came to seek for uniformity in the ultimate elements of which things and situations are composed because we were often disappointed in our efforts to find it in the complex things and situations as a whole. But it was in the complex mass that we looked for it at first. The first things that we notice and learn to separate from the confused mass around us are not definite details which we afterwards build up into more complex wholes, but more

or less vague wholes which we afterwards analyze into the more definite details: we can recognize a person as a whole before we can describe a single feature. And so the first uniformities we expect are not uniformities in the definite details that we have not noticed, but in the vague wholes that we have. When two wholes present about the same general appearance they call from us the same rather vague and clumsy act, and we never think of fine distinctions between either the wholes to which we react or the reactions themselves until we find that a reaction which gives us what we want in one case will not do so in another. Then perforce, we are driven to notice some detail in which the two cases differed; and by a repetition of this process we reach at last the ultimate things and relations to which we feel that the law of uniformity applies without exception.

But in spite of the disappointments which teach us that it is only in the ultimate details that we can be absolutely certain of our uniformities, we still continue to look for them with considerable confidence in the mass also. 'Appearances', or general impressions, would not be 'deceitful' if they never deceived; and they could not deceive if we had no tendency to trust them. We should not hear people say, 'It is strange how much difference a little thing often makes,' if they were not in the habit of ignoring the little things and looking at the larger wholes.

Thus we never lose the tendency to look for uniformity in the general appearance of things as well as in their ultimate relations, and thus it happens that when there are two possible explanations for some state of affairs which are equally satisfactory in the various details, we naturally choose the one which gives the greater impression of uniformity in the mass. The 'catastrophic' theory of the earth's history, which supposed the structure of its crust to have been produced by a series of changes more violent than those with which we are familiar, might have given a perfectly satisfactory account of all the details under consideration, and yet

it would still have been abandoned as soon as Lyell showed that these details could all be accounted for just as well—I do not say better, for that is not my point—by processes of the sort that are going on around us all the time.

This tendency to prefer accounts of things which make what is distant in time and space as similar as possible to what we perceive about us is due in part to the law of association. The feeling of belief is subject to this law like any other thought or feeling; and the more a new conception is like one we are accustomed to believe in, the more it tends to arouse the same feeling of belief by its very likeness. In the same way, if a new conception bears a strong total likeness to one which we are accustomed to reject, we tend through the law of association to reject it. We act towards ideas precisely as we act towards people, liking those that look like our old friends, and disliking those that bear a strong resemblance to our old enemies.

Another reason why it is easier to believe in things like those to which we are accustomed than in those that are not is that it is easier to conceive of them. If we have no ear for music, we cannot realize what it is, and so we can hardly help thinking that the enjoyment which others profess to get out of it is largely affected; if we have never been on the edge of nervous prostration, we are likely to believe that the utter exhaustion which others sometimes say they feel is merely 'imagination' or an empty excuse for laziness; if we have never suffered great pain, we are inclined to think that it cannot be so very terrible; and if we have never done a generous deed or sacrificed ourselves for a principle, we are likely to find nothing in stories of such devotion but veiled selfishness and hypocrisy. We cannot realize what these things mean, and so we do not believe in them.

What is true of these simple feelings and motives is true also of things and situations that are more complex. We sometimes read in books on psychology that if we have a separate knowledge of each of the elements that enter into

some situation, we can put them all together and thus imagine the situation as a whole. But this is no easy matter. We may know the meaning of every word in a description, but unless the thing described bears some total resemblance to something with which we are familiar it is extremely difficult to conceive of the separate details at once and hold them together in such a way as to get anything like a reliable conception of the whole. Experience begins with wholes and not with parts; and it is a great deal easier to construct a new whole by modifying an old one something like it than by laboriously fitting and holding together a large number of details. Indeed this is the way we almost always proceed. And from this it follows that the more total resemblance a new thing is supposed to bear to something with which we are familiar the easier it is to conceive of it, and the less total resemblance it is supposed to bear to anything with which we are familiar the harder it is to conceive of it.* And when it comes to a question of what we shall believe, the relatively familiar conception is likely

* This of course is the psychological law of *apperception*, and it explains the old rule that definition should be by genus and difference rather than by an enumeration of all the generic qualities as well as the specific in detail.

Complexity and novelty produce a certain inconceivability, but it is relative, and must be distinguished very sharply from the absolute inconceivability that was discussed a few pages back. Even when relations are so very complex that we feel sure no human mind could possibly picture them all together (*e.g.*, the position of every atom in an ink-bottle), nobody believes that this kind of inconceivability proves the relations to be impossible. Because we cannot think of a great many things at the same time we never think of denying that they can exist together. We recognize that the discrepancy is due to our own weakness ; to a certain fluidity of mind which makes it impossible to hold more than a few relations steadily before us at once. The kind of inconceivability that we discussed before seems, on the other hand, to be due rather to a certain mental firmness or rigidity : the relations keep their place quite steadily; and somehow we cannot make them bend to fit each other.

to be present and available when the less familiar has become too vague and shadowy to be taken hold of.

Thus the more some new state of affairs resembles one to which we are accustomed the easier it is to believe in it, if for no other reason, merely because it is easier to conceive of it.

When two things or situations resemble each other closely as a whole as well as in many of their details, there is a fair presumption that they resemble each other also in some other detail. The close resemblance between Mars and the earth in many known respects makes it more or less probable that Mars resembles the earth also in being inhabited; and the close resemblance between the lower animals and man in a multitude of other respects makes it probable that those animals resemble man also in possessing consciousness. Arguments of this sort are said to be 'from Analogy'. Such arguments are never absolutely conclusive. **Analogy.**

In general, the larger the proportion of respects in which the things compared are known to resemble each other and the stronger the resemblance in each, the stronger is the argument. But where the matter in question is known to be specially connected with some known point of resemblance or difference, general resemblance or difference does not count for so much. However much the moon may be like the earth in other respects, the one fact that it has no atmosphere is proof positive that it has no inhabitants like ourselves.

Many so-called arguments from analogy are little more than metaphors, and quite worthless as arguments: *e.g.*, the argument that a mother country has a right to regulate the internal affairs of a colony because a mother has a right to absolute control over her young daughter; or the argument that parliamentary government is always bound to fail because 'victories may be won by a poor general, but never by a debating society'.

Any decidedly new idea may be rejected as 'absurd'; but when it is not merely new but also contrary to our accustomed modes of thought the rejection is more emphatic and the term seems more appropriate. To believe in something that is merely different from what we have been accustomed to requires that we shall form a new habit of thought; to believe in something directly opposed to what we are accustomed to requires that we shall break up an old one, and for people old enough to have fixed habits this is often much more difficult. We are inclined to object to it in precisely the same way as we might object to having our things put away in some new place, or wearing skirts instead of trousers, or living under a new kind of government, or leaving old friends and associating with old enemies. The new arrangement seems intrinsically bad, but in reality the badness lies only in its relation to us. It is bad merely because it is new, and a person accustomed to it would find ours just as bad. So it is with beliefs. Apart from contradictions there is no such thing as an idea which is absurd in itself, and when we reject a new view as absurd it is not because our reason sees the absurdity; on the contrary, it seems absurd because our habits compel us to reject it. Thus we use Absurdity as a test of truth, not because we have concluded beforehand that the test is reasonable, but merely because we are creatures of habit and cannot help it.

Absurdity.

Often the absurdity of a new view appears not so much in the bare outline as in the details. We try to think them all out according to the new idea, but we keep putting in the old ones until at last we get a hopeless confusion of the new and the old, and then we say, See how inconsistent and ridiculous this new view is! Thus when Columbus suggested that there might be land on the other side of the world as a kind of counterpoise to Europe, his critics replied that that was impossible; for if there were land there, the inhabitants would have to walk upon their heads (since their feet would

be up, towards the sky above Europe, and their heads would be down). This filling out of the details according to old habits is often almost as true of those who embrace the new idea as of those who reject it. The new doctrine may fill them with a grand enthusiasm; but when it comes to working out the details the old ideas keep coming back to give the new words their meaning: 'New Presbyter is but old Priest writ large', and a revolution of any sort retains vastly more of the past than those who are in it realize.*

Because the impression of Absurdity (as distinguished from contradiction and true inconceivability) depends upon a breach with established ways of thinking rather than upon any intrinsic imperfection in the idea itself, there is hardly an idea which has not seemed absurd to somebody at some time. The King of Siam in the traveller's story refused to believe the absurd tale that water sometimes got solid; to Aristotle it was absurd to suppose that slavery should cease to exist, or looms work without hands; and so with others it seemed absurd to suppose that the tradition of the elders might be superseded; that a good thing could come out of Nazareth; that a man should love his enemy; that a law-observing Pharisee might be no better at heart than a law-breaking publican; that the earth moves; that a man might rightly disobey the king; that there are no ghosts or witches; that slavery is not a divine institution; that men and monkeys had the same ancestors; that steam could be made to do work; that it is possible to travel sixty or eighty miles an hour, talk with people hundreds of miles away, see through solid boards, or read at your breakfast-table in America what they did in Europe on the same day at noon.

These examples show how unreliable the test of absurdity is; and yet, in spite of its unreliability, we should be absolutely helpless without it: the victims of every passing sug-

* This explains, too, the many 'rebukes' that Jesus had to give his disciples. Again and again the old ways of thinking would return and they had to be told, "Ye know not what manner of spirit ye are of."

gestion, utterly devoid of all stability of thought or purpose and absolutely incapable of anything like steady progress.

Just as novelty gives the impression of absurdity, so familiarity removes it. Repeat a suggestion or a statement often enough and the sense of incongruity gradually wears away and the mind at last becomes 'open' to it. In this way mere habituation—the mere continued effort to see things from the new standpoint—often does more to produce conviction than argument. With such a thing, for example, as James's doctrine that the bodily 'expression' of an emotion determines the feeling, and not *vice versa*, students may think they *see* how much force there is in the argument at once, but they really do not *feel* its force until after they get used to the conception. When they have learned to find their way around in it they may accept it genuinely, but not before.

Sometimes it is possible to remove the sense of absurdity from a conception almost instantly by showing that though it seems strange it is really very much like some other to which we are accustomed. That the Roentgen rays should penetrate solid boards seems very marvellous, and had the facts been less well authenticated those who heard the story when it was new might well have said it was much too absurd to be true; but when we are reminded that common light-waves pass through glass as thick and solid as the boards, it is easier to believe the story of the Roentgen rays.

Simplicity.

When two explanations are equally good in other respects we tend to choose the simpler, and we feel that somehow or other the choice is justifiable.

We tend to believe in a simple state of affairs rather than a complex, just as we tend to believe in a familiar rather than an unfamiliar—for one thing—merely because the former is clearly remembered when the occasion for explanation arises, while the latter is confused or forgotten. This is one reason. But then, again, even when two explanations are conceived with equal clearness and equally well remembered we cannot help regarding the simpler one as 'better'. All explanation

is an attempt to find uniformity behind apparent disorder or anomalies, and the simpler the total state of affairs which we suppose to be there the more apparent and striking is the uniformity. A simple explanation is always more 'beautiful' than a complex. Then, again, in the third place, we ourselves, as creatures of limited time and strength, are always accustomed to accomplish our own ends by the simplest means possible and to see others do the same. Simplicity of means thus becomes a very practical ideal which we seek both to realize and to find in all human affairs; and when we come to deal with Nature the force of habit carries us on to seek for it there also, as though we knew beforehand that Nature were the product of some intelligent being working for ends like ours according to our ideals. Leibnitz says: "True physics must be derived really from the source of the divine perfections. It is God who is the final reason of things, and the knowledge of God is no less the principle of the sciences than his essence and his will are the principles of beings. The most reasonable philosophers agree to this, but there are very few of them who can make use of it to discover truths of importance. . . . Far from excluding *final causes* and the consideration of a being acting with wisdom, it is from thence that all must be derived in physics." *

I do not think that one needs to know about God or to believe in him in order to seek for ends and simplicity and other forms of rationality in Nature; but rather, on the contrary, that it is because we cannot help seeking for these that we do believe in him.

The rule of simplicity in explanation was laid down as a formal principle by the Franciscan monk William of Occam, in these words: *Entia non sunt multiplicanda præter necessitatem.* † This is known as Occam's Razor, or the Law of

* Letter to Bayle, 1687. Duncan's "Philosophical Works of Leibnitz", pp. 35, 36.

† 'We must not assume the existence of any more things than necessary.' Occam died in 1347. Petrus Areolus, another Franciscan, who

Parsimony. He used the razor to good effect to get rid of the abstractions which the recognized philosophy of his time took for things.* But nowadays, when explanations are concerned quite as much with the laws of action as with the things that are supposed to act, we may interpret the principle to mean not merely that the things we assume to exist or to be present must not be more numerous than necessary, but also that the laws must not be more complex. The things we assumed would be unnecessarily numerous if we supposed that God created the world through the efforts of a dozen different grades of inferior spirits, when all the facts would be explained just as well on the assumption that he did it all himself. The laws we assumed would be unnecessarily complex if we supposed that the planets moved in cycles and epicycles instead of in simple ellipses, or that elements did not combine chemically in simple proportions, but according to some very complex law that led to the same practical results.

When we try to make every explanation as simple as possible we are confronted by several dangers. The first of these is that we shall find simplicity by ignoring some essential part of the facts we are trying to explain. The theory that every voluntary act is done for the sake of gaining pleasure or avoiding pain is extremely simple and it can be made to explain a great many facts of conduct; but it certainly ignores the things that people sometimes do, knowing all the time that they will lead in the end to far more pain than pleasure. The theory that fear is due to a perception of danger will explain a great deal; but it cannot explain why children often suddenly and spontaneously begin to be afraid of things that have never hurt them, or why many adults show inordinate

died in 1321, had said the same thing in these words: *Non est philosophicum, pluralitatem rerum ponere sine causa; frustra enim fit per plura quod fieri potest per pauciora.*

* "*Sufficiunt singularia, et ita tales res universalia omnino frustra ponuntur.*"

fear of such things as cats, mice, caterpillars, and common snakes, which they know perfectly well to be harmless. Thus the alleged explanation is too simple; for it ignores an important part of the facts to be explained.

The second danger is that of attaining simplicity in explanation by ignoring the relations of the facts we are trying to explain to the rest of the world. The ideal of all science and philosophy is the simplest possible connected view of the whole universe; but in trying to attain it we must not be penny wise and pound foolish. There is a difference, as Kipling says, between a team of good players and a good team of players; and there is a difference between a set of good explanations and a good set of explanations. A simple explanation of some one little fact that will not fit in with our explanations of the rest of the world is not so satisfactory as a more complicated explanation that will. Suppose, for example, that the old witchcraft and demonology or that the modern telepathy and spiritualism gave or could be made to give a perfectly simple, coherent, and well-articulated explanation of the particular set of facts with which they deal. The explanation given would be extremely unsatisfactory, and would doubtless be rejected by many scientists as wholly unscientific, so long as there was no way of fitting the telepathic and spiritualistic conception of things in with the vast and ever-growing mass of facts that are being continually explained and coördinated more and more closely every day from other points of view. If the facts that the telepathists and spiritualists deal with can be explained piecemeal by principles recognized elsewhere, no scientist doubts that such an explanation is far better than one by thought-transference and spirits, although the latter might be a great deal simpler so far as the one set of facts was concerned.

It was some such feeling as this of opposition to the introduction of totally new principles of explanation serviceable in only one special field that Newton expressed in his celebrated dictum: *Hypotheses non fingo.* The cause which we assign

for any known effect must be, he said, *vera causa:* a true cause, or one for whose existence we have more evidence than can be found in the special facts that it is invoked to explain.

A third danger connected with the principle of simplicity is the opposite of that just explained. It is undoubtedly true that a complicated explanation of the facts in some one field, that fits in with what we know about all the rest of the world, is better than a simple explanation that does not, and it is just as true that we must not accept the new set of causes until we have made a reasonable effort to explain the various facts in question by old ones; and yet, on the other hand, we must not carry our rejection of strange causes to an extreme. If the alleged new principle does nothing else, it may serve to hold together a mass of facts many of which would otherwise escape us or become hopelessly confused, and to keep us puzzled until we find some other explanation that is better. In this way even a false hypothesis is often better than none at all. And then, again, it may turn out that our disconnected principle is not false after all. It is true that our scientific ideal is a simple, well-coördinated view of the world as a whole; but it is also true that we are a long way from attaining it. What we possess in the way of knowledge is not so much one field that is always growing broader as a large number of fields each of which is growing out towards the others so that sometimes they meet; but there are still plenty of gaps, and we must not always refuse to cultivate some new field because we do not see how it can be joined with the rest. It is nearly as bad to try to make our different hypotheses consistent with each other too soon as not to try to do so at all. The Greek philosophers except Aristotle were always thinking about the sum-total of things, and Aristotle is the only one of them who made anything like a respectable contribution to science. Descartes was thinking about the ultimate relations of mind and matter, and his general standpoint compelled him to say that the lower animals were mere automata: mechanical toys that cried when you

kicked them, just as a bell rings when you shake it, without any feeling whatever.

Thus the Principle of Simplicity or Parsimony is one that we are compelled to follow, but often it is hard to tell whether we will not reach it sooner in the end by leaving it for the moment.

Of course it is one thing to be so organized that the simple and familiar are more easily believed in than the complicated and unfamiliar, and a somewhat different thing to accept the formal principle that where there are two theories, equally good in other respects, that which assumes the simpler and more familiar state of affairs is the more likely to be true. And yet if we have the organization, we can hardly avoid the principle. To say that the simpler and more familiar is more easily believed in means that in most cases we do believe in it, or, in other words, that in most cases the relatively simple and familiar state of affairs is what we call the 'true' one; and now, a large number of particular cases being settled, all we have to do is to compare them and we reach our formal principle: The theory which supposes the simpler and more familiar state of affairs is more often true than the other.*

The right to assume these principles.

It is because we act on this principle of discrediting the new, whether we ever state it in words or not, that the phrase 'a very *strange* story' generally means a lie.

So much for the fact that we actually do use Consistency, Conceivability, and Uniformity and Simplicity as tests of truth, and even for the further fact that we come to think we have a right to. But have we this right? Is it possible to prove that there is such a relation in the world as com-

* We get the principle when we reflect enough to see the results of our own organization (*i.e.*, to see that all our explanations are relatively simple) but not enough to see that they are the results of that organization (*i.e.*, that these explanations are simple because the simple ones are those we chose). When we see that the question takes a new form.

patibility or incompatibility? Can we prove that anything real conforms or ought to conform to the laws of our imagination? Hardest of all, can we prove that because something is uniform or simple and therefore easy for us to think of, it is any easier on that account for it to exist? And if we cannot prove these things, have we any real right to use the tests? My answer so far as uniformity is concerned has been given already in Chapter XXII. I believe that we cannot prove these things, and therefore cannot prove our right to use the tests. We simply take them for granted and take the right to use the tests for granted along with them.

The time has gone by long since when wise men sought for a philosophy without assumptions; and if we cannot get along without them in philosophy, we certainly cannot get along without them in common life and in the logic that tries to serve as a guide for common life. If any one believes, as Kant did, that not only colors and sounds and smells and tastes, but also Space and Time with all their relations of shape, size, distance, direction, duration, coexistence, and succession, are purely human ways of imagining things and do not really belong to things in themselves at all, then that person ought not to use conceivability as a test of what things in themselves can or cannot be. If any one has meditated about these topics so long that it no longer seems to him absurd to doubt the simplicity and general rationality of Nature, I do not see how it is possible to restore his faith by demonstration; and if he has emancipated himself from the bonds of habit so completely that he really doubts the existence of those principles of uniformity that we indicate by the words Thing and Kind and Cause and Law, then there is certainly no way of proving to him that one explanation is better than another, or indeed that anything needs explanation at all. If, finally, he really and truly doubts—as Descartes tried to—the existence of everything but his own passing thought, I do not see how we could prove to his satisfaction that there is a real nature of things in virtue of

which one supposed fact is incompatible with another, and therefore that there really is such a thing as a contradiction.

If people actually doubt all these things, we do not argue with them: we lock them up instead, and by this resort to brute force we confess defeat in the field of pure argument. If, on the other hand, they profess to doubt them, but show by their acts or their argument—perhaps by the very fact of talking with us—that when they are off their guard they really take them for granted, the most we can do is to point out the inconsistency, with the hope that their faith in law and a nature of things is so strong at bottom that inconsistency in an argument will seem to them a fault. We cannot go beyond this appeal to faith. Thus we start with faith in experience as a whole, however vaguely we may conceive of this experience; but such a faith implies also faith in all the ultimate principles that that experience involves.

The limits of proof.

There are some questions which logic and its tests of truth will never help us to settle. If it is a question whether a certain state of affairs exists, and if there is no objection so far as consistency and conceivability are concerned to believing that it does, the only way of settling the question is to ask whether any one has observed the state of affairs itself or anything that can be recognized as its necessary cause or effect. If we cannot observe the state of affairs itself and if we cannot prove that anything which we do observe must be connected causally with something of the sort, we cannot prove that it exists. But if the state of affairs in question is one that might exist without being observed or producing unmistakable effects, we cannot prove either that it does not exist. It may be that there are mountains on the other side of the moon; but we cannot see them, and we do not know of any change that they would make in what we do see if they were there; consequently we cannot tell whether they are there or not. It may be that plants feel, that they enjoy the sunlight and the rain and suffer discomfort in the cold; but feelings can

never be perceived directly except in ourselves, and we have no idea what change their presence or their absence would make in the behavior of a plant, and so we can never know whether plants have them or not. It may be that every living soul existed on the earth before in the body of some man or beast although it has forgotten what took place in its life there; but no one knows how the life of a reincarnated soul should differ from that of one that never lived before, and so no one can ever tell from what he observes whether the theory of transmigration is true or not. It may be, finally, that the world was created in infinite love and wisdom and that all our human experiences are intended as preparations for some glorified life after death; but no one knows enough about infinite love and wisdom to say how a world made by it would differ from any other world, or how a life intended by the Creator as a preparation for another would differ from one which was not; and therefore nothing that we observe in the world can prove the matter one way or the other.

What we believe about questions like these is not inferred by logical processes from what we observe in the world; but is added to what we observe, as a matter of religious faith. Such a faith gives our experiences a new kind of significance; but no particular experience of the sort that science deals with can either confirm or refute that faith; and this is why different people often give diametrically opposite religious interpretations to the same concrete experiences.

It often seems indeed to persons of a certain temperament or character that some observable fact is a sufficient proof of what they believe about religion; and perhaps they make long abstract arguments to show it; but abstract arguments are hard to criticise in any case, and it requires an unusual amount of intellectual honesty and energy to seek for obscure fallacies in arguments that profess to prove what we already believe or wish to believe; and so this appearance of logical proof is easy enough to account for.

There is only one way in which logic or a knowledge of

scientific method can help us in matters of this kind. It can show us its own limitations, and save us in this way, on the one hand, from the trouble of constructing long and labored arguments that do not prove anything, and, on the other, from the error of supposing that the absence of proof in favor of certain beliefs can be taken as any argument against them.

Science and faith, particularly religious faith, are often supposed to be in some kind of logical conflict with each other. But this is a mistake. Science itself rests on faith, for it assumes, as we have seen, that there is such a thing as a world beyond one's own individual sensations, and that that world really possesses the uniformity and coherence which we feel impelled, as creatures of habit, to read into it. Indeed what we call scientific 'knowledge' is simply an interpretation that we give to our experience or some of it on the basis of this faith. But a faith in uniformity and coherence is certainly not inconsistent with a faith in purpose and wisdom and goodness also, and the mere fact that science does not attempt to unify experience from the standpoint of this latter faith as well as from that of the former is no reason why the scientist should not do so when he leaves his science and begins to think of something else. Religion cannot tell what the world looks like, and science cannot tell anything about its spiritual values; but for this very reason the one is no more inconsistent with the other than geometry or psychology is inconsistent with æsthetics or jurisprudence.

NOTE.

Some of the following examples for which no credit is given are used in many different books, and I have not taken the trouble to trace their origin. The initials after others should be interpreted as follows:

C, for J. E. Creighton's "Introductory Logic";

D, for N. K. Davis's "Theory of Thought";

F, for Thomas Fowler's "Elements of Deductive Logic";

H, for J. H. Hyslop's "Elements of Logic";

J, for W. S. Jevons's "Elementary Lessons in Logic", or his "Studies in Deductive Logic";

W, for Archbishop Whately's "Elements of Logic".

EXERCISES.

WITH every chapter students are advised to make use of the marginal summaries or the table of contents to assist them in review, thus: What are the two kinds of thinking? What is the nature of judgments and propositions, and how are they related to each other?

CHAPTER I.

1. Estimate the value of the following arguments:

(*a*) Tolstoi's interpretation of Christianity must be false; for if it is correct we and our ancestors have not been Christians at all.

(*b*) You may have an ape for an ancestor if you want to; but I don't care for one.

(*c*) "To believe that man was aboriginally civilized and then suffered utter degradation in so many regions, is to take a pitiably low view of human nature. It is apparently a truer and more cheerful view that progress has been much more general than retrogression; that man has risen, though by slow and interrupted steps, from a lowly condition to the highest standard as yet attained by him in knowledge, morals, and religion." (Darwin's "Descent of Man", end of Chap. V.)

(*d*) Of course you ought to be good, for you belong to a church and go to prayer-meeting; but I make no professions.

(*e*) If he thinks it is wrong, it is wrong for him; but with me the case is very different, for I never thought it was wrong.

(*f*) It is all very well for you to talk; but if these ideals of yours are so very beautiful, why don't you put them into practice?

(*g*) 'Is there a hell?'—That depends altogether upon your point of view.

(*h*) "But if there be no resurrection from the dead, then is Christ not risen; and if Christ be not risen, then is our preaching vain and your faith is also vain." (I. Cor. xv. 13, 14.)

(*i*) You say that 'of' is a preposition, but that cannot be; for in the very sentence in which you say so it is a noun.

(*j*) 'We know what we mean by clear water or a clear atmosphere, then why shouldn't we know what we mean by clear ideas?' 'I suppose you mean that a hazy atmosphere is one that you can see and that a clear atmosphere is one that you can not see, and that in the same way the clearest ideas are the only ones that we can not see.'

(*k*) Charlemagne is king; king is four letters; therefore Charlemagne is four letters.

(*l*) I cannot accept your opinion as true, for it seems to me that its general recognition would be attended with the most injurious consequences to society. (F.)

2. What is the best way to persuade the slothful man that there is no lion in the way?

3. Is there any objection to the following?—"General names are predicable of individuals because they possess certain attributes; to predicate the possession of those attributes is the same thing as to predicate the general name."

CHAPTER II.

1. What is the meaning of the word 'doubt' in the following lines?—

> Doubt that the stars are fire;
> Doubt that the sun doth move;
> Doubt truth to be a liar;
> But never doubt I love.

2. Explain the following:

> The good are envied of the bad, and glory finds disdain,
> And people are in constancy as April is in rain.

3. What is the meaning of the word 'or' in the following sentences?—'John or James will do'; 'Sodium chloride or common salt is always present'; 'Clergymen or lawyers always get in',

4. What is meant by the following statements, and in what sense is the second of them true?—

(*a*) Our team wins every year, except in off years.

(*b*) No one's burden is ever greater than he can bear.

5*a*. What is meant by 'the people' in the phrase "Government of the people, for the people, by the people"?

5*b*. What was meant by the statement that all men are born *free* and *equal?* In what sense of the words is it true?

5*c*. In the exercises to Chapter V are two passages from Bacon. Find the meaning of all the words used there in an obsolete or peculiar sense.

6. Define the terms 'Wealth', 'Capital', 'Society', 'Friend', 'Courage', 'House', 'Savage', 'Nice', 'Lady', 'Pious', 'Poor man', 'The weak', 'Educated', 'Circle', 'Triangle', 'Sophomore'. Why are the last three words more easily defined than the others?

7. Criticise the following arguments:

(*a*) The college chapel is sacred; for a chapel is a kind of church, and the Church is a divine institution.

(*b*) This stove saves half the ordinary amount of fuel; therefore two such stoves would save it all.

(*c*) A. B. is professor of religion in the university, but as he is very irreligious he does not possess the religion he professes; he is therefore a hypocrite.

(*d*) All presuming men are contemptible; this man therefore is contemptible, for he presumes to believe his opinions are correct. (J.)

(*e*) It is good to be great and it is right to be good; Napoleon was great; therefore it is right to be like Napoleon.

(*f*) A thoughtless person does not think; one who does not think cannot form a judgment; therefore a thoughtless person cannot form a judgment.

(*g*) It is wrong to criticise an inspired book; the historians apply criticism to the Bible; therefore they do wrong.

(*h*) No evil should be allowed that good may come of it; all punishment is an evil; therefore no punishment should be allowed that good may come of it.

(*i*) Whatever is dictated by Nature is allowable; devotedness to the pursuit of pleasure in youth, and to that of gain in old age, are dictated by Nature; therefore they are allowable. (W.)

8. A hunter is trying to shoot a squirrel that is clinging to a stump, and goes all around the stump to find him; but the squirrel always keeps on the opposite side of the stump. Does the hunter go around the squirrel?

9. Which hen is the mother of the chicken, the one that laid the egg, or the one that hatched it out and takes care of it?

10. What is the true significance of the last two questions?

11. Analyze the following explanations, arguments, and definitions:

(*a*) These two elements almost always unite when they are put together, for they have an affinity for each other.

(*b*) It is harder to slide on wood than on ice, because the friction is greater.

(*c*) Movement catches the attention, and therefore we are more likely to notice a moving object than one that remains stationary.

(*d*) It is always wrong to lie; for any departure, for any reason whatever, from the one invariable law of absolute veracity is always reprehensible.

(*e*) An archbishop is one who discharges archiepiscopal functions.

(*f*) Habit can be defined as 'the purgatory in which we atone for our past misdeeds'.

(*g*) "A definition is the exposition of the connotation of a term."

(*h*) Definition is the enumeration of the parts of an idea.

(*i*) The process by which the qualities that belong to an object is defined is called logical definition.

(*j*) "Despite the hot weather two cases of smallpox were discovered yesterday. . . . The discovery of these occasional cases, the health authorities say, is evidence that the plague is still prevalent in the city."

12. Make five arguments that turn upon ambiguous words, five purely verbal explanations, and five purely verbal definitions.

13. Define all the important words in the Declaration of Independence or some other connected discourse.

14. Is any fault to be found with any of the following rules for definition or the way in which they are stated?—

(*a*) A definition should contain the essential attributes of the word or object being defined.

(*b*) A definition should contain an adequate description of the term defined.

(*c*) The subject of a definition must be exactly equivalent to the predicate.

(*d*) A definition must not be defined by figurative or ambiguous terms: Death is an undisturbed *sleep* (figurative).

(*e*) The definition must be equivalent to the term defined: Rhubarb is a plant (which can be eaten).

(*f*) A definition must state the essential qualities of the term defined: A classical student is one who has studied Latin (Greek is also essential).

(*g*) Give all the essential qualities of the species to which the object defined belongs.

15. Criticise the following definitions:

(*a*1) A JUDGMENT is a conclusion reached after considering certain previous propositions upon the same subject but presenting different views.

(*a*2) Judgment is the conclusion arrived at after correct, logical analysis of a matter.

(*a*3) Judgment is estimating the relative value of things.

(*a*4) Judgment is the comparison of two or more simple apprehensions.

(*a*5) A judgment is an inference made from material gained by reasoning.

(*a*6) Judgment is logical conclusion drawn from given hypothesis.

(*a*7) Judgment is the process of clear thinking, of estimating accurately, and finally of reaching a decision.

(*a*8) A judgment is the expression of a belief, conclusion, determination, and the like.

(*a*9) Judgment is a statement which affirms or denies something.

(*a*10) Judgment is that act of the mind by which we deduce from two statements a third.

(*a*11) Judgment is anything which when stated will form a proposition.

(*a*12) Judgment is a decision as to the truth or falsity of a statement.

(*b*1) A PROPOSITION is a phrase that expresses a thought in form of a subject and predicate; a judgment, the predicate giving some relation, condition, or quality of the subject.

(*b*2) A proposition is a thought expressed in the form of a sentence, consisting of subject and modifiers, predicate and modifiers, and copula.

(*b*3) A proposition is a statement which contains a subject and predicate and forms a part of a syllogism.

(*b*4) A proposition is an equation consisting of a subject term, a predicate term, and a copula.

(*b*5) A proposition is a statement of a true or false fact.

(*b*6) A proposition is an assertion which expresses a declaration either positive or negative.

(*b*7) A proposition is a statement in the form of a grammatical sentence which affirms or denies a fact.

(*b*8) A proposition is a statement that asserts something.

(*b*9) A proposition is a statement which must contain a subject and predicate; in which the predicate asserts or denies or questions a relation existing between it and the subject.

(*c*1) SUBJECT of a proposition is that of which something is affirmed.

(*c*2) The subject of a proposition is the word about which something is said; *e.g.*, The *book* is lying on the table.

(*c*3) The subject of a proposition is the object or objects about which a statement is made.

(*c*4) The subject of a proposition is that of which something is affirmed or denied by the verb and rest of the proposition.

(*c*5) The subject of a proposition is the object of discourse.

(*c*6) The subject of a proposition is the name of an object or substance concerning which a statement is made in the predicate term.

(*c*7) The subject of a proposition is a word or group of words about which something is said.

(*c*8) The subject of a proposition is that about which some statement is made. It includes the word and all of its modifiers.

(*c*9) The subject of a proposition is a word, phrase, or clause of which something is affirmed or denied in the predicate.

CHAPTER III.

1. Summarize the opening lines of "Paradise Lost" in the plainest and most concrete language possible, getting rid of all the figures of speech and abstract terms, but giving their meaning in the simplest prose. Make summaries also of as many paragraphs as possible from Berkeley's "Principles of Human Knowledge", of the editorials in the morning paper, and of a couple of pages from some such book as Mrs. Eddy's "Science and Health". In making these summaries be careful to put in all the little connecting words, such as 'and', 'or', 'because', 'therefore', 'in other words', etc.; use your own language, not that of the author summarized; and take care to make your meaning absolutely clear and unambiguous, even if you have to give several alternative interpretations to a passage.

Why is it that some of these summaries are harder to make than others? Is it because the subject-matter is more difficult or because the writer's ideas are more hazy?

2. Explain the meaning of the law of gravitation; of the formulæ on pp. 170, 175, 263, 265, 278, and 292 of this book; and of any other formulæ that seem rather hard and complex.

3. Determine exactly what was meant by each one of the Ten Commandments, and show where we might commit a fallacy of Accident or Accent in interpreting them.

4. What fallacy did Bismarck commit in his abridgment of the Ems despatch? and what fallacy did Columbus commit when he proved that an egg could stand on end? (J.)

5. In what sense is it true and in what sense is it false that "There is nothing either good or bad, but thinking makes it so"?

6. What fallacy, if any, is to be found in the following arguments:

(*a*) You say now that you never believed in the war with Spain, and yet you voted for it; for you voted for the Republican ticket when their platform had already declared in favor of intervention, and everybody knew that intervention meant war.

(*b*) It is right to help the needy; therefore it is right to give to this beggar, for he is needy.

(*c*) In going around the world westward we keep gaining time, and the whole trip would gain us a full day; therefore if we could make the complete journey in twenty-four hours it would really take us no time at all.

(*d*) It is not true that on the first of next month John Smith will become the husband of Miss Brown; he will become the husband of Mrs. John Smith.

(*e*) No cat has nine tails; one cat has one tail more than no cat; therefore one cat has ten tails.

(*f*) Keeping a promise is right; Herod kept his promise when he killed John the Baptist; therefore he did right.

(*g*) Peter is a saint; therefore his denial of his Lord was the act of a saint.

(*h*) You are not what I am; I am a man; therefore you are not a man. (J.)

(*i*) He that says you are an animal speaks truly; he that says you are a donkey says that you are an animal; therefore he speaks truly.

(*j*) "Blessed are the poor in spirit." This man is a thief and a liar, but he is poor in spirit; therefore he is blessed.

(*k*) It is a well-recognized maxim that the king can do no wrong. Unless the maxim is false it was not wrong for the king to say what he did.

(*l*) He who is most hungry eats most; he who eats least is most hungry; therefore he who eats least eats most. (Aldrich.)

(*m*) Nuisances are punishable by law; a noisy dog is a nuisance; therefore a noisy dog is punishable by law. (D.)

(*n*) Haste makes waste, and waste makes want; a man therefore never loses by delay.

(*o*) A brick house is cooler than a frame house; therefore this house is cooler than that one.

(*p*) Whatever restricts liberty is bad; government restricts liberty; therefore government is bad.

(*q*) Interference with another man's business is illegal; underselling interferes with another's business; therefore underselling is illegal. (D.)

7. Criticise the following definitions:

(*a*) EQUIVOCATION is the use of the same word in two different senses in a syllogism.

(*b*1) A FALLACY OF ACCIDENT occurs when a conclusion about a special object is drawn from a general premise,

(*b*2) From a general to a special case.

(*b*3) The fallacy of accident is drawing a conclusion concerning the species from propositions concerning the genus.

(*b*4) Fallacy of accident is when in an argument you go from a general rule to a special case.

CHAPTER IV.

1. Criticise the following divisions:

(*a*) Human beings into men, women, negroes, boys, females, and cranks.

(*b*) Religions into true and false. (C.)

(*c*) Animals into bipeds, quadrupeds, birds, monkeys, and crawly things.

(*a*) Quadrilateral figures into squares, rectangles, parallelograms, and rhomboids. (F.)

(*e*) The fine arts into painting, drawing, sculpture, architecture, poetry, and photography. (F.)

(*f*) Governments into monarchies, tyrannies, oligarchies, and democracies. (F.)

(*g*) The sciences into physical, social, ethical, logical, and metaphysical. (F.)

(*h*) Men into fools and knaves.

(*i*) Skin-diseases into those that are cured by zinc ointment and those that are not.

2. "The following were the classes of persons which were in 1868 qualified to vote in one or other of the United States of America: Male citizen, male inhabitant, every man, white male citizen, white freeman, male person, white male adult, free white male citizen, free white man. Form a scheme of logical division which shall have a place for each of the above classes." (Jevons' "Studies in Deductive Logic".)

3. Criticise the following definitions:

(*a*1) GENUS is a class or group which contains all the fundamental qualities which are necessarily possessed by the species in order to make it one of a certain genus.

(*a*2) A genus is a class of objects made up of a collection of other classes, these objects being in their separate classes because of common characteristics.

(*a*3) Genus is a class of higher objects.

(*a*4) Genus is a class made up of several specific classes.

(*a*5) A genus is a name applied to a number of things made up of several (two or more) species.

(*a*6) Genus is a classification of objects which can be divided into species. It is the largest division of objects.

(*a*7) Genus is the name of a class of objects which may be divided into species and subdivided.

(*a*8) Genus is a name of a class of objects which contain two or more species.

(*a*9) Everything is a species of some larger class genus. Thus genus is class made up of smaller class species.

(*b*1) SUMMUM GENUS is the largest division possible of a certain class of objects.

(*b*2) Summum genus is the greatest of all genuses in extension.

(*b*3) By summum genus is meant highest division which can be made according to any given basis. For instance, on a basis of intelligence, man is the summum genus of living beings.

(*b*4) By summum genus is meant the highest division, *i.e.*, the division which embraces the greatest number of divisions which may be subdivided, *e.g.*, Animal.

(*b*5) Summum genus or genus generalissimum is the higher class or division which is divided into species.

(*c*1) DIFFERENTIÆ are the attributes of the individual and not common to the class.

(*c*2) Differentiæ are fundamenta divisionis.

(*c*3) Differentiæ are the attributes by which objects or classes of objects are distinguished from each other.

(*c*4) Differentiæ are the attributes by which one genus differs from another.

(*c*5) Differentiæ is the thing upon which depends the putting or leaving out an object from a class.

(*c*6) Differentiæ are differences which objects of a certain class have from each other.

(*c*7) Differentiæ is the name given to the difference found in the attributes.

(*c*8) The differentiæ of a word or object express the difference between the object and the class to which it belongs.

(*c*9) Differentiæ is the name of the attributes in virtue of which a species is subdivided.

(*c*10) Differentiæ are the attributes which must be known to

distinguish one species from another which belongs to the same genus.

(*c*11) Differentiæ distinguish the species of the genus.

(*c*12) Differentiæ are the terms by which one object is distinguished from another.

(*d*1) ACCIDENT is that quality which distinguishes one genus from another.

(*d*2) Accident is a quality which belongs to the class genus, but may also belong to some other class.

(*d*3) Accident is a quality which certain terms have and certain other terms do not.

(*d*4) Accident is a property which a thing may or may not have and does not affect the class to which the thing belongs.

(*d*5) An accident may be found in one species of a class but not necessarily in the others.

(*d*6) Accident is that which may be true of one object of a class, and not true of the other objects of that class, and is not needed to define that object.

(*d*7) An accident is a quality which belongs to an object permanent or temporary.

(*d*8) An accident is a relation which belongs to an object temporarily or which may belong to that particular object on account of circumstances.

(*d*9) An accident is an attribute which may or may not belong to individual members of the same species.

(*e*1) PROPERTY is a term applied to the qualities denoted by the term.

(*e*2) Property is a quality which is found in every species of a genus.

(*e*3) Property consists of the quality or qualities which must be added to the species to make it a genus.

(*e*4) Property is a quality that belongs to a genus, but not one for which the genus is especially known.

(*e*5) A property of a word is one of its qualities.

(*e*6) Property is a quality or qualities which an individual must possess in order to be distinguished from any other individual.

(*e*7) A property of a term is something that belongs to it but must not necessarily be included in the definition.

(*e*8) Property is a quality that belongs to every member of a class.

(*e*9) Property is a quality which a class of objects may possess, but which does not need to be defined.

(*e*10) Property is something which is true of an object, but which does not define that object.

(*e*11) Property of a genus or species is a quality that belongs to that genus or species alone and distinguishes it from all other genera and species.

(*f*1) A CROSS-DIVISION is the classification of a term into its genus and species.

(*f*2) Cross-division is where the word may belong to more than one species at once and these different species cross each other and make it difficult to get any good order out of them.

(*f*3) In making a division of a class of things or genus into species, if one of these species is made to include another there is a cross-division.

(*f*4) Cross-division is analysis from a general term to a specific, and from that specific used as a general term to its specific, until the last real specific can be found, but by not, in one or more cases, taking the next specific nearest to the general term.

(*f*5) Division consists in placing different things into classes on some one basis of division. Cross-division occurs when division is made on two or more bases.

CHAPTER V.

1. Consider each of the following words as it occurs on page 217 and determine whether it is a term or not, demonstrative or descriptive, connotative or non-connotative, singular or general, collective or distributive, abstract or concrete, positive or negative, relative or absolute:

Taxes, Supporting, The church, Party, Fact, The forgotten man, Woman, Five, Letters, Corset-stitchers, Seventy-five cents, Machine, Grade, Prohibitory, As, To, All, Time, Labor, Enhancement, Price, The tax, Best terms, Art, Commerce, Today.

2. Criticise the following definitions:

(*a*1) A TERM is a word or group of words that denote a quantity, state, action, or relation of an object or class of objects about which something is affirmed or denied.

(*a*2) Term is the name of an object or group of objects, the

quality or action or condition of the object about which we are thinking.

(*a*3) A term is the name of an object or group of objects denoting a quality, state, action, relation, or condition about which we are talking or thinking.

(*a*4) A term is the name of the quality, of the thought of the relation of the thing about which we are talking.

(*a*5) A term is the name of an object or a group of objects about which something is affirmed or denied.

(*a*6) A term is the name of a state, condition, or relation which is affirmed or denied of an object.

(*a*7) A term is the name of a quality, relation, or state under discussion. In a proposition the subject and predicate form the two terms.

(*a*8) A term is a name of an object about which we are thinking, asserting, or questioning some relation, quality, or condition.

(*b*1) A GENERAL TERM is one that applies to all the objects belonging to the class to which the term applies.

(*b*2) A general term is one in which all the objects included under a given general name are designated.

(*b*3) A general term is one applicable to each of a certain number of similar things.

(*b*4) A general term is one which distinguishes a particular class of individuals as such.

(*b*5) A general term is one which can be applied to any one of a number of specified individuals.

(*b*6) A general term is one expressing all the individuals as such.

(*b*7) A general term is one which does not designate an aggregate of similar and separable things considered as a temporary unit.

(*b*8) A general term is one denoting as such the kind of objects.

(*b*9) A general term is one used to designate a class of objects which are not separable.

(*b*10) A general term is one which designates all of a class or group of objects.

(*c*1) A SINGULAR TERM is one which has but one signification.

(*c*2) A singular term is one used to apply to any one of a class of objects.

(*c*3) A singular term is one which points to one specified object or class of objects.

(*c*4) A singular term is a name applied to one object to distinguish it from the other objects of its class.

(*c*5) A singular term is one which is applied to a single individual as such.

(*c*6) A singular term is one which is applied to a singular individual as such.

(*c*7) A singular term concerns a particular individual as such.

(*c*8) A singular term is one expressing a specified individual as such.

(*c*9) A singular term is one which is applied to a particular designated individual.

(*c*10) A singular term is the name applied to a certain designated object.

(*c*11) A singular term is a name applied to a certain distinguished object.

(*c*12) Singular term is applied purely arbitrary to individuals.

(*c*13) A singular term is one which is applied to a certain designated object and no other in the same sense. "This man is tall." Man is a singular term.

(*c*14) A singular term is one which applies to only one individual or object.

(*d*1) A COLLECTIVE TERM is a term made up of individuals thought of as separate independent objects.

(*d*2) A collective term represents a number of singular units taken as a whole.

(*d*3) A collective term is a term applied to a number of similar units, as in case of a mob.

(*d*4) A collective term is a term applied to a group of individuals.

(*d*5) A collective term is one in which a class of objects taken as a whole is designated.

(*d*6) A collective term is one applicable to a whole class taken together.

(*d*7) What is said of a collective term is said of all the objects comprised in the term taken as a whole.

(*a*8) A collective term is one which may be applied to a number of objects taken as a whole on the ground of some common likeness.

(*d*9) A collective term is one which includes a number of singular terms.

(*d*10) A collective term is one consisting of an aggregate of separable and single individuals considered as forming some sort of temporary unit.

(*d*11) A collective term is one used to designate a group of objects which may be separated.

(*d*12) A collective term expresses an aggregate of a number of similar objects, regarded as a temporary unit.

(*d*13) A collective term is one which can be applied to a group of specified individuals taken as one.

(*d*14) A collective term is one which is made up of a number of similar terms.

(*e*1) A DISTRIBUTIVE TERM is one which expresses universality; as, All men are mortal. *All men* is distributive.

(*e*2) A distributive term is one which includes all the objects of the class specified.

(*e*3) A distributive term is one which we do not think of as being made up of a number of similar elements.

(*e*4) A distributive term is the name applied to a number of similar objects thought of as a whole.

(*e*5) A distributive term is a name applied to a number of objects not thought of as separate, individual, similar things.

(*e*6) A distributive term is a name applied to objects thought of as made up of a number of similar, separable, and independent elements.

(*e*7) A distributive term indicates some of the objects of a class as such.

(*f*1) An ABSTRACT TERM is one which expresses an indefinite relation or object, such as disobedience or power.

(*f*2) Abstract has in it the idea of quality and attribute.

(*f*3) Abstract terms are nouns which express some sort of relation.

(*f*4) Abstract term is that which is not the name of matter but expresses an active relation.

(*f*5) An abstract term is a noun which involves relations between things without expressing or naming them.

(*f*6) Abstract signifies a quality, act, or state of a thing.

(*f*7) An abstract term is one which cannot be thought of without its relation to something else being considered.

(*f*8) An abstract term is one, used in a grammatical sentence,

which does not necessitate the introduction of an object to which it is applied into the thought of the proposition.

(*f*9) An abstract term is the name of a quality not involving the mentioning of the thing itself.

(*f*10) Abstract term is a term that can be used without the object being mentioned to which it applies.

(*g*1) A CONCRETE TERM is an adjective, verb, adverb, or preposition completing the meaning of a verb.

(*g*2) Concrete terms are names of things, verbs, adjectives, and other parts of speech.

(*g*3) Concrete term is the names of nouns, adjectives, and verbs.

(*h*1) A RELATIVE TERM is one that expresses a relation, as father and son.

(*h*2) A term may be relative in three ways. 1st, the relation between itself and an invariable standard; 2d, the comparison with a certain variable standard.

(*h*3) A relative term is one whose relation with another term is so close as to always involve the apprehension of that term as a part of a larger whole.

(*h*4) Relative term is one that expresses one party of a relation usually active.

(*h*5) Relative term is one which denotes relationship.

(*h*6) If the standard changes when the same term is applied to different things, that term is relative.

(*h*7) Terms are relative when they express comparison with a variable standard, and also a relation between two objects which are mutually dependent in meaning.

(*h*8) Relative terms show a dependence upon, or connection with, some other term or object.

(*i*1) An ABSOLUTE TERM is one which expresses quality belonging absolutely to that object.

(*i*2) An absolute term is one which does not express any relation.

(*i*3) If the standard remain the same when the same term is applied to different things, the term is absolute, *i.e.*, perfect.

(*i*4) An absolute term states a relation between things referred to a standard which does not vary.

(*i*5) An absolute term is one stating the relations of the object in itself.

(*j*1) The CONNOTATION of a word is the special meaning

which it conveys to the mind of each person in addition to the common definition.

(*j*2) Connotation is the significance of a term as distinguished from others.

(*j*3) The connotation of a term are the attributes belonging to it.

(*j*4) Connotation is the summing up of all the attributes of the term to be defined.

(*j*5) Connotation is the consideration of attributes.

(*j*6) The connotation of a name is expressed by the attributes by virtue of which the name is applied.

(*j*7) The connotation of a class is the name applied to the common attributes of the members of the class, by virtue of which attributes the members belong to the class.

(*j*8) Connotation is the relation of the common attributes to the general term.

(*j*9) The connotation of a name is the concept of the object designated by that name.

(*j*10) Connotation is the enumeration of attributes which an object must have in order to be known by the general name.

(*j*11) The connotation of a term is the attribute or attributes which are applied to the object about which you are speaking.

3. Give the exact equivalent of the following in concrete terms:

(*a*) Charity never faileth.

(*b*) Life is real, life is earnest.

(*c*) Whether we realize it or not, our lives always have a moral purpose.

(*d*) But flesh with the life thereof, which is the blood thereof, shall ye not eat. (Gen. ix. 4.)

(*e*) He stretcheth out the north over empty space, and hangeth the earth upon nothing. (Job xxvi. 7.)

(*f*) Our demand is for justice to silver!

(*g*) By the passes which he made he drew the pain from the patient's forehead out into his own hands.

(*h*) The energy released by this process soon travelled from A to B and put it in motion, or rather, put motion into it.

(*i*) Labor is oppressed by the money power.

4. Rewrite these paragraphs, so as to preserve the sense but get rid of as many of the abstract terms as you can. In the case of the last two be careful to interpret the words as the

author meant them to be interpreted, not as they are usually interpreted nowadays.

(*a*) "What . . . recommends the geometrical method to Spinoza is, not only its apparent exactness and the necessity of its sequence, but, so to speak, its disinterestedness." (Ed. Caird.)

(*b*) "Men generally confound this distinction, and regard all their settled opinions or beliefs as knowledge. This is not merely false, but absurd. . . . And this is apparent also from the nature and generation of our opinions. For, in general, these come to us not from any conscious process, but naturally and spontaneously from many sources, as, *e.g.*, from testimony, from authority, from inaccurate observation or careless reasoning, and even largely from mere prejudice or bias. Hence, familiar to us as our opinions are, their origin in general is as unknown to us as were anciently the sources of the Nile; nor have we any just notion of the grounds on which they rest, or of the nature and justice of their demands on our belief. Hence, until some means of verifying our opinions be found and applied, we can have no assurance of their rectitude. The first step in science or philosophy must, therefore, be to distinguish between verified and unverified opinions. The former constitutes true knowledge or science; the latter—though it is in fact the stuff out of which most of the current philosophy is woven—has no just pretension to the name." (G. H. Smith's "Logic".)

(*c*) "Notwithstanding, certain it is that if those schoolmen, to their great thirst of truth and unwearied travail of wit, had joined variety and universality of reading and contemplation, they had proved excellent lights, to the great advancement of all learning and knowledge; but as they are, they are great undertakers indeed, and fierce with dark keeping: but as in the inquiry of the divine truth, their pride inclined to leave the oracle of God's word, and to vanish in the mixture of their own inventions; so in the inquisition of nature, they ever left the oracle of God's works, and adored the deceiving and deformed images, which the unequal mirror of their own minds, or a few received authors or principles, did represent unto them.' Bacon, "Advancement of Learning", Bk. I.

(*d*) "There fall out to be these three distempers, as I may term them, of learning: the first, fantastical learning; the

second, contentious learning; and the last, delicate learning; vain imaginations, vain altercations, and vain affectations; and with the last I will begin. Martin Luther, conducted no doubt by a higher Providence, but in discourse of reason finding what a province he had undertaken against the Bishop of Rome and the degenerate traditions of the church, and finding his own solitude being no ways aided by the opinions of his own time, was enforced to awake all antiquity, and to call former times to his succors, to make a party against the present time. So that the ancient authors, both in divinity and in humanity, which had long time slept in libraries, began generally to be read and revolved. This by consequence did draw on a necessity of a more exquisite travail in the languages original, wherein those authors did write, for the better understanding of those authors, and the better advantage of pressing and applying their words. And thereof grew again a delight in their manner of style and phrase, and an admiration of that kind of writing, which was much furthered and precipitated by the enmity and opposition that the propounders of those primitive but seeming new opinions had against the schoolmen; who were generally of the contrary part, and whose writings were altogether in a differing style and form; taking liberty to coin and frame new terms of art to express their own sense, and to avoid circuit of speech, without regard to the pureness, pleasantness, and, as I may call it, lawfulness of the phrase or word." Bacon, loc. cit.

5. 'This cheap quack thought he could gull the public, so he put some dirty water in a bottle and then turned all his low cunning to the invention of a lying advertisement in which he claims that one bottle of his nasty stuff will cure a man of everything under the sun except his credulity.' When a person makes such a statement as this, what probably are the ascertained facts in the case?

6. Pick out the demonstrative and the descriptive words and phrases in the following:

(*a*) And this is my little daughter! But she is my big daughter now.

(*b*) The swift-footed Achilles is lame.

(*c*) Smith and Jones, manufacturers of Mrs. Kelly's homemade pickles.

(*d*) Many oriental antiques are now manufactured in Chicago.

7. What fallacy, if any, is to be found in the following?

(*a*) Books are a source both of instruction and amusement; a table of logarithms is a book; therefore it is a source both of instruction and amusement. (J.)

(*b*) Three and two are two numbers; five is three and two; therefore five is two numbers. (Ray.)

(*c*) I hate the English and I always shall; for was it not they who fought against us in the Revolutionary War and again in 1812?

(*d*) Unless his cold works out through the throat or nose, how can he hope to get rid of it?

(*e*) Do not let him learn all those useless facts; they will overload his mind.

(*f*) You say that A is taller than B and shorter than C; but that is absurd, for the same person cannot be both taller and shorter at the same time.

(*g*) It is all very well for you to talk in prayer-meeting about your badness, but when any one tells you outside that you are bad you resent it.

(*h*) The people of the country are suffering from famine; and as you are one of the people of the country you must be suffering from famine. (J.)

(*i*) 'You gave him all the information he had!' Then you were very foolish; for you had none too much for yourself.

(*j*) All the trees in the park make a thick shade; this poplar is one of them, and therefore it makes a thick shade. (J., altd.)

(*k*) It is true that in a democracy the people vote, but it is absurd to say that they rule; for you and I are people and everything we vote for is defeated.

(*l*) In a republic a majority of the people rules. That is what we mean by the word. Therefore it is a contradiction to say that the majority of the people did not rule in the South African Republic.

(*m*) 'You said that these cakes were hot, and when I bought them I found that they were frozen.' 'I never said they were hot; I simply called: Hot cakes!—that is the name of them.'

(*n*) The civilization of this people, like every other organism, was developed gradually from small beginnings and some day it is bound to decay.

(*o*) The ancient Greeks produced the greatest masterpieces of eloquence and philosophy; the Lacedæmonians were ancient Greeks; therefore they produced the greatest masterpieces of eloquence and philosophy. (J.)

(*p*) We shared his money and then we shared his sorrows; so that if we did not leave him very much of the former we did not leave him much of the latter either.

(*q*) He who believes himself to be always in the right in his opinion, lays claim to infallibility; you always believe yourself to be in the right in your opinion; therefore you lay claim to infallibility. (W.)

(*r*) The life is gone, and, since there are no vacant places in Nature, something has come to take its place, namely, death.

(*s*) No soldiers should be brought into the field who are not well qualified to perform their part; none but veterans are well qualified to perform their part; therefore none but veterans should be brought into the field. (W.)

(*t*) The accumulated experiences which we inherit from our ancestors have always shown that the wages of sin is death.

(*u*) You say that you did not understand that this move was contemplated; and yet it was recommended by the directors, and you are one of them.

(*v*) 'Love my neighbor!' Yes. But that is no reason why I should not get ahead of one of these soulless corporations if I can.

(*w*) How can I be religious when religion has done so much harm in the world? What makes the Mohammedans so intolerant? What makes the people of India so helpless? What made the great wars of history? What keeps Ireland and England always at strife? What made the Inquisition? What has opposed every fundamentally new scientific conception for centuries? Always religion!

(*x*) "No reason, however, can be given why the general happiness is desirable, except that each person, so far as he believes it to be attainable, desires his own happiness. This, however, being a fact, we have not only all the proof which the case admits of, but all which it is possible to require, that happiness is a good, that each person's happiness is a good to that person, and the general happiness, therefore, a good to the aggregate of all persons." (Mill's "Utilitarianism.")

(*y*) "Little Indian, Sioux or Crow,
Little frosty Eskimo,
Little Turk or Japanee,
O! don't you wish that you were me?

. . . . . .

You must often, as you trod,
Have wearied *not* to be abroad.
You have curious things to eat,
I am fed on proper meat.
You must dwell beyond the foam,
But I am safe and live at home."

(R. L. Stevenson.)

CHAPTER VI.

1. Take O. W. Holmes' "Last Leaf" or some other piece of poetry or prose and show the relations expressed by each word or phrase, thus:

I saw him once before
As he passed by the door,
And again
The pavement stones resound,
As he totters o'er the ground
With his cane.

'I'—individual identity.

'Saw'—noetic relation; also time; also perhaps a suggestion of causation, since I could not have seen him unless he had acted in some way upon me.

'As he passed by the door'—time; also causation if the phrase implies that his passing the door was what made me see him. 'Passed by the door'—motion; i.e., change of place, involving the relations of both time and place.

'The door'—individual identity, if the term is really intended to mean the movable thing that we use to close an aperture. In this case the word also suggests the complex relation of means and end if we stop to think about it. But in this context the term is more probably used to mean the open doorway. If this be so the relation expressed is mainly one of position in space, though the term and context also suggest the absence

of a causal relation,—he came into a certain position where nothing prevented me from seeing him.

2. Criticise the following definitions :

(*a*1) A REAL PROPOSITION tells the name of the object. It does not tell what state the object is in.

(*a*2) A real proposition asserts something of a term.

(*a*3) A real proposition is one in which there is identity, also description.

(*a*4) Real terms relate to things.

(*a*5) A real proposition is one which states a relation.

(*a*6) Real propositions state some additional fact about an object.

(*b*1) A VERBAL PROPOSITION is one which defines an object.

(*b*2) A verbal proposition is one which explains the meaning of nouns or the terms of the proposition.

(*b*3) A verbal proposition is one which defines an object.

(*b*4) A verbal proposition is the meaning of a name.

(*b*5) Verbal propositions define the object itself.

(*b*6) A verbal proposition is one which explains a part or the whole of some thing.

(*b*7) A verbal proposition explains the meaning of a word within the proposition.

(*b*8) Verbal terms relate to words.

(*b*9) A verbal proposition explains the term or some part of it.

3. Consider how far each of the five fundamental relations given in the text is independent of the others. Is it possible, for example, for anything to have individual identity without also being the subject of states, and *vice versa ;* can we know a thing unless it acts ; can a thing be said to have a state or quality unless some one knows it ? etc.

CHAPTER VIII.

1. What is the subject, what is the copula, and what is the predicate of each of the following propositions;* what is its quality and quantity; and if the predicate has any quantity what is it ? If the proposition is ambiguous or peculiar in any

* Taken for the most part from the collection in Jevons's "Studies in Deductive Logic", p. 38. (Macmillan, 1884.)

other way, point out the fact; and if the terms are abstract make them concrete.

(*a*) All Athenians are Greeks.

(*b*) They never pardon who have done the wrong.

(*c*) Great is Diana of the Ephesians.

(*d*) No abracadabras are gasteropods.

(*e*) All, all are gone, the old familiar faces.

(*f*) He that is not for us is against us.

(*g*) Men mostly hate those whom they have injured.

(*h*) Old age always involves decrepitude.

(*i*) Nothing morally wrong is politically right.

(*j*) It is not good for man to be alone.

(*k*) A certain man had two sons.

(*l*) There's something rotten in the state of Denmark.

(*m*) All cannot receive this saying.

(*n*) Few men are free from vanity.

(*o*) He that fights and runs away may live to fight another day.

(*p*) There is none good but one.

(*q*) Two straight lines cannot enclose a space.

(*r*) Familiarity breeds contempt.

(*s*) Only the ignorant affect to despise knowledge.

(*t*) All is not gold that glitters.

(*u*) Love is not love which alters when it alteration finds.

(*v*) A friend should bear a friend's infirmities.

2. 'Few books are at once learned and amusing.' This is treated in the passage quoted from Jevons on p. 98 as proposition O. Consider whether it might be regarded as equivalent to both O and I, or to an exclusive or exceptive proposition.

3. Give hypothetical and disjunctive equivalents for all the universal propositions in the first exercise on this chapter.

4. Are the following propositions disjunctive or hypothetical? Give their categorial equivalents; also their exclusive and exceptive equivalents.

(*a*) If you are a man you are mortal.

(*b*) Either you are a man or you are mortal.

(*c*) Either you are a man or you are not mortal.

(*d*) If you are not mortal you are a man.

(*e*) You will not go to heaven unless you are good.

(*f*) You will not go to hell if you are not bad.

5. When I say 'No men are infallible' how many men am I

talking about? When I say 'All men are not infallible' how many am I talking about?

6. Examine, and if necessary correct, the statement on p. 102 that 'Almost any Turk hates a Greek' is universal with reference to the Greeks.

7. Criticise the following definitions:

(*a*1) SINGULAR PROPOSITION denotes an individual as such.

(*a*2) A singular proposition is one which specifies a single one of several relations.

(*a*3) A singular proposition is one in which singular terms are used.

(*a*4) A singular proposition is one which deals with or treats of a singular term.

(*a*5) A singular proposition is one which makes a statement about some one of a class of objects not especially designated.

(*a*6) A singular proposition is applicable to certain designated members of a class as such.

(*a*7) A singular proposition is one that states something about certain designated members of a class.

(*a*8) A singular proposition is one that makes a statement about an individual object.

(*a*9) Singular proposition is one in which we find certain specified objects pointed out.

(*a*10) Singular proposition is one which states something about an individual and leaves no doubt as to whether it is referring or is meant to refer to a thing or is applicable or meant to apply to a thing.

(*a*11) A singular proposition is one which refers to certain specified individuals and leaves no doubt as to who is under discussion.

(*b*1) UNIVERSAL PROPOSITION is one in which the predicate is affirmed of the whole subject.

(*b*2) A universal proposition is one which distributes its subject but not its predicate.

(*b*3) Universal propositions are those which involve statements about all the objects included in the subject term.

(*b*4) Propositions are universal when they make statements about all the objects mentioned in the subject term.

(*b*5) A universal proposition is a statement which affirms or denies something about all the objects in the class to which the subject term belongs.

(*b*6) A universal proposition is one in which a statement is made of some particular object definitely specified.

(*b*7) Universal as applied to a proposition is the quality of embracing all of the objects belonging to the class specified in the subject of the proposition.

(*b*8) A universal proposition is one in which an affirmation is made concerning every object of the kind denoted by the subject term.

(*b*9) A proposition is universal when the statement made is true of every object indicated by the subject term.

(*b*10) Universal as applied to propositions refers to all the objects indicated by the subject term.

(*c*1) A PARTICULAR PROPOSITION is one which has for its subject a singular term.

(*c*2) A particular proposition is one whose predicate does not apply to all of the subject.

(*c*3) A particular proposition is one in which the predicate is stated of part of the subject.

(*c*4) A particular proposition is one in which the predicate is not affirmed or denied of all the subject.

(*c*5) A particular proposition is one which makes an assertion about an indefinite part of a subject.

(*c*6) A particular proposition is one in which the predicate affirms or denies only a part of an indefinitely designated group of objects.

(*c*7) A particular proposition is one in which a statement is made concerning a part of an indefinite number of objects.

(*c*8) A particular proposition is one that asserts something of an indefinite number of objects.

(*c*9) A particular proposition is one in which the predicate makes a statement about any part of the thing talked about.

(*c*10) A particular proposition is one which does not make a statement about all of a class of objects, but about some particular member of a class.

(*c*11) A particular proposition is a proposition which has a subject which does not include a whole class, but only some.

(*c*12) A particular proposition is one in which a statement is made regarding some portion of the class of objects designated by the subject of the proposition.

(*d*1) The QUANTITY of a proposition refers to the fact whether it is negative or affirmative.

(*d*2) Quantity is the character of a proposition which states whether it is singular, universal, or particular.

(*d*3) The quantity of a proposition is its form as to whether it affirms something about the whole of a subject or something about undesignated members; in other words, whether it is universal, particular, or singular.

(*d*4) Quantity is the affirmation or denial of a thing as universal or particular.

(*d*5) Quantity is that character of a proposition as distinguishing the universal from the particular.

(*d*6) The quantity of a proposition is that property of a proposition by which a proposition is considered as universal, referring to all of a class, singular, referring to some specified part of a class, or particular, referring to some unspecified part of a class.

(*d*7) Quantity has reference to the kind of propositions, whether they be universal or singular, as distinguished from other kinds.

(*d*8) Quantity is that characteristic of a proposition which marks it as universal, particular, singular, or indefinite.

(*d*9) Quantity is the character of a proposition which states something about all the members of a class, undesignated member or members, or a specified individual or member.

(*e*1) AN AFFIRMATIVE PROPOSITION is one in which the predicate affirms or asserts something about the subject.

(*e*2) An affirmative proposition is one where a certain relation is said to exist between the subject and predicate.

(*e*3) An affirmative proposition is one which expresses a certain agreement between the subject and predicate so that the qualities of the predicate belong to the subject.

(*e*4) An affirmative proposition is one which tells some fact about the object named in the subject.

(*e*5) An affirmative proposition is a statement of a fact and whose predicate refers to the name of the subject.

(*e*6) An affirmative proposition is one which makes a positive assertion.

(*e*7) An affirmative proposition is one which asserts the predicate as having qualities belonging to the object which the subject represents.

(*e*8) An affirmative proposition is one which affirms a certain relation or condition between two terms.

(*e*9) An affirmative proposition is one in which the predicate is asserted to belong to either the whole subject or a part of the subject.

(*e*10) An affirmative proposition is one in which the qualities of the predicate are asserted as belonging to the subject.

(*f*1) QUALITY has to do with whether a proposition is singular, particular, or universal.

(*f*2) Propositions are said to differ in quality when they are affirmative or negative.

(*f*3) Propositions differ in quality when one affirms and the other denies a statement.

(*f*4) The quality of a proposition indicates whether it is affirmative or negative.

(*f*5) The quality of a proposition is in regard to its nature as affirmative or negative.

(*f*6) The quality of a proposition is the relation between A E I O.

(*f*7) The quality of a proposition is the attribute or attributes of the subject term taken together.

(*f*8) The quality of a proposition is the character of a proposition as it affirms or denies.

(*f*9) Quality as applied to propositions is that attribute of the proposition which determines whether the proposition is affirmative or negative.

(*f*10) The quality of a proposition is its character as to the affirmation or negation of the statement made in the proposition.

(*f*11) Quality of a proposition is its form as to whether it is affirmative or negative.

(*g*1) AN EXCEPTIVE PROPOSITION affirms or denies something of an object mentioned.

(*g*2) In an exceptive proposition something is affirmed or denied of the object excluded.

(*g*3) An exceptive proposition refers to some unspecified objects.

(*g*4) An exceptive proposition is one in which something is affirmed or denied of some part of the subject specified.

(*g*5) Exceptive propositions are those which affirm the predicate of all the subject except certain well-defined cases to which the subject does not belong.

(*g*6) An exceptive proposition is one that states something about all specified members of a class except certain ones.

(*g*7) An exceptive proposition is one that states something about all but the unspecified member or members of a class.

(*g*8) Exceptive term as applied to propositions is used to mean that the predicate affirms the subject in all cases with one exception.

(*h*1) AN EXCLUSIVE PROPOSITION affirms or denies something of an object not mentioned.

(*h*2) An exclusive proposition is one in which something is told about the individuals specified in the subject term.

(*h*3) An exclusive proposition is one which tells something about a designated part of the subject term.

(*h*4) An exclusive proposition is one which denies the truth or falsity of a part of the object spoken of.

*h*5) An exclusive proposition states something about certain specified members of a class.

(*h*6) An exclusive proposition is one which states something only about a specified member of a class.

(*h*7) An exclusive proposition uses the words "only", "none but" in asserting the predicate of the subject.

(*h*8) An exclusive proposition is the statement of one of a class to the exclusion of all other members of that class.

8. Criticise the following arguments:

(*a*) Every lover of truth is an admirer of Huxley; every bishop in the established church is not; therefore the bishops of the established church are not lovers of truth.

(*b*) Those who have sacrificed their health and fortune for a principle deserve the gratitude of mankind; few men have done this; therefore few men deserve the gratitude of mankind.

(*c*) All poets are not imaginative; some philosophers are poets; therefore some philosophers are not imaginative. (Ray.)

(*d*) The Cretans are liars; A, B, C are Cretans; therefore A, B, C are liars. (Hamilton.)

(*e*) All that glitters is not gold; tinsel glitters; therefore it is not gold. (W.)

(*f*) Warm countries alone produce wines; Spain is a warm country; therefore Spain produces wines. (W.)

(*g*) Revenge, Robbery, Adultery, Infanticide, etc., have been countenanced by public opinion in several countries; all the crimes we know of are Revenge, Robbery, Adultry, Infanticide, etc.; therefore all the crimes we know of have been countenanced by public opinion in several countries. (W.)

(*h*) The learned are pedants; A is a learned man; therefore A is a pedant. (Ray, quoted.)

(*i*) Testimony is a kind of evidence which is very likely to be false; the evidence on which most men believe that there are pyramids in Egypt is testimony; therefore the evidence on which most men believe that there are pyramids in Egypt is very likely to be false. (W.)

(*j*) All the miracles of Jesus would fill more books than the world could contain; the things related by the Evangelists are the miracles of Jesus; therefore the things related by the Evangelists would fill more books than the world could contain. (W.)

(*k*) No trifling business will enrich those engaged in it; a mining speculation is no trifling business; therefore a mining speculation will enrich those engaged in it. (W.)

(*l*) Every Turk hates a Greek; Dr. Constantinides is a Greek; therefore every Turk hates Dr. Constantinides.

(*m*) The English and Germans are quarrelling; I am English and you are German; therefore you and I are quarrelling.

CHAPTER IX.

1*a*. Assume the truth of each in turn of the propositions A, E, I, O, Singular Affirmative and Singular Negative, and inquire what follows in each case about the truth or falsity of the others.

1*b*. Assume the falsity of each in turn of these propositions and inquire what follows about the truth or falsity of the others.

2. If you wish to put an opponent in the wrong, what kind of statements should you try to get him to make, and what kind of statements should you yourself try to avoid making?

3*a*. 'None but the strong survive.' Resolve this into two ordinary propositions.

3*b*. Make a list of all the propositions with which it is inconsistent, and from these pick out its contradictory.

4. Find the contradictory or contradictories of the following: 'If a person is a good citizen, he will not smuggle', 'Heavy objects do not necessarily fall when they are thrown into the air.'

5. Estimate the truth of the following statements and correct them if they are wrong.

(*a*) If one proposition is true, then any other proposition from which it follows is true also, and if the proposition in question is false, then the other from which it follows is false also.

(*b*) If one state of affairs exists, then any other state of affairs which its existence involves exists also, and if the first state of affairs does not exist, neither does the other.

(*c*) If the truth of A involves the truth of B, and the falsity of B involves the falsity of A, then the falsity of A must also involve the falsity of B, and the truth of B the truth of A.

6. What in each case is the smallest amount of information necessary to disprove the following?—

(*a*) Great men are always large of stature.

(*b*) Lazy people are never useful.

(*c*) A certain celebrated statesman died in a lunatic asylum.

(*d*) Some dogs are afraid of ghosts.

7. Criticise the following definitions:

(*a*1) OPPOSITION of propositions occurs when one is false and the other true.

(*a*2) Opposition of propositions is the name applied to the process by which, taking a given proposition as true, we determine the truth, falsity, or doubtfulness of the other propositions.

(*b*1) The term 'CONTRADICTORY' as applied to propositions means that in one proposition a statement cannot be both affirmed or denied in regard to a certain object.

(*b*2) A contradictory proposition is one in which the absence of something is affirmed or denied whose presence was denied or affirmed in the original proposition.

(*b*3) Contradictory propositions are those which differ either in quantity or in quality or in both.

(*b*4) Term contradictory as applied to propositions. A negative particular proposition is the contradictory of a positive general.

(*b*5) A contradictory proposition is one which affirms or denies the falsity or truth of the proposition with which it is compared.

(*b*6) Contradictory propositions are those which differ in both quantity and quality.

(*b*7) Two propositions are contradictory when they make statements opposite in meaning.

(*b*8) Contradictory propositions are two propositions which cannot both be true at the same time.

(*b*9) A contradictory proposition is one in which one statement is true and the other false.

(*b*10) Statements are said to be contradictory when one affirms and the other denies in any way a certain thing said about an object.

(*b*11) When we have one proposition which is universal affirmative and one which is particular negative, these two propositions are contradictory. Or if we have a universal negative and a particular affirmative, they are contradictory. That is, when we have two propositions such that when one is true the other must be false, they are called contradictory.

(*c*1) CONTRARY propositions are those which directly refute each other. If one is true, the other is false.

(*c*2) Two propositions are contrary when one is in the universal affirmative form and the other in particular negative form.

(*c*3) Contrary propositions are those which assert the utmost variety of circumstances.

(*c*4) Contrary propositions are those in which there is a contrary relation expressed, as, 'Wretched people are unfortunate' is the contrary proposition of 'Happy people are fortunate'.

(*c*5) Contrary propositions are those which deny a state of things asserted by a previous proposition.

(*c*6) Contrary propositions are those in which what is affirmed of all the subject or some of the subject in one proposition is denied of all the subject or some of the subject in the other proposition.

(*c*7) A universal negative and a universal affirmative are called contrary propositions; one is false and one is true.

(*c*8) Contrary propositions are propositions which are directly opposite in quality.

CHAPTER X.

1. Explain the following so as to make clear in each case what relation (cause, motive, premise, law) is indicated by the italicized words:

(*a*) *Why* did you go to London? (W.)

(*b*) *Why* is this prisoner guilty? (W.)

(*c*) *Why* does a stone fall to the earth? (W.)

(*d*) *Why* do the germs of diphtheria do so much harm?

(*e*) This ground is rich *because* the trees on it are flourishing. (W.)

(*f*) The trees flourish *because* the ground is rich. (W.)

(*g*) "He that is of God heareth God's words: ye *therefore* hear them not, *because* ye are not of God." John viii. 47.

(*h*) "Why do ye not understand my speech? Even *because* ye cannot hear my word." John viii. 43.

(*i*) "And *because* I tell you the truth, ye believe me not." John viii. 45.

(*j*) "And no man laid hands on him; *for* his hour was not yet come." John viii. 20.

(*k*) "And ye will not come to me *that* ye might have life." John v. 40.

(*l*) "How can ye believe, *which* receive honour one of another, and seek not the honour which cometh from God only?" John v. 44.

(*m*) Two cerebral centres once connected tend to keep that connection, *since* we tend to have the same thoughts again that we have had before.

2. Obvert the following:

(*a*) This writer is inconsistent.

(*b*) The authors of these books are specialists.

(*c*) Good for this date only.

(*d*) We have met the enemy and they are ours.

(*e*) I was not alone.

(*f*) This is not the worst filter in the market.

(*g*) It was not a bad dinner.

(*h*) It is not unreasonable to suppose that a man with all his experience and all his success would be abundantly able to perform so simple a task.

(*i*) The luxuriousness of the trees is not the cause of the soil's fertility.

(*j*) This man is my brother.

(*k*) We climbed the mountain.

3. Criticise the following definitions:

(*a*) OBVERSION is the affirming or denying of a statement which has previously been denied or affirmed.

(*b*) An obverse proposition is one which denies or affirms what has been previously affirmed or denied and *vice versa*.

(*c*) Obversion consists in affirming or denying the absence of

qualities whose presence has been affirmed or denied in the original proposition.

(*d*) In obversion the same subject is kept, but that is affirmed or denied of the opposite of that which was denied or affirmed in the previous proposition.

(*e*) Obversion is the moving of the sign of negation from the copula to the predicate or from the predicate to the copula.

(*f*) Obversion is making a proposition include all it excluded before and *vice versa*.

CHAPTER XI.

1. Apply the traditional conversion to all the propositions in the first exercise to Chapter VIII.

2. Obvert each of the following, convert what you get, and keep on alternately obverting and converting until you cannot go any further. Do the same again, beginning with conversion.

(*a*) All good children are happy.

(*b*) No abracadabras are hypotheses.

(*c*) "Who noble ends by noble means obtains
Or, failing, smiles in exile or in chains, . . .
That man is blest indeed."

(*d*) He is not happy who is dead.

3. If it is not true that John is no heavier than James, what can we say about James, things heavier than James, things not heavier than James, things heavier than John, things not heavier than John, and the relation 'heavier'?

4. What is the relation of opposition, conversion, and obversion between each two of the following propositions?—

(*a*) All the trout are in this pond.

(*b*) The fish in the other pond are not trout.

(*c*) Some of the fish in this pond are not trout.

5. Give (*a*) the converse of the obverse, and (*b*) the obverse of the converse of this proposition: Every pig has a cloven hoof. Then apply conversion by negation to the resulting propositions.

6. Criticise the following definitions:

(*a*) CONVERSION is telling something in a new proposition about objects predicated in another proposition.

(*b*) The conversion of a proposition is when the subject and predicate change places.

(*c*) The converse of a proposition is one in which the predicate is taken for the subject and the subject for the predicate.

(*d*) Conversion is saying something in a new proposition about the objects that were or might have been indicated by the predicate of the original proposition.

7. Criticise the following arguments:

(*a*) None but the industrious deserve to succeed; I am industrious; therefore I deserve to succeed.

(*b*) None but the industrious deserve to succeed; I deserve to succeed; therefore I am industrious.

(*c*) The express train alone does not stop at this station; and as the last train did not stop it must have been the express train. (J.)

(*d*) When we hear that all the righteous people are happy, it is hard to avoid exclaiming, What! are all the unhappy persons we see to be thought unrighteous? (J.)

(*e*) All equilateral triangles are equiangular, and consequently all equiangular triangles must be equilateral. (F.)

CHAPTER XII.

1. Estimate the correctness of the following statements:

(*a*) No inference can be drawn unless two objects are compared with a third.

(*b*) No inference can be drawn if any of the relations in question are heterogeneous.

2. Criticise the following definitions:

(*a*) MEDIATE inference is inference drawn from more than two statements.

(*b*) Mediate inference is an inference which requires a middle term.

(*c*) Mediate inference is inference from a set state of affairs—under specific conditions.

(*d*) In mediate inference the mind has more than one act to perform. It arrives at the conclusion by means of other statements than just the one in regard to which the inference is made. Mediate inference is used chiefly in syllogisms.

(*e*) Mediate inference is reasoning from one premise to another by means of an intervening constructive process.

(*f*) Mediate inference is that form of reasoning which re-

quires some intermediate statements before the conclusion may be drawn. These intermediate processes are called premises.

(*g*) Mediate inference is the total of a series of conclusions considered according to specifications, part of which are considered in the major premise and part in the minor.

(*h*) Mediate inference is inference drawn from more than one premise.

CHAPTERS XIII—XVI.

1. Arrange the propositions in the following syllogisms so as to have the major premise first, the minor premise second, and the conclusion last; tell the quantity and quality of each proposition; tell also the figure of each syllogism, whether it is valid, and what caution it breaks if it is not.

(*a*) All A's are B; all B's are C; therefore all A's are C.

(*b*) No A is C; for no A is B and all B's are C.

(*c*) Some A's are B and some B's are C; therefore some A's are C.

(*d*) All A's are B and some B's are C; therefore some A's are C.

(*e*) No B is A; all C's are B; therefore some C is not A.

(*f*) All B's are C; all A's are C; therefore all A's are B.

(*g*) This cannot be S's boy Jack; for I saw Jack ten minutes after he was born and his body was covered with dark hair, and this child's is not.

(*h*) Every A is C and some B's are not C; therefore some A's are not B.

(*i*) It is not true that a man cannot do a great work without a strong physique; for the philosopher Kant did a great work and his physique was anything but strong.

(*j*) B has the kindest of feelings towards everybody; therefore the man you saw in a violent altercation with his wife could not have been he.

(*k*) There must be some invariable connection between red hair and a hasty temper, for every red-haired person I know is quick-tempered.

(*l*) All B's are C; no A is C; therefore no A is B.

(*m*) A and B each examined all the specimens he could find, and A noticed that in every case a certain feather in the tail was bent or broken, while B noticed that in nearly every case

there was a scar near one of the eyes. Whatever the cause, it is evident that these two strange peculiarities often coincide.

2*a*. What caution or cautions are embodied in the following proverbs :

(1) All that glitters is not gold.

(2) One swallow doesn't make a summer.

2*b*. Find or make corresponding proverbs for as many of the other cautions as you can.

3. A is B and B is C, therefore A is C. Reasoning on the same principle, may we not say : Polytheism is superstition, and superstition is everywhere, therefore polytheism is everywhere ?

4. What fault, if any, is to be found with the following arguments ? Show how they can be tested by Euler's diagrams.

(*a*) Some men are heroes ; J. S. is a man ; therefore J. S. is a hero.

(*b*) All men are heroes ; J. S. is not a man ; therefore J. S. is not a hero.

(*c*) All men are heroes ; J. S. is a hero ; therefore J. S. is a man.

(*d*) He must come from B. ; for every one that comes from there has that same way of pronouncing his r's.

(*e*) Some men are wise, and some men are good ; therefore some wise men are good.

(*f*) No man is perfect ; this goat is no man ; therefore this goat is perfect.

(*g*) No men are thoroughly unselfish ; some women are ; therefore some men at least are not women.

(*h*) All men are animals and no animals are myths ; therefore no men are myths, and Homer really lived.

(*i*) Few towns in the United Kingdom have more than 300,000 inhabitants : and as all such towns ought to be represented by three members of Parliament, it is evident that few towns ought to have three representatives. (J.)

(*j*) Only animals are sentient beings ; fishes are animals ; therefore fishes are sentient beings. (Ray.)

5. Give all the conclusions that can be drawn from each of the following pairs of premises :

(*a*) Every A is B, and no B is C.

(*b*) Every A is B, and this C is not A.

(*c*) No A is B, and some C's are A.

(*d*) No A is B, but this C is B.

(*e*) Every A is B, but some C's are not B.

(*f*) Some B's are A and some B's are C.

(*g*) Some B's are A, but no B's are C.

6. Supply the missing premise in the following syllogisms:

(*a*) All A's are C; for all A's are B, and—

(*b*) Some A's are not C; for some A's are B, and—

(*c*) None but non-A's are non-C's; for all non-B's are non-A's, and—

(*d*) All A's are non-C's; for no A's are non-B's, and—

(*e*) Some A's are not non-C's; for no non-C is B, and—

7. Show that the 'rules of the syllogism' given in the footnote to page 177 can be inferred from the 'cautions' given in preceding chapters.

8. Make a set of arguments each one of which breaks a different caution, and show how each of them also breaks the 'rules of the syllogism'.

9. Is it possible for both major and minor terms to be particular at the same time in the premises? If so, construct an argument where this is the case. (C.)

10. Reduce the following argument in the fourth figure to each in turn of the other three:

Some B is A·
No C is B;
Therefore some A is not C.

CHAPTER XVII.

1. Criticise the following definitions:

(*a*1) An ENTHYMEME is an imperfect syllogism.

(*a*2) An enthymeme is an incomplete syllogism.

(*a*3) An enthymeme is a syllogism some one or two of whose premises or conclusion are imperfectly expressed.

(*a*4) An enthymeme is a misstated syllogism; that is, one in which one of the premises is wanting.

(*b*1) A DILEMMA consists in assuming two assertions, but proves something in either case.

(*b*2) A dilemma is a disjunctive proposition in which there are two alternatives.

(*b*3) A dilemma is when there is no escape from two propositions. If one is false, the other must be true, and vice versa; there can be no half-way ground.

(*b*4) A dilemma is a combination of the disjunctive and hypothetical, such that no true conclusion can be drawn.

(*c*1) By SORITES is meant a syllogism composed of the three propositions O, I, E.

(*c*2) Sorites contains a series of syllogisms imperfectly expressed.

(*c*3) Sorites is a compound syllogism and may be decomposed into single syllogisms.

(*d*1) An EPICHEIREMA is a syllogism one of whose premises proves or gives a reason for another premise.

(*d*2) An epicheirema is a syllogistic statement so framed as to contain the conclusion of one part as the prosyllogism of another, and so on.

(*d*3) Epicheirema is a syllogism furnishing reason for premise of another syllogism, or a syllogism one of whose premises is the conclusion of another.

(*d*4) Epicheirema is the name given to the process of proving a statement by a prosyllogism.

(*d*5) An epicheirema is a statement which gives a reason for that statement.

(*d*6) An epicheirema is a syllogism that depends upon an incomplete episyllogism.

(*d*7) Epicheirema is a syllogism in which the propositions are not all present.

(*d*8) Epicheirema is a syllogism that gets its reason for a proposition from the syllogism before it.

(*e*1) A DESTRUCTIVE DILEMMA is a disjunctive proposition in which either horn of the dilemma is not true.

(*e*2) A destructive dilemma is a syllogism in hypothetical form, only with two different antecedents in the major premise for the same consequent. It is called destructive because the consequent is denied.

(*e*3) A destructive dilemma is the one known as the modus ponendo tollens, where by affirming the consequent is denied.

(*e*4) A destructive dilemma is a syllogism in which the major term is a disjunctive proposition and the minor term is a categorical proposition; from which the conclusion is derived by modus ponendo tollens.

(*e*5) A destructive dilemma consists of two syllogisms which break the validity of each other.

(*e*6) A destructive dilemma is a complex dilemma and consists of two alternatives called horns, both of which are denied in the course of the argument.

(*e*7) A destructive dilemma consists of a complex dilemma, that is, a syllogism in which the major premise consists of a hypothetical proposition with two or more antecedents, and a minor premise consisting of disjunctive proposition.

2. Give two examples each of a constructive hypothetical syllogism, a destructive hypothetical syllogism, a disjunctive syllogism, a simple constructive dilemma, a complex constructive dilemma, a destructive dilemma, an epicheirema, an enthymeme, and a sorites. Make sure that each of these is valid.

3. 'If they do not work, they go hungry.' Assuming this to be true, what happens when one of the following statements is also true?—

(*a*) They do not work;

(*b*) They do not go hungry;

(*c*) They work;

(*d*) They go hungry.

(4) Reduce the following arguments to a recognized form and determine whether they are valid or not:

(*a*) All D's are E, all C's are D, all B's are C, and no A's are B; therefore no E is A.

(*b*) All J's are L, all C's are non-V, all J's are W, all L's are C; and therefore some W's are not V's.

(*c*) Some E's are A; for all non-E's are non-D, no C's are non-D, all B's are C, and all non-B's are non-A's.

5. With each of the following tell the kind of argument. Is it valid? If not, what caution is broken or what fallacy is committed? Reduce the argument to categorical form if it is capable of such reduction; and also if it is hypothetical reduce it to disjunctive form, and *vice versa*. Show what fallacy the argument commits in these new forms.

(*a*) If his conscience disapproved of the act, it was certainly wrong for him to do it, but his conscience did not disapprove, therefore it was not wrong for him to do it.

(*b*) No honest man can advocate a change in the creed of his church; for he must either believe it or not believe it, and if he believes it he cannot honestly help to change it, while if he

does not believe it he cannot honestly belong to the church at all.

(*c*) Protection from punishment is plainly due to the innocent; therefore, as you maintain that this person ought not to be punished, it appears that you are convinced of his innocence. (W.)

(*d*) If men are not likely to be influenced in the performance of a known duty by taking an oath to perform it, the oaths commonly administered are superfluous; if they are likely to be so influenced, every one should be made to take an oath to behave rightly throughout his life. But one or other of these must be the case; therefore either the oaths commonly administered are superfluous, or every man should be made to take an oath to behave rightly throughout his life. (W.)

(*e*) The child of Themistocles governed his mother; she governed her husband; he governed Athens; Athens, Greece; and Greece, the world: therefore the child of Themistocles governed the world. (W.)

(*f*) It is worth while to teach the elements of Latin in the public schools if this gives the minds of the pupils a good general training, or if it gives most of the pupils a useful insight into Roman life and literature, or if most of them will continue the study in a college course. But none of these alternatives is true; therefore it is not worth while to teach the elements of Latin in the public schools.

(*g*) If every ghost story is to be believed, we must accept the general standpoint of the 'spiritualists'; but we cannot accept their general standpoint; therefore we cannot believe ghost stories.

(*h*) If transportation is not felt as a severe punishment, it is in itself ill suited to the prevention of crime; if it is so felt, much of its severity is wasted, from its taking place at too great a distance to affect the feelings, or even come to the knowledge, of most of those whom it is designed to deter; but one or other of these must be the case: therefore transportation is not calculated to answer the purpose of preventing crime. (W.)

(*i*) If virtue is voluntary, vice is voluntary; virtue is voluntary; therefore so is vice. (Aristotle.)

CHAPTER XVIII.

Criticise the following definitions:

(*a*1) A FALLACY is a statement which is not true.

(*a*2) When wrong meaning is given to a statement it is called a fallacy.

(*a*3) Fallacy is a mistake in term.

(*a*4) A fallacy is a false conclusion drawn from two premises.

(*a*5) A fallacy is the incorrect use of the laws governing thought.

(*a*6) A fallacy is an untruth—a flaw. If you detect a flaw in a proposition, you have found a fallacy in the proposition.

(*a*7) If a method of reasoning is so used as to make statements or conclusions contradictory, a fallacy is made.

(*a*8) A fallacy is a syllogism in which an incorrect conclusion is drawn.

(*a*9) A fallacy is a false judgment.

(*b*1) A MATERIAL FALLACY is a fallacy which arises from the use of more than three terms, more than three propositions, or ambiguous words, or an improperly stated syllogism.

(*b*2) A material fallacy is a fallacy in the word or words of a proposition.

(*b*3) A material fallacy is one in which there is a breach in the subject-matter.

CHAPTER XIX.

1. Criticise the following definitions:

(*a*1) PETITIO PRINCIPII is the fallacy of begging the question.

(*a*2) The fallacy of petitio principii is using one of the premises in the conclusion.

(*b*2) IGNORATIO ELENCHI is a fallacy of imperfect reasoning; that is, what follows is not true.

(*b*2) Ignoratio elenchi is a fallacy in which the premises and conclusion have no distinct connection.

(*b*3) Ignoratio elenchi is the fallacy committed when a conclusion is drawn which may have some bearing upon what has been said in the premises, but which does not logically follow.

2. Consider whether the assumption of a false premise may be fairly regarded as a case of begging the question.

3. Make or find two examples besides those given in the text of each of the fallacies there explained.

4. Criticise the following arguments or implied arguments:

(*a*) Written examinations are not an absolutely fair test of a student's scholarship—much less of his industry and intelligence. It is therefore wrong to base his grade upon them.

(*b*) Municipal ownership of street-cars would be beneficial to the people, and what is beneficial to the people should be put into practice; therefore the street-cars under private ownership should not be patronized.

(*c*) It is wrong to take the life of a fellow man, for God has distinctly commanded us not to, and it is wicked to disobey his commandments. If any one pretends to doubt that this commandment really did come from God, I can only appeal to his own conscience and his own common sense. When God gave the commandments to his people is it likely that he would have omitted the most important of them all—a commandment which only expresses the natural feeling of every normal human being?

(*d*) "America has still a long vista of years stretching before her in which she will enjoy conditions far more auspicious than any European country can count upon. And that America marks the highest level, not only of material well-being, but of intelligence and happiness, which the race has yet attained, will be the judgment of those who look not at the favored few for whose benefit the world seems hitherto to have framed its institutions, but at the whole body of the people." (James Bryce.)

(*e*) "It no doubt wounds the vanity of a philosopher who is just ready with a new solution of the universe to be told to mind his own business [laissez faire]. So he goes on to tell us that if we think that we shall, by being let alone, attain to perfect happiness on earth, we are mistaken. The half-way men—the professorial socialists—join him. They solemnly shake their heads, and tell us that he is right—that letting us alone will never secure us perfect happiness." (W. G. Sumner.) What fallacy do they commit?

(*f*) 'Every particle of matter gravitates equally.' 'Why?' 'Because those bodies which contain more particles ever gravitate more strongly, *i.e.*, are heavier.' 'But', it may be urged, 'those which are heaviest are not always more bulky'. 'No, but

still they contain more particles, though more closely condensed.' 'How do you know that?' 'Because they are heavier.' 'How does that prove it?' 'Because, all particles of matter gravitating equally, that mass which is specifically the heavier must needs have the more of them in the same space.' (W.)

(*g*) Those are your arguments against the course of conduct I propose; and yet the fact remains that when you were in my position you did the very thing that you are now advising me not to do.

(*h*) Everybody ought to contribute something to the support of the unfortunate; therefore there is no harm in a law which compels him to do so.

(*i*) "Again, we are not inclined to ascribe much practical value to that analysis of the inductive method which Bacon has given in the second book of the Novum Organum. It is, indeed, an elaborate and correct analysis. But it is an analysis of that which we are all doing from morning to night, and which we continue to do even in our dreams." (Macaulay.)

5. Examine the arguments or implied arguments in the following passages from Trumbull's "Lie never justifiable". The first of them may be regarded as an introduction to the history of the discussion.

(*a*) "Because of the obvious gain in lying in times of extremity, and because of the manifest peril or cost of truth-telling in an emergency, attempts have been made, by interested or prejudiced persons, all along the ages, to reconcile the general duty of adhering to an absolute standard of right, with the special inducements, or temptations, to depart from that standard for the time being." (p. 81.)

(*b*) "All the refinements of casuistry have their value to those who admit that a lie may be right under certain conceivable circumstances; but to those who, like Augustine and Aquinas, insist that a lie is a sin *per se*, and therefore never admissible, casuistry itself has no interest as a means of showing when a sin is not sinful." (p. 114.)

(*c*) "When he attempts the definition of a lie, however, Jeremy Taylor would seem to claim that injustice to others and an evil motive are of its very essence, and that, if these be lacking, a lie is not a lie." (p. 117.)

(*d*) "As to falsifying to a sick or dying man, he [Dorner]

says 'we overestimate the value of human life, and, besides, in a measure usurp the place of Providence, when we believe we may save it by committing sin.'" (p. 132.)

(*e*) "A lie being a sin *per se*, no price paid for it . . . would make it other than a sin. . . . It was a heathen maxim, 'Do right though the heavens fall,' and Christian ethics ought not to have a lower standard than that of the best heathen morality." (pp. 78, 79.)

(*f*) "God cannot justify or approve a lie. Hence it follows that he who deliberately lies in order to secure a gain to himself, or to one whom he loves, must by that very act leave the service of God, and put himself for the time being under the rule of the 'father of lies'." (p. 79.)

(*g*) "It is a physician's duty to conceal from a patient his sense of the grave dangers disclosed to his professional eye, and which he is endeavoring to meet successfully. And in well-nigh every case it is possible for him to give truthful answers that will conceal from the patient what he ought to conceal; for the best physicians do not know the future, and his professional guesses are not to be put forward as if they were assured certitudes." (p. 75.)

CHAPTER XX.

1. Criticise the following definitions:

(*a*) A fallacy of MANY QUESTIONS is one in which many irrelevant questions are asked which cannot be answered directly by Yes or No and which confuse the argument.

(*b*) The fallacy of many questions is the use of more data than are needed to prove the premises.

(*c*) A fallacy of many questions is the result of a false supposition.

(*d*) The fallacy of many questions consists in asking two or more questions at the same time so that it is impossible to give a correct answer.

(*e*) The fallacy of many questions is using so many questions in the argument that the conclusion does not stand out clearly.

(*f*) The fallacy of many questions consists in asking two or more questions in one.

2. What fallacies, if any, are committed or exposed in the following:

(*a*) 'We were put in this world to help others.' 'And what are others here for?'

(*b*) "If any one thinks that there are or ought to be in society guarantees that no man shall suffer hardship, let him understand that there can be no such guarantees, unless some other men give them." (W. G. Sumner, op. cit.)

(*c*) Old age is wiser than youth. We should therefore pay great deference to the convictions of the ancients.

(*d*) 'There never was an Irishman so poor that he did not have a still poorer Irishman living at his expense.'

(*e*) 'Habit is the cable which has grown so strong by use that its victim cannot depart from it.'

(*f*) Thought is nothing but a movement of the brain.

(*g*) "There is a beautiful notion afloat in our literature and in the minds of our people that men are born to certain 'natural rights.' . . . If there were such things as natural rights, the question would arise, Against whom are they good? Who has the corresponding obligation to satisfy these rights? There can be no rights against Nature, except to get out of her whatever we can, which is only the fact of the struggle for existence stated over again. The common assertion is that the rights are good against society; that is, that society is bound to obtain and secure them for the persons interested. Society, however, is only the persons interested plus some other persons; and as the persons interested have by the hypothesis failed to win the rights, we come to this, that natural rights are the claims which certain persons have by prerogative against some other persons. Such is the actual interpertation in practice of natural rights—claims which some people have by prerogative on other people." (W. G. Sumner, op. cit.)

(*h*) And if a spark has been kindled by the exercises of this day, let us water that spark.

(*i*) The sultan dreamed that all his teeth had fallen out, and summoned the soothsayers to tell him what it meant:

> "At last an old soothsayer, wrinkled and gray,
> Cried, 'Pardon, my lord, what I have to say;
>
> 'Tis an omen of sorrow sent from on high;
> Thou shalt see all thy kindred die.'
>
> Wroth was the sultan; he gnashed his teeth,
> And his very words seemed to hiss and seethe,

And he ordered the wise man bound with chains,
And gave him a hundred stripes for his pains.

The wise men shook as the sultan's eye
Swept round to see who next would try ;

But one of them, stepping before the throne,
Exclaimed in a loud and joyous tone :

' Exult, O head of a happy state !
Rejoice, O heir of a happy fate !

For this is the favor thou shalt win,
O Sultan, to outlive all thy kin.'

Pleased was the sultan and called a slave,
And a hundred crowns to the wise man gave."

(*j*) The claimant has undoubtedly many peculiarities of gait and manner which were characteristic of the missing baronet. Are not these therefore proofs of identity equivalent to the evidence of imposture afforded by the absence of tattoo-marks which the genuine man is proved to have possessed? (J.)

(*k*) Human powers are bounded only by the infinite. (D. altd.)

(*l*) Hoc unum scio, quod nihil scio.

(*m*) God separated the different races of men; then why should we encourage immigration that will mix them up again?

(*n*) "A lie is inconsistent with confidence; and the knowledge that a lie is, under certain circumstances, deemed proper by a man, throws doubt on all that the man says or does under any circumstances. No matter why or where the one opening for an allowable lie be made in the reservoir of public confidence, if it be made at all, the final emptying of that reservoir is merely a question of time." (Trumbull, p. 227.)

(*o*) According to Professor Sumner "the possession of capital is an indispensable prerequisite of educational, scientific, and moral goods". Now a man who possesses no moral goods is not a good man, and a man without educational goods is not educated. From all of which it follows that according to Professor Sumner the capitalists are the only people in the community who are either good or educated.

(*p*) A man who has money enough to obtain the best food, the best tools, and the best care when he is sick has a great

advantage over his rivals in the struggle for existence. But what is to the advantage of one man in this struggle is to the disadvantage of his competitors; and thus it happens that by the very fact of bringing life to the rich man the money which he possesses often brings death to his poorer rival.

(*q*) All the great financial crimes that we read about are committed by bankers and stock-brokers. Therefore the country would be a great deal better off if banks and stock exchanges were abolished.

(*r*) 'They tell us that we are weak, unable to cope with so formidable an adversary; but when shall we be stronger?'

(*s*) "When I have grown to man's estate
I shall be very proud and great,
And tell the other girls and boys
Not to meddle with *my* toys."
(R. L. Stevenson.)

(*t*) Whatever is universally believed must be true; the existence of God is not universally believed; therefore it is not true. (W.)

CHAPTERS XXI—XXV.

1. "A, B, C, D, and E are the only German students I know; they are all men of considerable intellectual attainments, and consequently I may infer that all German students are men of considerable intellectual attainments." (F.) Is this argument inductive or deductive? Is it valid? If so, how much confidence can be attached to the conclusion?

2. Why are we able to reach many conclusions by induction that cannot be reached by deduction?

3. 'I have no confidence in a Chinaman, for the Chinese government is one of the most corrupt in the world.' Estimate the value of such an inference.

4. Should we seek for the causes of *things*, or merely of their states and relations? Is there any objection to speaking of a thing as 'cause of itself'? (See definition in Spinoza's "Ethic".)

5. What fallacy is committed by a man who passes counterfeit money on the ground that the public has given it to him and the public ought to get it back again?

6. Is there any objection to the following explanation?—

'People get tired because everything they do involves some action on the part of the cells in their brain. These cells, like all other cells, are probably living creatures, and as such they are subject to the universal law of fatigue.'

7. "The analysis of a fact consists in the process of distinguishing *mentally* between its different details (the various episodes of an event, the characteristics of an institution), with the object of paying special attention to each detail in turn; that is what is called examining the different 'aspects' of a fact,—another metaphor." (Langlois-Seignobos.) In the light of this, consider the possibility of modifying statements in Chapter XXII in which the word 'aspect' is used.

8. Give some examples of explaining in a circle—'circulus in explicando.'

9. Give some examples of a 'Perfect Induction' and of an Inductio per Enumerationem Simplicem.

10 Give some examples of your own to show how we assume relations of identity and general laws in discovering relations of causation, and similarly with each of the other two.

11. How would you prove that the rotation of the earth is not due in large measure to the constant influence of the fixed stars? How would you prove, on the other hand, that the rotation is not due to the influence of some demon inhabiting it?

12. Give examples of your own showing how causal analysis gives definiteness of conception and greater reliability of inference.

CHAPTERS XXVI—XXXI.

1. With the following combinations of antecedents and consequents, find the cause of K. What is the method?—ACDEKLMN, ACEKMN.

2. What causal relation is suggested by the following combinations? Is it suggested only, or proved to exist? What is the method?—ABCFNOPQ, CDEGLMNT, CHIJNRS, BCENOT.

3. Find a set of causal relations that will account for the following combinations. What is the method?—ABCDNOPQ, BCDOPQ, EFRS, ABENR, AFDPQS, BEGRTU.

4. Do the same with this: ABCDUNQ, ABEFUNOPR, CDEFQTOPR, BEGNOS.

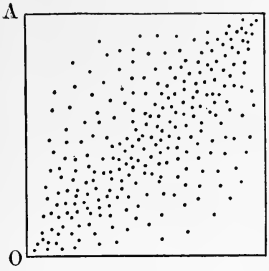
5. Do the same with this: ABCDZ, ACEPQRX, BCEQXZ, ACFPRVX, DEFQV, ADFPV.

6. Do the same with this: ABCPQST, ACDPSTO, CDESTOV, BDEPQOV, ADEPSOV, ACEPSTV.

7. Do the same with this: ABCPST, ABDPQS, ABEPS, ABFOPRS, ACEST, ACFORST, ADEQS, ADFOQRS, BCDPQT, BCEPRT, BCFOPRT, BDEPQR, BDFPQRO, CDEQRT, CDFQT, CEFORT, DEFOQR.

8. Do the same with this: ABCPR, ABDO, ABEP, ACDOPZ, ACEPZ, ADEOPZ, BCDOPRX, BCEPRX, BDEOPX, CDEOP, ABCDOPR, ABCEPR, ABDEOP, ACDEOPZ, BCDEOPRX.

9. What inference can be drawn from the following diagram? The distance of any dot above the line OB represents the proficiency of some one student in one kind of work, A; the distance from the line OA represents the proficiency of the same student in another kind of work, B.



10. What inference could have been drawn from the diagram if the dots had been found to group themselves about a diagonal running from A to B? If they had been grouped about a horizontal line? If they had been scattered irregularly over the whole square?

11. What is the significance of the italicized words in the following sentences?—*When* he came I left; He arrived *and then* there was trouble; The rain *followed* the lightning.

12. Examine the reasoning expressed in the following:

(*a*) "Depend upon it, the best antiseptic for decay is an active interest in human affairs: those live longest who live most."

(*b*) When this pond was first fished in the fish were very numerous and very easily caught; but now so many of them have been caught that the rest have grown very wary.

(*c*) Children are a good deal like apples—the first to ripen are usually the first to decay; and therefore a wise teacher will do what he can to delay the development of those that seem most precocious.

(*d*) We all drank the water and none of us got sick; so this outcry about the danger of typhoid is all nonsense.

(*e*) Any one who examines the records will soon find out for himself that those students who 'scatter' most in their choice of studies are those who accomplish least in any of them; and when he sees this he ought to realize the harm that can be done by a system of absolutely free electives.

(*f*) Trains run with Blank's oil have made the fastest time known amongst railway men.

(*g*) "A letter forged! Saint Jude to speed!
Did ever knight so foul a deed! . . .
Thanks to Saint Bothan, son of mine,
Save Gawain, ne'er could pen a line." (Scott.)

(*h*) 'The rain-maker is going to fire off his gunpowder to-morrow, but he can't fool us; we have a written contract that if there is no rain there is to be no pay.' Examine the same kind of reasoning with reference to patent medicines and 'tips' on investments: 'no cure no pay'—'no profits no pay.'

13. Give two examples of your own of each of the inductive methods explained in these chapters.

14. Give some examples of 'combined' and of 'compounded' causes.

15. With A as engineer and M as fireman a certain locomotive can average sixty-eight miles an hour; with A as engineer and N as fireman it averages sixty-three miles; with B as engineer and M as fireman it averages sixty; and with B as engineer and N as fireman it averages fifty-five. What part of the speed in each case would you credit to the locomotive, and what to each of the men?

16. The weight of boys and girls of different ages in the public schools of the United States is about as follows: Boys of six, 43½ lbs.; of seven, 47½; of eight, 52½; of nine, 57½; of ten, 62½; of eleven, 68½; of twelve, 73½; of thirteen, 80; of fourteen, 88; of fifteen, 100; of sixteen, 114. Girls of six, 42; of seven, 46; of eight, 50½; of nine, 55; of ten, 60½; of eleven, 65½; of twelve, 73; of thirteen, 83½; of fourteen, 94; of fifteen, 103; of sixteen, 110; of seventeen, 115½. From these data it is usually inferred that girls have a period of rapid growth between the ages of twelve and fourteen or fifteen, and that

boys have a similar period a couple of years later. Is there any other way of accounting for the figures?

17. "More men than women die every year. This is due to the greater mortality attending the life of the male" (Mayo-Smith, "Statistics and Sociology"). What does this mean? How can it be possible? 'In Germany 109 men die each year for every 100 women.' What can we conclude from this? Why is it better to know that in Germany 28.6 out of every thousand males die each year and 25.3 out of every thousand females? Since everybody must die sooner or later how is it possible that there should be in any country (or in the whole world) a permanently greater death-rate for one sex than for the other?

CHAPTERS XXXI—XXXII.

1. In many gymnasiums a prize is given for 'symmetrical development'. How should this be estimated? Is there any objection to giving the prize to the candidate whose proportions approach nearest to the average of all students measured?

2. In a study of great men it is found that for thirty-nine fathers and twenty-five mothers the average age of the parents at the time of the birth of the great man is 37.78 years for the former and 29.8 years for the latter. "Although there are few cases, the results are interesting, because they are in agreement with those of Galton, who finds the average age of the parents of one hundred English men of science to be thirty-six years for the fathers and thirty years for the mothers. . . . Both studies would seem to show that the child born of parents in the prime of physical life has the better chance of greatness. Both results conflict with Lombroso's theory, which he took from Mosso, that 'the number of men of genius and even of talent issued from aged fathers is very great.'" (A. H. Yoder in Ped. Sem. III, 137.) Can any fault be found with these conclusions?

3. A got a certain story from B, B from C, C from D, and D from E. In each case there was an even chance that the hearer 'got it wrong'. What are the chances that A got E's story exactly as he told it?

4. Is there any difference between saying that a certain horse is the most likely to win the race and saying that it is likely to win?

5. "What an author expresses is not always what he believed, for he may have lied; what he believed is not necessarily what happened, for he may have been mistaken. These propositions are obvious. And yet a first and natural impulse leads us to accept as true every statement contained in a document, which is equivalent to assuming that no author ever lied or was deceived." (Langlois-Seignobos.) Is there any objection to this statement?

6. Account for the saying that lightning never strikes twice in the same place.

7. Is there any objection to this reasoning?—'The event in question must have been either A, B, C, or D. Of these A is the most probable. Therefore the event in question was probably A.'

8. Scarlet fever is one of the most contagious of diseases, and yet an eminent authority says he 'can find no instance recorded where it has been transmitted through two healthy persons'. If we assume that such cases do not exist, what can we infer about the number of germs usually present in the vicinity of a patient and about the number necessary to transmit the disease?

9. What reply can be made to the following?—'You say that the prisoner is probably guilty. I grant it. But this only means that the prisoners in most cases of this sort are guilty. It does not mean that this particular prisoner has even a touch of guilt. Your very use of the word 'probable' is a confession that for all you know he may be absolutely innocent. How then can you ask the jury to condemn him to an awful fate?'

10. What reply can be made to the spiritualist who demands assent to his explanation of certain phenomena on the ground that it is the most probable of all the many explanations which have been offered? Can we deny his conclusion without denying the premises?

11. Assuming that the difference between several different measurements of the same quantity is due to a large number of different variables, each one of which affects each of the measurements to a slight degree, prove that the mean of all the measurements is far more likely to be nearly correct than either of the extremes.

CHAPTERS XXXIII—XXXVI.

1. How much inference is there in the following 'observations'?—

(*a*) A player catching a baseball at the far side of the field;

(*b*) The observer himself shooting a bird;

(*c*) The effect of a certain speech upon the hearer.

2. Give some instances in which you yourself have 'observed' or 'remembered' an event that did not take place.

3. Can you recall any cases in which you have made records for your own use and then been unable to find them or to interpret them? If so, what was wrong?

4. Recall any cases you may know about in which circumstantial evidence seemed convincing, though the conclusion to which it pointed was afterwards found to be wrong.

5. "Violenta presumptio is many times plena probatio; as if one be run thorow the bodie with a sword in a house, whereof he instantly dieth, and a man is seen to come out of that house with a bloody sword, and no other man was at that time in the house." (Lord Coke.) Explain exactly what this means, and then see how many alternative explanations you can find for the occurrence mentioned in illustration.

6. Estimate the value of the following arguments:

(*a*) There must be something in oracles; for Herodotus tells many tales that show the wonderful powers of the oracle at Delphi, and Herodotus was a careful and critical historian.

(*b*) It is fair to assume that he was guilty of at least some misconduct, for where there is smoke there is fire, and he has certainly succeeded in getting himself criticised by everybody.

(*c*) The doctrine of the divine right of kings was certainly current in the time of Elizabeth; for we read in Shakspere:

> "Not all the water in the rough rude sea
> Can wash the balm off from an anointed king."

(*d*)

> "To thine own self be true,
> And it must follow, as the night the day,
> Thou canst not then be false to any man."

These are the words of Shakspere, perhaps the greatest observer of human life that ever existed; and we should value them accordingly.

(*e*) You say it was not Homer who wrote about the travels of Odysseus. What of it? Somebody must have known about them, and what difference does it make whether we call him Homer or call him something else?

(*f*) We must not expect too much happiness in this world, for the Bible warns us that "man is born unto trouble, as the sparks fly upward."

7. What is the significance of the statement that the Iliad and Odyssey were not written by Homer, but by another man of the same name? What would be the significance of a similar statement about Moses and the Pentateuch?

8. From the "Life of Professor Huxley", the "Letters of J. R. Green", and any other sources of information that you can command, find out exactly what it was that Huxley said in his retort upon the Bishop already mentioned on page 11.

9. Estimate the following arguments:

(*a*) If you believe in the survival of the fittest, you must believe that this old manuscript was one of the best of its time, for it is the only one that has survived.

(*b*) This book is authentic; then why should we not believe what it says?

(*c*) This text of Cicero dates from the twelfth century and that only dates from the fourteenth, then why isn't this a better one to go by than that?

(*d*) This text has great value for the historian, for it was restored at infinite pains, and there is now every reason to believe that it is substantially correct.

(*e*) 'Come and have your fortune told by Blank's system of palmistry. No man of science has ever disputed the claims of this system.'

10. Herodotus tells of certain sailors who circumnavigated Africa and said on their return that as they went westward around the southern extremity of the land the sun was on their right. Herodotus said he did not believe this. What can we infer about Herodotus, and what can we infer about the alleged voyage?

11. During a thunder-storm the cook in a certain house rushed from the kitchen exclaiming that she had seen a ball of fire enter one window and go out of the other. What probably were the facts?

12. Two students who have never been suspected of dis-

honesty sit near each other at an examination and each of them writes these very words: "Henry George was the great orator of the Revolution; it was he who said in Faneuil Hall, 'Give me liberty or give me life.'" What inference can be drawn from this coincidence? How much should the inference be affected by the protests of the students that they were perfectly honest, or by their explanation that they had studied together?

13. Estimate the value of the following arguments:

(*a*) Vacillating and cruel as our treatment of the Indians has often been, there can be no doubt that the European conquest of America has been a good thing for the world. Certain it is that we have not a single eminent writer of history who doubts it.

(*b*) We must choose between A's account of this affair and B's. A's has never been seriously disputed; B's has been disputed for centuries. How then can we hesitate between them?

(*c*) I know that I might have made up my mind to act otherwise; for I was conscious at the time that there was nothing in me to prevent it.

(*d*) The great influence of the mind over the body is well illustrated by such facts as the following: People often dream that they are falling from some great height, and if the dream stops there it does them no harm; but if any one dreams that he strikes the ground, the mental shock is too great, and it kills him.

14. Pick out and summarize the bare, concrete, observable facts (real or imaginary) stated in the following paragraphs; point out any passages in which matters of fact and matters of opinion are confused; assuming the concrete facts to be as stated, show what they prove; show what must be assumed in order that the conclusion drawn should follow from the concrete facts; and estimate the reasonableness of such assumption or assumptions.

'A careful observer of the people of India says: More systematic, more determined liars than the people of the East cannot, in my opinion, be found in the world. Yet, strange to say, some of their works and sayings represent a falsehood as almost the unpardonable sin. Take the following example: "The sin of killing a Brahman is as great as that of killing a hundred cows; and the sin of killing a hundred cows is as great as that of killing a woman; and the sin of killing a hun-

dred women is as great as that of killing a child in the womb; and the sin of killing a hundred [children] in the womb is as great as that of telling a lie." '

'The duty of veracity is often more prominent among primitive peoples than among the more civilized. Among those Hill Tribes of India which have been most secluded, and which have retained the largest measure of primitive life and customs, fidelity to truth in speech and act is still the standard, and a lie is abhorrent to the normal instincts of the race. The Bheels, a race of unmitigated savages, are yet imbued with a sense of truth and honor strangely at contrast with their external character. The Sowrahs do not know how to tell a lie. The Arabs are more truthful in their primitive state than when they are influenced by "civilization". The word of a Hottentot is sacred.'

'It is found, in fact, that in all ages, the world over, primitive man's highest ideal conception of deity has been that of a God who could not tolerate a lie; and his loftiest standard of human action has included the readiness to refuse to tell a lie under any inducement, or in any peril, whether it be to a friend or to an enemy. This is the teaching of ethnic conceptions on the subject. The lie would seem to be a product of civilization, and an outgrowth of the spirit of trade and barter, rather than a natural impulse of primitive man.'

'It would seem to be clear that the best moral sense of mankind everywhere deems a lie incompatible with the idea of a holy God, and consistent only with the spirit of man's arch-enemy—the embodiment of all evil.' (Condensed from Trumbull: "A Lie Never Justifiable.")

15. "You say that development drives out the Creator; but you assert that God made you: and yet you know that you yourself were originally a little piece of matter, no bigger than the end of this gold pencil-case." (Huxley.) What kind of argument is this? Estimate its force.

16. "Nevertheless, I did not formerly consider sufficiently the existence of structures which, as far as we can at present judge, are neither beneficial nor injurious; and this I believe to be one of the greatest oversights as yet detected in my work. I may be permitted to say, as some excuse, that I had two distinct objects in view: firstly, to show that species had not been separately created, and, secondly, that natural selection had

been the chief agent of change, though largely aided by the inherited effects of habit, and slightly by the direct action of the surrounding conditions. I was not, however, able to annul the influence of my former belief, then almost universal, that each species had been purposely created; and this led to my tacit assumption that every detail of structure, excepting rudiments, was of some special, though unrecognized, service." (Darwin's "Descent of Man", part 1, chap. 2.) What tendency does this illustrate?

17. "The assertion of a fact wholly beyond the reach of evidence, for or against, is to be held as untrue." (Bain's "Logic", p. 382.) Discuss this.

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